Competition and the signaling role of prices*

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Abstract

In a market where sellers are endowed with heterogeneous qualities of the same good and are more informed than buyers, high quality sellers' chances to trade might depend on their ability to inform buyers about the quality of the goods they offer. We study under what conditions and to what extent sellers of high quality goods are able to overcome such a need for communication by means of pricing decisions in the context of a market populated by a large number of strategic price setting sellers and a large number of buyers. The unique robust equilibrium outcome - in terms of prices, quantities and qualities traded - depends upon market conditions. If demand is strong so that sellers face weak competitive pressure, sellers of high quality goods are able to use prices as a signaling device and this enables them to trade. However, not all sellers of high quality are ex-post able to sell. In particular, if demand is sufficiently strong, high quality sellers become rationed. If demand is weak, competition among sellers inhibits the role of prices as signals of high quality, and high quality sellers are driven out of the market.

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1 Introduction

A buyer interested in a specific digital camera could find out the list of retailers' price quotes at shopper.com in just a few seconds. For almost any model, such a list would invariably contain substantially dispersed prices.[1] Why would such price quotes not obey the law of one price? After all, if consumers have access to market prices at a negligible cost, one would expect homogeneous products to trade approximately at the same price.

The Internet allows consumers to observe prices of any specific camera at almost no cost. Whether the Internet is as informative about other relevant characteristics of the product they are interested in, is far less clear. For example, information about the quality of customer services such as delivery, assistance, and the like, is much less available than price quotes, and is rather opaque anyway. Can such an asymmetry of information be the source of the observed persistent price dispersion? Could such price dispersion be the result of sellers' attempts to signal quality through price decisions?

When sellers are more informed than buyers, the ability of sellers endowed with high quality goods to inform buyers about the quality of their goods could be crucial in keeping these sellers from being wiped out by price competition. We study under which conditions sellers are able to conduct such communication by means of pricing decisions in the context of a lemons market populated by many price setting sellers and many buyers. We find that the effectiveness of prices as a communication device is inversely related to the competitive pressure faced by the sellers. Correspondingly, the degree of price dispersion among sellers is inversely related to the competitive pressure. When competition is weak, that is the potential demand exceeds the potential supply of low quality goods, high quality sellers can credibly communicate their quality to the market and are able to sell their goods. In this case, the market features persistent price dispersion as different qualities of the good trade at different prices in the prevailing equilibrium. Conversely, when competition is strong, that is the potential supply of low quality goods exceeds potential demand, the only way to credibly communicate high quality is to announce prices that are too high to attract any buyer. In other words, prices are not an effective communication tool and sellers endowed with high quality goods will be driven out of the market. Only sellers of low quality goods are able to sell and the equilibrium features minimum price dispersion.

This conclusion is reached in a model where there are two qualities of the same good, and therefore two types of sellers. However, as shown in the appendix, the results carry through the more general case of a market with a generic number of qualities of the same good. The stronger the competitive pressure faced by the sellers, the lower the number of qualities that will be traded and the associated degree of price

[1] Various studies document the presence of persistent significant price dispersion in internet markets for final goods. See, for instance, Clay, Krishnan and Wolff (2001)for the case of electronic bookstores and Baye, Morgan and Sholten (2004) for the case of consumer electronic products.

dispersion.

Prices serve as a signal only when different types of sellers have different incentives to announce a particular price. How could this depend on the competitive pressure faced by sellers? When competition among sellers is strong, undercutting implies that, in equilibrium, sellers of low quality goods announce a price such that they make zero profits. Sellers of high quality goods would be willing to sell only at prices higher than that as long as reservation utility is increasing in quality. Yet, in an equilibrium in which low quality sellers make zero profits, no trade must occur at such higher prices. Therefore, high quality sellers must be unable to trade and also make zero profits. Since both types of sellers make zero profits, the incentives to announce any off-equilibrium price higher than high quality sellers' reservation utility are the same across sellers' types. Such prices do not serve as a signaling device to high quality sellers. These sellers are thus unable to communicate the quality of their good to buyers and are driven out of the market. As the equilibrium dominance literature suggests, differences in the level of equilibrium profits help to determine the informative content of deviations to prices that are not announced in equilibrium. When equilibrium profits are zero for both types, the informative content of price deviations is undetermined.

When competition among sellers is weak, buyers compete. This implies that, in equilibrium, low quality sellers make positive profits. For this reason, prices become an effective communication device for sellers of high quality goods. Consider, for instance, a candidate equilibrium in which only the low quality is traded. In this situation, sellers of high quality, who in equilibrium are out of the market, have an incentive to announce any off-equilibrium price greater than their reservation utility, whatever positive chance of selling at that price there is. Low quality sellers would announce such price if and only if the chances to sell were sufficiently high, since they would make strictly positive profits by announcing the equilibrium price. Therefore, there will be some off-equilibrium prices that sellers of high quality goods are more likely to benefit from announcing than sellers of low quality goods. Such prices can effectively signal that the sellers who announce them own a high quality good. This avoids that high quality sellers are driven out of the market by price competition. When competition among sellers is weak, the robust equilibrium is one in which both qualities are traded and the higher quality trades at a higher price.

Interestingly, incentive compatibility for the low quality sellers requires that the probability to sell at the higher price should be lower than one. Independently of the strength of demand, some high quality sellers will always be unable to sell. This is true even when the price of high quality goods exceeds sellers' reservation price. The price should fall to equate demand and supply, but imperfect information inhibits such a market-clearing role of prices. Thus, sellers of high quality could be rationed in a sense similar to Stiglitz and Weiss (1981).

Following the original work by Akerlof (1970), for the case of a competitive market with price setting sellers and asymmetric information, the possibility of price dispersion being the result of a separating equilibrium emerges from the extensive analysis by Wilson (1979 and 1980). However, "the absence of restrictions on the expectations of agents outside the set of [equilibrium] prices actually announced" [Wilson, 1980, page 126] implies a huge degree of indeterminacy. Many types of equilibria could actually exist, each associated with a particular degree of price dispersion.[2] Such indeterminacy is the result of the lack of restrictions on beliefs associated with off-equilibrium actions. As Wilson puts it

> "We would like the equilibrium to have the [robustness] property that even if sellers occasionally experiment by announcing a new price, their experience will never lead them to revise their expectations in such a way that they permanently alter their equilibrium behavior. [...] The problem is that it is no longer obvious what it means to restrict expectations to be 'correct' at prices at which no trade takes place." [Wilson, 1980, pp. 126-127].

Subsequent advances in the analysis of strategic stability (Kohlberg and Mertens, 1986) address this issue. In the model analyzed in the present paper, off-equilibrium beliefs are restricted following Banks and Sobel (1987) and Cho and Kreps (1987). The result is a set of robust equilibria, which share all the same unique outcome in terms of prices, quantity and quality of trade. Interestingly, such a property of uniqueness matches that found in Gale (1992) where the Walrasian approach to markets with adverse selection yields a stable set of refined equilibria all characterized by the same outcome.

As a result of the reduced indeterminacy, the predictive power of the theory is enhanced to a great extent. The unique prediction is that whether high quality sellers are driven out of the market or not depends on the competitive pressure faced by sellers. This, in turn, is determined by the overall supply and demand conditions of the market, as described above. Since different qualities always trade at different prices in robust equilibria, price dispersion is also uniquely determined by the overall market conditions.

The role of strategic pricing in the presence of asymmetric information has been subject of extensive research and a summary of the related literature is clearly beyond the scope of this paper. Representative contributions have focused on the cases of a single seller (Milgrom and Roberts 1986, Laffont and Maskin 1987, Overgaard 1993), multiple sellers (Laffont and Maskin 1989), free entry (Cooper and Ross 1982), search costs (Wolinsky 1983), and durable goods (Janssen and Roy 2002). However,

[2] It should be noted that Laffont and Maskin (1986) study an oligopolistic market where sellers can signal quality through their prices and conclude that, even in the case of just two firms, the problem of characterizing all perfect Bayesian equilibria seems intractable.

to our knowledge, this is the first study linking the degree of price dispersion and the information conveyed through prices to the competitive pressure faced by sellers.

The reduced indeterminacy also makes possible a welfare comparison between a market with strategic price setting and a market where agents are price-takers and trade occurs at a single price set by an auctioneer. The market where the auctioneer sets the price generates a level of welfare which is either lower or equal to that generated by the market with strategic price setting. This occurs unless competition among sellers is sufficiently weak, in which case the market where agents are price takers could outperform the market where sellers are price setting.

Recent contributions use the mechanism design methodology to study the maximum level of welfare achievable in an economy characterized by asymmetric information. Gul and Postlewaite (1992) study the conditions under which an economy characterized by asymmetric information can achieve efficiency as it becomes large. Muthoo and Mutuswami (2005) characterize the second best solution in markets with quality uncertainty where sellers are more informed than buyers. They find that the degree of competition among sellers affects the second best level of welfare in a non-trivial way.

Our approach is complementary to theirs to the extent that we study the prevailing equilibrium associated with a specific price convention. This enables us to find a relationship between competition and observable features of the market such as price dispersion. Moreover, we can assess how the use of prices as a communication device could affect the level of welfare generated by a market á la Akerlof. In particular, while such a role of price could help high quality sellers to trade, it does not always lead to a welfare improvement. Equilibria in which prices are uninformative (pooling) might generate higher welfare. However, we show that when sellers are price setting, these equilibria will be deviated. In spite of these potential inefficiencies, markets in which sellers post their prices are ubiquitous. For instance, many of the Internet markets are characterized by posted prices rather than other conventions such as, for instance, bargaining. Indeed, as put forward by Bester (1993), posted prices is likely to emerge as the prevailing convention, rather than bargaining, in those markets with quality uncertainty and asymmetric information that are characterized by low search costs.

The paper is organized as follows. In section two we present the model. Section three focuses on the equilibrium concept and the approach to equilibrium refinements. In sections four and five we characterize the set of robust equilibria. Section six describes the features of the unique equilibrium outcome associated with the set of robust equilibria. Section seven analyzes the efficiency implications of the model. A final section concludes the paper.

2 The Model

We consider a competitive market populated by a large number of buyers, B, and a large number of sellers, S. The set $\mathcal{S}$ of sellers is indexed by $s = 1, ..., S;\ s \in \mathcal{S}$. Each seller is endowed with one unit of good. Goods come in two different qualities, $q \in \{h, l\}$, where l (h) stands for *low* (*high*). The general case of a finite number of qualities is analyzed in the appendix. We refer to sellers endowed with quality q as sellers of quality (or type) q. The quality of each seller is decided by nature: each seller has a probability λ to be of quality h and probability $1 - \lambda$ to be of quality l. The law of large number applies, so that λ is interpreted as the fraction of type h sellers. The distribution of qualities is common knowledge. However, buyers cannot observe individual qualities. Moreover, quality is not verifiable ex post. The monetary utility that individual sellers of type q derive from their good is $v(q) > 0$, with $v(h) > v(l)$.

The set $\mathcal{B}$ of buyers is indexed by $b = 1, ..., B;\ b \in \mathcal{B}$. Each buyer consumes either one unit of the good or nothing. Buyers share identical preferences defined by the monetary utility function $u(q) > 0$, with $u(h) > u(l)$. We impose $u(q) > v(q)$ for $q \in \{h, l\}$, which implies that under full information there are always gains from trade to be realized. For expositional purposes, we also impose $u(l) < v(h)$: buyers are never willing to buy a low quality good at any price that is feasible for a high quality seller.

The market functions as follows. At stage zero, each seller s observes his quality. Endowed with this piece of information, sellers move first, by simultaneously choosing their action, while buyers do nothing. The action p_s^0 played by individual seller s consists in announcing a price $p \in [0, \overline{p}]$, where $\overline{p}$ is finite and strictly greater than $u(h)$, so that in equilibrium trade never occurs at $\overline{p}$. For simplicity, we adopt the convention that sellers who choose not to trade always announce $\overline{p}$. A strategy α_s^0 for seller s is a map from $\{l, h\}$ into the set $\mathbb{A}_s$ of probability distributions over $[0, \overline{p}]$. An action profile for the sellers is a collection $p^0 \equiv \{p_1^0, ..., p_S^0\}$, with $p_s^0 \in [0, \overline{p}]$. We also define $\alpha^0 \equiv \{\alpha_1^0, ..., \alpha_S^0\}$, with $\alpha_s^0 : \{l, h\} \rightarrow \mathbb{A}_s$, as a strategy profile for the sellers.

At stage 1, after observing the prices announced by sellers at stage zero, each buyer selects the price at which she is willing to buy (automatically rejecting all other price offers), while sellers do nothing. Selecting a price $p \notin p^0$ that has not been announced by any seller is equivalent to choosing not to trade since announced prices are assumed to be take-it-or-leave-it offers. We adopt the convention that buyers choosing not to trade select a price equal to zero, and obtain zero surplus. We denote with $p_b^1 \in [0, \overline{p}]$ the action of an individual buyer, b, and with p^1 an action profile for the buyers. A strategy α_b^1 for buyer b is a map from the set of all possible sellers' action profiles, $\mathcal{P}^0$, into the set of probability distributions $\mathbb{A}_b$ over $[0, \overline{p}]$. We denote with $\alpha^1 \equiv \{\alpha_1^1, ..., \alpha_B^1\}$, $\alpha_b^1 : \mathcal{P}^0 \rightarrow \mathbb{A}_b$, a strategy profile for the buyers.

At stage 2, buyers and sellers wishing to sell and to buy at the same price are matched. If at any price there is excess demand (supply), the purchase (sale) is randomly assigned with uniform probabilities. Hence, the probability to make a sale at any price p, $J(p, p^0, p^1)$, is given by the minimum between one and the ratio of buyers to sellers willing to trade at p. Symmetrically, the probability to make a purchase at p, $K(p, p^0, p^1)$, is the minimum between one and the ratio of sellers to buyers willing to trade at that price. The choice of this matching mechanism is motivated by several considerations. First, it has been widely employed in the literature on market for lemons (see for instance Wilson [1980]). Second, it is symmetric in the sense that the probabilities to trade only depend on the price at which agents are willing to trade and never on the identities of the agents. This is consistent with the conventional idea of a large market. Finally, it facilitates the comparison with the standard market for lemons where agents are price-taker.

3 Equilibrium analysis

Buyers' prior beliefs assign a probability λ to the event that an individual seller s is of type h. By contrast, upon observing the price p_s^0 announced by a seller s, and given the sellers' action profile p^0, buyers' (ex post) beliefs that the seller s is of type q are given by the conditional probability function $\sigma_s(q|p_s^0, p_{-s}^0)$. Given the belief function $\sigma_s(.,.)$, the expected utility of a buyer b who buys the good from seller s announcing $p_s^0 = p$ is

$$\mu_s^b(p, p_{-s}^0, \sigma) \equiv u(h)\sigma_s(h|p, p_{-s}^0) + u(l)\sigma_s(l|p, p_{-s}^0), \tag{1}$$

Accordingly, $\mu_s^b(p, p_{-s}^0, \sigma)$ measures the maximum willingness to pay of the buyer.

A seller of type q who sells his good at a price p receives a net payoff $p - v(q)$. Hence, he makes non-negative profits if and only if $p \geq v(q)$. For a given pair $\{p^0, p^1\}$, the expected payoff of a seller s of type q announcing $p_s^0 = p$ is

$$\pi_s(p, p_{-s}^0, p^1|q, \sigma) = J(p, p^0, p^1)[p - v(q)] \tag{2}$$

Given p^0, the expected payoff of any individual buyer b playing $p_b^1 = p$ is

$$\pi_b(p, p_{-b}^1|p^0, \sigma) = K(p, p^0, p^1)[\mu^b(p, p^0, \sigma) - p] \tag{3}$$

where $\mu^b(p, p^0, \sigma)$ is the unweighted average of $\mu_s^b(p, p_{-s}^0, \sigma)$ among the sellers s who announced p. Let $\{\alpha^0, \alpha^1\}$ be a strategy profile for sellers and buyers. Abusing notation, we denote with $J(p, \alpha^0, \alpha^1)$ the expected value at stage 0 of the probability to sell at a given p, evaluated by taking the expectation of $J(p, p^0, p^1)$ over the probability distribution associated with $\{\alpha^0, \alpha^1\}$. Given the realization p^0 of α^0, we denote with $K(p, \alpha^1, |p^0)$ the expected value at stage 1 of the probability to buy at p, evaluated by taking the expectation of $K(p, p^0, p^1)$, over the probability distribution associated with α^1. Similarly, we denote with $\pi_s(\alpha_s^0, \alpha_{-s}^0, \alpha^1|q, \sigma)$ and $\pi_b(\alpha_b^1, \alpha_{-b}^1|p^0, \sigma)$ the sellers' and buyers' expected payoffs in mixed strategies, respectively, obtained by taking expectations in (2) and (3).

Endowed with the above notation, we can define the equilibrium:

Definition 1. *A symmetric Perfect Bayesian Equilibrium (PBE) is a profile of strategies* $\{\alpha^{0*}, \alpha^{1*}\}$ *and a belief function* $\sigma_s^*(q|p_s^0, p_{-s}^0)$*,* $s \in \mathcal{S}$ *common to all buyers, with* $\sum_{q\in\{h,l\}} \sigma_s^*(q|p_s^0, p_{-s}^0) = 1$*, such that:*

i. Buyers and sellers are playing their best responses:

a. $\pi_s(\alpha_s^{0*}, \alpha_{-s}^{0*}, \alpha^{1*}|q, \sigma^*) \geq \pi_s(\alpha_s^0, \alpha_{-s}^{0*}, \alpha^{1*}|q, \sigma^*)\ \forall s \in \mathcal{S}, q \in \{l, h\}$*,* $\alpha_s^0 : \{h, l\} \to \mathbb{A}_s$*;*

b. $\pi_b(\alpha_b^{1*}, \alpha_{-b}^{1*}|p^0, \sigma^*) \geq \pi_b(\alpha_b^1, \alpha_{-b}^{1*}|p^0, \sigma^*)$*,* $\forall b \in \mathcal{B}$*,* $p^0 \in \mathcal{P}^0$*,* $\alpha_b^1 : \mathcal{P}^0 \to \mathbb{A}_b$ *;*

ii. Buyers' beliefs are derived from sellers' strategies using Bayes rule where possible;

iii. Strategies are symmetric: $\alpha_s^{0*} = \alpha_r^{0*}$*,* $\forall s, r \in \mathcal{S}$*, and* $\alpha_b^{1*} = \alpha_v^{1*}\ \forall b, v \in \mathcal{B}$*.*

iv. For all s*,* p^0*,* $\hat{p}^0$*, and* $q \in \{l, h\}$*, buyers' beliefs satisfy* $\sigma_s^*(q|p_s^0, p_{-s}^0) = \sigma_s^*(q|\hat{p}_s^0, \hat{p}_{-s}^0)$ *if* $p_s^0 = \hat{p}_s^0$*.*

Conditions i)-ii) do not require any explanation. As for condition iii), symmetry in strategies is usually imposed to simplify the equilibrium analysis. However, with price-setting sellers, such a condition is also motivated by the fact that buyers' beliefs are derived from sellers' strategies. If strategies were not symmetric, buyers could assign different probabilities to be of a given type to sellers announcing the same price. This is at odds with the conventional idea of a large market in which trade is not affected by the identity of individuals. Moreover, since we allow for mixed strategies, asymmetries in actions can always emerge in equilibrium. Thus, the restriction to symmetric strategies does not imply a great loss of generality in terms of agents' behavior. Symmetry, together with the assumption that buyers' beliefs obey Bayes' rule, imply that buyers should assign the same probability to be high quality to any pair of sellers taking the same action. We also restrict our attention to the equilibria in which agents randomize over actions by using distributions with finite support. This restriction, together with the assumption that all buyers and sellers adopt the same strategies, permits to exploit the law of large numbers. Finally, condition iv) implies that beliefs about seller s are independent from other sellers' actions, even in the presence of deviations (Fudenberg and Tirole, 1991, p. 332). This condition follows from the fact that sellers different from s have no information about s's type that is not also available to the buyers.

Given definition 1, in the remainder of the paper, we denote the belief function simply as $\sigma(q|p)$. Symmetry and the law of large numbers imply that in equilibrium the fraction of sellers of type q among sellers announcing p is equal to $\sigma^*(q|p)$. Also, whenever there is no risk of confusion, strategy profiles and beliefs will be omitted from the probabilities to make a sale or a purchase at price p prevailing in equilibrium. These will be denoted as $J^*(p)$ and $K^*(p)$ respectively. In a symmetric equilibrium with a large number of individuals, $J^*(p)$ and $K^*(p)$ converge to certain values for all p. That is, the change in the probabilities to buy and sell induced by a change in the actual realizations of: a) Nature's random selection of each individual seller's type; b) Individual players' randomizations, is negligible. Finally $\pi_s^*(q)$ will denote equilibrium profits of seller s of type q.

Equilibria can take different forms:

a. *Separating equilibria*, in which, by definition, different seller-types take different actions;

b. *Pooling equilibria*, in which all seller-types take the same action;

c. *Partially separating* or *Hybrid equilibria*, in which heterogenous poolings of sellers take different actions.

Associated with this variety of equilibria is a great deal of inderterminacy with respect to the market's outcome in terms of prices, traded quantities and qualities, as well as with respect to the associated expected payoffs of market's participants. For this very reason it is important to investigate how a robustness analysis helps restricting the set of possible equilibria.

3.1 Restrictions on off-equilibrium beliefs: Preliminaries

The high degree of indeterminacy is due to a typical "unsent message" problem: if a seller deviates to a price p that is announced with probability zero in equilibrium, Bayes' rule cannot determine the posterior beliefs of the buyers. Thus, upon observing a profile p^0 containing price p, buyers could hold arbitrary beliefs about the quality of the seller who is announcing p.

In the spirit of equilibrium refinements in signaling games, the procedure we adopt to test the robustness of the different equilibria goes as follows. First, off-equilibrium beliefs associated with some deviation p are refined by eliminating type-action pairs that are "less likely" to occur. Second, we compute the best response of each individual buyer, given a deviation p and the refined beliefs, under the assumption that all other buyers stick to their equilibrium strategies.[3] Finally, given buyers' best response, we consider whether a deviation p is profitable for an individual seller under the assumption that no other seller is deviating.

Assuming that seller s deviates and announces a new price p, each buyer decides whether to buy at p, or at any of the other announced prices. If sellers different from

[3]This implies that, in determining the hypothetical best response to the deviation, the probability to being able to buy the good at p equals one.

s stick to their equilibrium strategy, buyer b's decision is based on the comparison between the expected payoff at the unchanged old equilibrium prices with the expected payoff from buying at p.[4]

A potential problem refers to the beliefs that buyers assign to the sellers who did not deviate. Let p^0 be a profile of prices that comprises the deviation p. Since p is never announced in equilibrium, the realization p^0 occurs with probability zero which, in principle, might have an effect on the beliefs that buyers assign to sellers who stick to the old equilibrium prices. However, the standard requirement stated in point iv) of definition 1 implies that no situation in which buyers' beliefs about sellers who did not deviate are changed by a deviation can constitute equilibrium play. The intuition is that the seller who deviated does not possess any information about the quality of sellers who did not, that is not available also to the buyers. Therefore, the deviation must not change buyers' beliefs about these sellers.

Finally, a seller will strictly benefit from a deviation p if his equilibrium payoff is lower than the payoff associated with p. Whether this is the case or not depends on the probability to sell at p resulting from buyers' response. This in turn depends on buyers' beliefs upon observing p. Thus, our next step is to refine off-equilibrium beliefs. The following section derives the rule according to which buyers' off-equilibrium beliefs should be refined based on a commonly used equilibrium refinement: D1 (Cho and Kreps 1987). The reader not interested in technical details can focus on section 4, where the associated test of equilibrium robustness is presented and applied.

3.2 Restrictions on off-equilibrium beliefs: D1

The approach we take is to follow Banks and Sobel (1987) notion of Universally Divine Equilibrium and Cho and Kreps (1987) version of this concept, known as D1. We remand to these works for a general discussion of these refinements in standard signaling games. Here, we limit ourselves to the discussion of how D1 translates in this model.

Consider an equilibrium and assume that an individual seller deviates and announces price p. At the off-equilibrium price p, beliefs are given by σ. The expected payoff of buyer b who is willing to buy at p for a given profile α^1_{-b} of other buyers' strategies is:

$$\pi_b(p, \alpha^1_{-b}|p^0, \sigma) = K(p, \alpha^1_{-b}|p^0)\{\sigma(h|p)[u(h) - p] + (1 - \sigma(h|p))[u(l) - p]\}, \quad (4)$$

where p^0 comprises the deviation p. We say that the mixed strategy α^1_b played by buyer b is a mixed best response to p^0 if there exist beliefs $\hat{\sigma}$ such that $\hat{\sigma}(h|p) + \hat{\sigma}(l|p) = 1$

[4] Differently from standard signaling games, in the model discussed here there are many sellers. Thus, in the presence of a deviation of a single seller, buyers retain the option to buy from all the sellers who did not deviate.

and a profile α^1_{-b} such that:

$$\pi_b(\alpha^1_b, \alpha^1_{-b}|p^0, \hat{\sigma}) \geq \pi_b(\alpha^{1'}_b, \alpha^1_{-b}|p^0, \hat{\sigma}) \; \forall \alpha^{1'}_b : \mathcal{P}^0 \rightarrow \mathbb{A}_b \tag{5}$$

Let $MBR(\hat{\sigma}, p^0)$ denote the set of mixed best responses when p^0 is observed and beliefs are given by $\hat{\sigma}$. Let also

$$MBR(p^0) \equiv \bigcup_{\hat{\sigma}:\hat{\sigma}(h|p)+\hat{\sigma}(l|p)=1} MBR(\hat{\sigma}, p^0) \tag{6}$$

denote the set of mixed responses to p^0 (common to all buyers) that are mixed best responses for some $\hat{\sigma}$. Finally, we define the set of profiles of mixed best responses as:

$$MBRP(p^0) \equiv \{\alpha^1 : \alpha^1_b \in MBR(p^0) \forall b \in \mathcal{B}\} \tag{7}$$

Any profile α^1 that belongs to $MBRP(p^0)$ is a strategy profile such that each buyer b plays some strategy α^1_b that is a best response to p^0 for some profile of other buyers' strategies and some beliefs $\hat{\sigma}$ (not necessarily the same across all buyers). In other words, all buyers play a strategy in response to p^0 that can be justified by some beliefs and some strategy profile for the other buyers.

Define $\{\hat{\alpha}^0, \hat{\alpha}^1\}$ a profile of strategies that makes low quality sellers indifferent between deviating to p and announcing an equilibrium price. Let $J^l(p, \hat{\alpha}^0, \hat{\alpha}^1)$ be the correspondent probability to sell if announcing $p > v(l)$, so that

$$\pi^*_s(l) = J^l(p, \hat{\alpha}^0, \hat{\alpha}^1)[p - v(l)], \tag{8}$$

where $\pi^*_s(l)$ is the equilibrium payoff of the seller. Then, whenever the probability to sell when deviating, $J(p, \alpha^0, \alpha^1)$, is greater than J^l, low quality sellers would always deviate. Similarly, high quality sellers would always deviate if

$$J(p, \alpha^0, \alpha^1) > J^h(p, \tilde{\alpha}^0, \tilde{\alpha}^1) \tag{9}$$

where, for $p > v(h)$,[5]

$$\pi^*_s(h) = J^h(p, \tilde{\alpha}^0, \tilde{\alpha}^1)[p - v(h)] \tag{10}$$

and $\{\tilde{\alpha}^0, \tilde{\alpha}^1\}$ is the profile that makes high quality sellers indifferent between the deviation and their equilibrium payoff. Based on equations (8) and (10) we define:

$$R_1(l|p) \equiv \{\alpha^1 \in MBRP(p^0) : \; J(p, \alpha^0, \alpha^1) \geq J^l\} \tag{11}$$

and

$$R_2(h|p) \equiv \{\alpha^1 \in MBRP(p^0) : J(p, \alpha^0, \alpha^1) > J^h\}. \tag{12}$$

[5]In general, high quality sellers never deviate to any $p < v(h)$ while low quality sellers never deviate to any $p < v(l)$. This implies that, upon observing a deviation to a price $p \in (v(l), v(h))$, sellers should conclude that the deviation comes from low quality sellers.

According to D1, off-equilibrium beliefs should satisfy the following restriction: whenever $R_1(l|p) \subset R_2(h|p)$, $\sigma(l|p) = 0$ must hold. In fact, when $R_1(l|p) \subset R_2(h|p)$, high quality sellers benefit from the deviation p whenever low quality sellers weakly benefit.

Clearly, D1 does not impose any restriction on beliefs when $R_2(h|p)$ is empty. The next lemma gives necessary and sufficient conditions for $R_1(l|p) \subset R_2(h|p)$ when $R_2(h|p)$ is nonempty.

Lemma 1. *Assume $R_2(h|p)$ is non-empty. Then, $R_1(l|p) \subset R_2(h|p)$ if and only if $J^l \equiv \frac{\pi_s^*(l)}{p-v(l)} > J^h \equiv \frac{\pi_s^*(h)}{p-v(h)}$.*

Proof See appendix.

Notice that $\pi_s^*(q)/(p-v(q))$ is the ratio between the opportunity cost of deviating, measured by the equilibrium profits, and the potential gain from the deviation. The above lemma states that if this ratio is larger for low quality sellers, sellers who deviated should be assigned a probability to be low quality equal to zero.

4 Robust equilibria

The essence of the D1 refinement is that if the opportunity cost of a deviation p relative to the potential gain is larger for a low quality seller than for a high quality one, high quality sellers are more seemly to benefit from such a deviation than low quality sellers. If so, upon observing p, buyers' beliefs should assign a probability zero to the event that the seller undertaking the deviation p is of low quality.

Accordingly, an equilibrium fails D1 if, based on beliefs refined in such a way, a buyer and a seller would both profit from trading at some off-equilibrium price p. Conversely, an equilibrium is robust to D1 if either of the two is true: 1) There is no deviation p at which buyers' refined beliefs are such that the seller announcing p is of low quality with probability zero, 2) If such p exist(s), then trade at p must not be mutually beneficial.

For a seller of type q, the opportunity cost of deviating to p is measured by his equilibrium profits, $\pi^*(q)$, and the potential gain is $p - v(q)$. Therefore, checking the robustness to D1 reduces to verifying whether the condition

$$\frac{\pi_s^*(l)}{p - v(l)} \leq \frac{\pi_s^*(h)}{p - v(h)} \tag{13}$$

holds, for any p such that

$$u(h) - p > K^*(p^*)\{\sigma^*(h|p^*)[u(h) - p^*] + (1 - \sigma^*(h|p^*))[u(l) - p^*]\} \tag{14}$$

where p^* is any price at which trade occurs in the equilibrium. The RHS is the expected payoff of a buyer when she selects any price announced by the sellers who did not deviate (this is the same across all prices at which trade occurs in the equilibrium).[6] The LHS of inequality (14) represents buyer's expected payoff when she selects p and believes that the seller who announced p is of type l with probability zero. Note that an individual buyer is always able to buy at p with probability one when all other buyers stick to their equilibrium strategy. Inequality (14) requires that a buyer strictly benefit from the deviation, given her refined beliefs.

For the purposes of the analysis of robustness we distinguish the possible equilibria into two broad categories on the basis of how many qualities are traded:

Definition 2. *A type I equilibrium is a PBE where both qualities are traded. A Type II equilibrium is a PBE where only the low quality is traded.*

It is worth noting that, in general, there would be a third category, which includes those equilibria in which no quality is traded. However, as it turns out, the low quality is always traded given the model's assumptions.[7]

4.1 Type I equilibria: both qualities are traded

Type I equilibria can take two forms: a. Separating equilibria (SE) in which low and high quality sellers take different actions thereby fully revealing their type; b. Pooling equilibria (PE) and Hybrid equilibria (HE) in which either all sellers take the same action (PE) or different poolings of sellers take different actions (HE). Rather than characterizing all equilibria and then discard those which fail D1, we characterize only the equilibria that pass D1.

Let us analyze PE and HE, first. In any PE, by definition, there is a single equilibrium price p^* at which both high and low qualities are traded. In HE sellers of the same type may announce different prices and there is at least one price that is announced with positive probability by both types.

It is well known that D1 tends to select SE (see Cho and Sobel (1990)). The next lemma shows that also in the present model, within the set of type I equilibria, D1 discards PE and HE:

Lemma 2. *No pooling/hybrid equilibrium of type I survives D1.*

Proof. See Appendix.

In any PE or HE where both qualities are traded, sellers of type h, who face a higher opportunity cost of selling ($v(h) > v(l)$) make lower equilibrium profits than

[6] The RHS is equal to the equilibrium payoff for a buyer. This follows since there are a large number of sellers and buyers, equilibrium strategies are symmetric (definition 1, iii), and beliefs about the sellers who did not deviate are not affected by a deviation (definition 1, iv).

[7] Consider an equilibrium where all sellers announce prices at which trade does not occur, i.e. $p^0 = \{\overline{p}, \overline{p}, ..., \overline{p}\}$. Buyers would be willing to buy at any price $p < u(l)$ for any possible off-equilibrium beliefs. Then, deviating and announcing $p \in (v(l), u(l))$ is always profitable for a seller of type l.

sellers of type l. Hence, sellers of type h have a lower opportunity cost of deviating. Being aware of this, upon observing a deviation, buyers infer that it comes from a high quality seller. Therefore, sellers of type h would find it optimal to announce a price that is slightly higher than the equilibrium price at which both qualities are traded. This would enable high quality sellers to stand out from the crowd and attract buyers. Accordingly, no PE or HE of type I is ever robust: equilibria of type I that are robust to D1 could only include SE.

In general, a SE is a PBE in which low quality sellers announce a price p_l and high quality sellers announce a price $p_h \neq p_l$. Prices constitute a perfect signal of quality: $\sigma^*(h|p_h) = 1$, and $\sigma^*(h|p_l) = 0$. When a good is exchanged at p_q, $q \in \{l, h\}$, the buyer obtains $u(q) - p_q$. In any SE, p_h and p_l satisfy

$$[p_l - v(l)]J^*(p_l) \geq [p_h - v(l)]J^*(p_h) \tag{15}$$

$$[p_h - v(h)]\, J^*(p_h) \geq [p_l - v(h)]J^*(p_l), \tag{16}$$

These two inequalities represent the Incentive Compatibility Condition (ICC) for low and high quality sellers, respectively. Any SE in which quality $q \in \{l, h\}$ is traded must also: 1) satisfy the participation constraint of sellers ($p_q \geq v(q)$) and buyers ($p_q \leq u(q)$); 2) ensure that buyers obtain at least the same expected payoff as that they would obtain from quality $q' \neq q$:

$$K^*(p_q)[u(q) - p_q] \geq K^*(p_{q'})[u(q') - p_{q'}] \tag{17}$$

Clearly, if both qualities are traded, p_h and p_l should guarantee the same expected payoff to the buyers:

$$[u(l) - p_l]K^*(p_l) = [u(h) - p_h]K^*(p_h) \tag{18}$$

The above conditions combined with the fact that in any SE the low quality must be traded with positive probability imply that any SE satisfies $p_h > p_l$ and $J^*(p_h) < J^*(p_l)$.

For a SE, the equivalents of the robustness conditions (13) and (14) are

$$\frac{[p_l - v(l)]\, J^*(p_l)}{p - v(l)} \leq \frac{[p_h - v(h)]\, J^*(p_h)}{p - v(h)} \tag{19}$$

and

$$u(h) - p > [u(l) - p_l]K^*(p_l), \tag{20}$$

where we note that, given (18), the RHS of inequality (20) is equal to $[u(h)-p_h]K^*(p_h)$ whenever the high quality is traded. In order to assess the robustness of a SE we should verify whether condition (19) is satisfied for any p such that (20) holds. The following result holds for all robust separating equilibria.

Lemma 3. *In any D1-robust SE, the ICC of low quality sellers is satisfied with equality unless* $p_h = v(h)$.

Proof. See appendix.

We note that this result applies both to type I and to type II equilibria.[8] Here, we discuss the intuition in the case of type I equilibria. In any SE of type I the probability to sell at p_h must be less than 1. Otherwise, the ICC of low quality sellers, see equation (15), could not be possibly satisfied. Therefore, a high quality seller who is announcing at p_h would be willing to deviate and announce a price p slightly less than p_h whenever the gains from the increase in the probability to sell outweigh the loss due to the small reduction in the announced price. If the ICC_l does not hold with equality, low quality sellers strictly prefer p_l to p_h. Therefore, they are not willing to deviate unless the chances to sell at p become relatively high. Because of that, buyers infer that the deviation p must come from a high quality seller, which in turn gives high quality sellers the incentive to deviate. By contrast, when the ICC_l holds with equality, low quality sellers are indifferent between p_l and p_h. Therefore, they are willing to deviate whenever the high quality are, which implies that buyers' off-equilibrium beliefs cannot be restricted.

The question we are interested in is under which conditions, if any, is a SE of type I (i.e. where $J^*(p_h) > 0$ holds) robust to D1. A key parameter in our discussion is the ratio between potential demand, given by the number of buyers B, and potential supply, given by the number of sellers, S. Such ratio, denoted with $\theta \equiv B/S$, is a measure of the competitive pressure faced by buyers and sellers.

The next result explains under which condition SE of type I may emerge:

Lemma 4. *If* $1 - \lambda \geq \theta$ *there is no SE of type I. If* $1 - \lambda < \theta$ *D1-robust SE must be of type I.*

Proof. See appendix.

Consider a SE of type I: sellers of type l announce p_l and sellers of type h announce $p_h \geq v(h)$. If $1 - \lambda > \theta$, low quality sellers are relatively more numerous than buyers (i.e. they are the long side of the market). In other words, sellers compete to sell, while buyers face no competitive pressure. Accordingly, the only possible equilibrium value for p_l is $v(l)$ so that low quality sellers make zero profits. Otherwise, if $p_l > v(l)$, low quality sellers would undercut each other in order to increase their chance to sell their good.[9] However, if $p_l = v(l)$, the ICC of type l sellers can never be satisfied for

[8] Interestingly, type l sellers' ICC binding is a necessary (but not sufficient) condition for the second best (see Muthoo and Mutuswami 2005).

[9] Note that undercutting would always induce a buyer to buy since: i. The worst belief that buyers can assign to a seller announcing a price lower than p_l is that he is of type l with probability 1, and buyers are buying quality l at p_l; ii. Given others' equilibrium strategies, the probability to buy at the off-equilibrium price equals one for the individual buyer.

any $p_h \geq v(h)$ unless the probability to trade at p_h were equal to zero, which would contradict the hypothesis of a type I SE. Therefore, if $1-\lambda > \theta$, there exist no type I equilibria.

Consider now the case $1-\lambda < \theta$. Low quality sellers are on the short side of the market. Buyers, on the other hand, compete in order to buy. If prices are such that buyers are making a positive surplus, then all buyers should be willing to buy. However, if $1-\lambda < \theta$, low quality sellers are not enough to satisfy demand. Therefore, they will raise their prices until some of the buyers will buy (from high quality sellers) at a price $p_h \geq v(h)$. If, on the other hand, buyers obtain zero surplus, then $p_h > v(h)$, and lemma 3 ensures that the high quality is traded. Yet, the probability to sell at p_h must be low enough to ensure that low quality sellers do not have incentive to announce p_h.

We now turn to the characterization of robust type I equilibria. Define

$$\delta \equiv \frac{GFT_l}{u(h)-v(l)} \in (0,1) \tag{21}$$

and

$$\gamma \equiv \frac{\Delta GFT}{v(h)-v(l)}, \tag{22}$$

where, for one unit of quality q, $GFT_q \equiv u(q) - v(q)$ measures the gains from trade, and $\Delta GFT = GFT_l - GFT_h$. Note that δ represents the overall gains from trading the low quality scaled by the range of feasible prices $u(h)-v(l)$, while γ is the difference in the gains from trade between the two qualities, ΔGFT, over the difference in the seller's evaluation of the two qualities $v(h)-v(l)$. When $\gamma > (<)0$, the gains from trading the low quality are higher (lower) than those from trading the high quality.

Let $\hat{\theta} \equiv 1-\lambda+\delta\lambda$, and $\theta_\gamma \equiv 1-\lambda+\gamma\lambda I_{\{\gamma>0\}}$ where $I_{\{\gamma>0\}} : \mathbb{R} \to \{0,1\}$ is an indicator function that takes value 1 if $\gamma > 0$ and zero otherwise. Note that since $\gamma < \delta$ holds, $\hat{\theta}$ is always strictly greater than θ_γ.

Proposition 1. *D1-robust equilibria of type I emerge if and only if* $1-\lambda < \theta$. *In all these equilibria: i)* $J^*(p_l) = 1$, *and* $J^*(p_h) = \min\left[\frac{\theta-(1-\lambda)}{\lambda}, \delta\right]$, *ii)* p_l *and* p_h *are uniquely determined:*

i. $p_h = u(h)$, $p_l = u(l)$, *if* $\theta \in [\hat{\theta}, \infty)$;

ii. $p_h = v(l) + \frac{\lambda[u(h)-u(l)]}{1-\theta}$, $p_l = v(l) + \frac{\theta-(1-\lambda)[u(h)-u(l)]}{1-\theta}$ *if* $\theta \in (\theta_\gamma, \hat{\theta})$;

iii. $p_h = v(h)$, $p_l = u(l) - [u(h)-v(h)]$ *if* $\theta \in (1-\lambda, \theta_\gamma]$.

Proof. See appendix.

Proposition 1 implies that the equilibrium outcome in terms of prices and traded quantities and qualities is uniquely determined and crucially depends on the buyers to sellers ratio, θ. If θ is very large, i.e. greater than $\hat{\theta}$, trade (of both qualities) occurs at buyers' reservation prices, $u(h)$ and $u(l)$ (case *i*). If θ is only moderately large, i.e. greater than $1-\lambda$ but lower than $\hat{\theta}$, trade (of both qualities) occurs at prices that guarantee a positive surplus to the buyers (cases *ii* and *iii*). However, note that, provided that $\gamma \leq 0$, prices will exceed type h sellers' reservation prices in any robust SE of type I; that is whenever $1-\lambda < \theta$. This, even though the probability of selling at p_h, $J^*(p_h)$, is always less than one, which would suggest that high quality sellers should undercut each other. This is a standard effect of asymmetric information. Price competition among the high types is impaired by buyers' fear that low types may deviate and announce p_h if the probability to sell at p_h becomes too large. Hence, price competition among sellers of type h comes to a halt when the demand at p_h is such that low quality sellers are indifferent between announcing p_h and announcing p_l. Limited price competition causes high quality sellers' profits to remain positive, even if $J^*(p_h) < 1$.

Things change substantially if the gains from trading the low quality exceed those from trading the high quality ($\gamma > 0$) and $\theta \leq \theta_\gamma$ (if $\theta > \theta_\gamma$ the previous discussion applies). In this case, the ICC of low quality sellers cannot hold with equality. Thus, the price announced by high quality sellers must drop to $v(h)$. In other words, high quality sellers must forgo their profits in order to trade. Low quality sellers will then announce the highest possible price at which buyers (weakly) prefer to buy the low quality, given the option to buy the high quality at $v(h)$.

4.2 Type II equilibria: Only the low quality is traded

In this section we turn attention to the typical lemon-market situation in which the high quality is driven out of the market (type II equilibria). We will characterize the (unique) robust outcome of type II equilibria and show that such equilibria arise if and only if no robust type I equilibrium exists.

The following proposition characterizes robust type II equilibria:

Proposition 2. *D1-robust equilibria of type II emerge if and only if* $1-\lambda \geq \theta$. *In all these equilibria, the fraction of quality* l *traded is* $\theta/(1-\lambda) \leq 1$. *All trade occurs at a unique price* p^*, *which is equal to* $v(l)$ *if* $1-\lambda > \theta$.[10]

Proof. See appendix.

In order to gather intuitions on proposition 2, notice that when $1-\lambda > \theta$, low quality sellers are on the long side of the market and therefore face competitive pressure. They compete to sell their goods, which drives their profits to zero ($p^* = v(l)$). If, on the other hand, $1-\lambda < \theta$, sellers of low quality are on the short side and buyers

[10] The analysis of the equilibrium price for the special case $1-\lambda = \theta$ is presented in the proof. There, it is shown that a discontinuity arises when $\gamma > 0$.

compete to buy. Thus, when $1-\lambda<\theta$, low quality sellers would announce $p^*=u(l)$. At this price, they would extract all the surplus from the buyers and make strictly positive profits. It is easy to see why these equilibria fail D1. Whenever sellers of low quality make strictly positive profits, their individual opportunity cost of deviating, measured by $\pi^*(l)$, is larger than that of high quality sellers, which equals zero since they are not trading. Accordingly, while high quality sellers are never made worse off by deviating, low quality sellers can be. It then follows that, upon observing a deviation $p \geq v(h)$, buyers should infer that the seller deviating is of high quality.

By contrast, given a type II equilibrium, low quality sellers make zero profits when they are the long side of the market. Hence, the opportunity cost of deviating is the same for low and high quality sellers, and both types of sellers are never worse off if deviating. Thus, buyers cannot rule out that an observed deviation is coming from a low quality seller and the equilibrium is therefore robust. It is interesting to notice that when low quality sellers are the long side of the market, the equilibrium requires that all sellers earn zero profits. Such a zero-profit condition makes all sellers indifferent between trading and not trading. Thus, deviating to prices at which no trade occurs in equilibrium is as cheap a way to signal quality for the low type as it is for the high type. Therefore, it is not surprising that deviations to higher prices fail to signal higher quality.

Finally, notice that robust type II equilibria can be either SE or hybrid equilibria (HE). In SE all low quality sellers announce p^* while high quality sellers announce $\overline{p}$. In HE, all sellers of quality h and a fraction smaller than $1-\lambda-\theta$ of low quality sellers announce $\overline{p}$ (and do not trade) while the rest announce p^*.

The effects of asymmetric information become more evident, although not necessarily more harmful, under the type II equilibria just described where high quality sellers are unable to trade their goods. Since these equilibria have been given special relevance in the literature, before turning to the discussion of the implications of our analysis, we briefly review the robustness of these equilibria by means of alternative criteria based on refinements weaker than D1.

4.2.1 Robustness of type II equilibria to weaker refinements

D1 is sometimes considered a strong refinement. In particular, the fact that it eliminates all equilibria which do not assign probability zero to type l whenever type h has a greater incentive to deviate may appear extreme. Here, we give a brief account of the robustness of type II equilibria to less powerful refinements such as Divinity [Banks and Sobel (1987)] and Sequential Perfection [Grossman and Perry (1986)].[11] A

[11] Interestingly, the Intuitive Criterion (Cho and Kreps 1987) has no bite in this model. The Intuitive Criterion requires that, given some off-equilibrium price which could possibly benefit type h sellers, type l sellers should be made worse off by announcing such a price if beliefs are to assign probability 0 to type l. This must hold for any beliefs buyers may hold (and therefore for any best

formal analysis is contained in the Appendix.

Although they are built on somewhat different intuitions, both Sequential Perfection and Divinity yield the same results in this model. Since Divinity is weaker than D1, type II equilibria where low quality sellers are the long side of the market are always robust to Divinity. In the appendix, we show that they pass Sequential Perfection as well. By converse, unlike D1, Divinity and Sequential Perfection eliminate type II equilibria where low quality sellers are the short side of the market only when λ exceeds the critical value $\hat{\lambda} = \frac{v(h)-u(l)}{u(h)-u(l)}$. In other words, the fraction of high quality sellers in the population must be large. It is interesting to note that this condition is equivalent to the condition for the existence of an equilibrium where all qualities are traded in the standard textbook's model of adverse selection, where trade takes place at a single price set by an auctioneer. The condition $\lambda > \hat{\lambda}$ in fact implies that there exists a price at which buyers are willing to buy a pooling of the two qualities.

5 Uniqueness

We now discuss the uniqueness properties of the equilibrium outcome and analyze the implications for the amount of high quality traded.

Cho and Sobel (1990) show that in signaling games that satisfy specific monotonicity and sorting conditions, D1 selects a unique equilibrium, which is a SE. In the model we analyze, given any two prices p and $p' < p$ and associated probabilities to sell $J(p)$ and $J(p')$ their sorting condition would be

$$J(p)[p - v(l)] \geq J(p')[p' - v(l)] \Rightarrow J(p)[p - v(h)] > J(p')[p' - v(h)]. \tag{23}$$

Whenever low quality sellers benefit from announcing a higher price, high quality sellers would strictly benefit from doing the same. Condition (23) is of course satisfied for all prices at which trade occurs, i.e. provided that $J(p) > 0$ and $J(p') > 0$. However, at prices at which the probability to sell is zero the net payoff is independent of the announced price and seller's type. Hence, at such prices, no sorting is possible and (23) is not satisfied. These observations help explaining why in the model we analyze the set of $D1$ robust equilibria does not include only separating equilibria and generally contains more than one equilibrium. Nevertheless, as it directly follows from the combination of proposition 2 and lemmata 2 and 4, $D1$ guarantees separation among sellers' types at prices at which trade occurs. Pooling can only occur at prices at which trade does not occur. In particular, pooling survives $D1$ in equilibria of type II where only a fraction of low quality sellers announce $p^* = v(l)$ at which trade occurs, while high quality sellers and the rest of low quality sellers who decide not to

response). In this model, by converse, one can usually find buyers' best responses which make type l sellers better off by announcing a higher price.

trade announce $\overline{p}$. However, such HE yield the same equilibrium outcome in terms of quantities and qualities traded and agents' interim payoffs as the robust SE of type II in which all sellers of type l announce p^*. This observation, together with the result about uniqueness of equilibrium outcomes associated with type I robust equilibria stated in proposition 1 leads to the following

Proposition 3. *Given the values of* λ, $\theta \neq 1-\lambda$, $u(l)$, $v(l)$, $u(h)$, $v(h)$, *all the resulting D1-robust equilibria yield the same unique outcome in terms of prices of traded goods, quality and quantity of trade. In particular: i. The fraction of quality* l *goods being traded (over the total supply of quality* l*) is* $f(l) = \min[\theta/(1-\lambda), 1]$*; ii. The fraction of quality* h *goods being traded (over the total supply of quality* h*) is* $f(h) = \max\left[0, \min\left[\frac{\theta-(1-\lambda)}{\lambda}, \delta\right]\right]$.

Proof. See appendix.

According to proposition 3, while in the present model $D1$ does not select a unique separating equilibrium, it still yields a unique equilibrium outcome in terms of prices, quantities and qualities exchanged, and interim payoff of market participants. Because of this uniqueness property, the model implies a very precise relationship between the traded quantity of high quality goods and the market conditions as measured by the ratio between potential demand and potential supply, θ.

If low quality sellers are relatively more numerous than buyers ($1-\lambda < \theta$), competition among these sellers drives the low quality price to its minimum level. As a consequence, the only D1-robust equilibrium is one in which all sellers make zero profits and only the low quality is traded; all buyers are able to buy. By converse, if buyers are relatively more numerous than low quality sellers, buyers will compete to buy so long as only low quality goods are traded. In this case, robustness implies that both low quality and high quality sellers are able to sell their goods with positive probability. All low quality sellers are able to sell while only a fraction of sellers of high quality is able to find a buyer.

The prices of traded goods and the traded fraction of high quality goods are nondecreasing, nondifferentiable functions of θ, as illustrated in Figures 1 and 2. For values of $\theta \leq 1-\lambda$, the fraction of high quality traded is constant with respect to θ and equal to zero. If $\theta > 1-\lambda$, the fraction of high quality traded linearly increases in θ until it reaches the value $\delta < 1$ where θ equals the critical value $\hat{\theta}$. Once θ has reached $\hat{\theta}$, further increases in θ do not affect any longer the fraction of high quality traded. Notice that, while all buyers are able to buy one good if $\theta \leq \hat{\theta}$, a fraction $\lambda\delta$ of buyers do not obtain any good when the reverse (strict) inequality holds. This in spite of the fact that high quality sellers must be selling with probability $\delta < 1$ (otherwise the ICC of low quality sellers would be violated). Therefore, there might be buyers and sellers who do not trade even when trade would be mutually beneficial. Having said that, we should also point out that D1 selects the equilibrium where the amount of trade is maximized among all SE. Hence, the prevailing SE is the one in which the potential inefficiency related to the quantity of trade is minimized.[12]

It is important to note that δ is decreasing in $u(h)$. When demand is sufficiently high, the higher is $u(h)$ the higher must be the price of high quality goods. However, for the SE equilibrium to be robust, the incentive compatibility constraint of low quality sellers must hold with equality. This implies that the probability to sell high quality goods must be decreasing in the price. The result is that the more buyers value high quality goods, the lower must be the maximum fraction of high quality sellers able to sell their good. The model thus displays a curse on high quality sellers.

6 Competitive pressure and the signaling role of prices

The uniqueness property of the set of robust equilibria we identified in the previous analysis emerges as the consequence of the signaling role played by sellers' strategic pricing decisions, both off and along the equilibrium path.

The signaling effectiveness of sellers' strategic pricing decisions crucially depends on the competitive pressure they face. Off the equilibrium path, the information content of a deviation to a price higher than the equilibrium price changes according to whether sellers face weak competition (θ exceeds $1-\lambda$) or strong competition ($1-\lambda$ exceeds θ). If competition among sellers is weak, starting from an equilibrium in which only the low quality is traded, a deviation to a higher price allows high quality sellers to reveal themselves (provided buyers hold refined beliefs). As a consequence, the initial equilibrium unravels. By converse, strong competition among sellers forces the equilibrium profits of both types of sellers to zero. Therefore, both types have identical incentives to deviate. As a result, the deviation does not serve as a signal for high quality sellers. Hence, the initial equilibrium holds.

In equilibria that are robust to D1, the effectiveness of the price system at reflecting information along the equilibrium path also depends on the extent of competition among sellers. As we have shown, when competition is weak, both qualities are traded in the robust equilibrium. Each quality is traded at a different price. That is, prices are fully informative and enable high quality sellers to trade. However, when competition is strong, only low quality sellers are able to trade. The price system associated with this equilibrium is still fully informative, but the only credible way for high quality sellers to reveal their type is to decide not to trade (i.e. to announce a price at which

[12] We finally note that proposition 3 gives a strong economic rationale for the use of D1 as the equilibrium concept. By selecting the SE with the highest possible amount of high quality traded and by imposing $p_h = u(h)$ whenever there are ex-post unsatisfied buyers, D1 ensures that there is no ex-post incentive for unsatisfied buyers to buy from the high quality sellers who were not matched with any buyer.

trade does not take place). In other words, when competition is strong, there is no equilibrium price system that is both informative and would allow high quality goods to be traded.

These observations lead us to the the following conclusion. In lemon markets, whenever competition among sellers is strong, the traditional role of prices as a device for competing against rival sellers impairs the effectiveness of prices as a device for conveying information to the buyers.

Wilson (1980) first argued that, in a market for lemons with price setting sellers, trade may occur at a distribution of prices rather than at a unique price. The results of our analysis have precise implications regarding the conditions under which a distribution of prices should arise in a lemon market. In particular, the empirical prediction following propositions 1-3 is that the degree of price dispersion is inversely related to the competitive pressure faced by sellers. The stronger the competition, the less price dispersion and variety of trade we should observe, and viceversa. This implication holds not just in the case of a market with two types of sellers and two qualities of goods, but also in the general case of any finite number N of qualities, which is analyzed in the appendix.

7 Welfare

It is well established that asymmetric information could have negative effects on the level of welfare generated by a market. In this section we assess the degree of inefficiency associated with the D1-robust equilibria in the market with strategic price setting sellers (MSPS). For this purpose, we compare these equilibria with the equilibrium that would arise in a perfectly competitive market (MPC). We also compare the D1-robust equilibria of the MSPS with the equilibrium outcome of the standard textbook case of a competitive market for lemons where trade occurs at a single price set by an auctioneer (MA). An equilibrium in the MA is a price at which demand equals supply. We assume that, in the presence of multiple equilibria, the auctioneer is always able to select the equilibrium that maximizes trade. This further comparison is meant to shed light on how strategic price setting affects the adverse selection mechanism.

Asymmetric information leads to two potential sources of inefficiency: the quality of trade could be inefficient and so could the quantity. As already noted, in the MSPS there are always some high quality sellers who are unable to trade, even when they are relatively less numerous than buyers. Hence, the MSPS would always lead to an inefficient quality of trade, compared to the MPC, if the gains from trading quality h exceed those of trading quality l. Such an inefficiency would instead disappear if gains from trading quality l were to exceed those related to quality h. Furthermore, since in the MSPS high quality sellers become rationed when demand is sufficiently high, such a model could also lead to an inefficient amount of trade, compared to the MPC.[13] This inefficiency, however, arises if and only if demand is sufficiently strong ($\theta > 1 - \lambda + \delta\lambda$).

These results emerge from the comparison between the level of welfare generated by the MPC relative to the maximum market size, S, as measured by

$$W_{FI} = \begin{cases} GFT_l \min(\theta, 1) - \Delta GFT \max(0, \min(\theta, 1) - (1 - \lambda)) & \text{if } \Delta GFT \geq 0 \\ GFT_l \min(\theta, 1) - \Delta GFT \min(\theta, \lambda) & \text{if } \Delta GFT < 0. \end{cases} \tag{24}$$

and the same measure of welfare for the MSPS, which, from proposition 3, is

$$W = GFT_l \min(1 - \lambda + \lambda\delta, \theta) - \Delta GFT \max(\min(\lambda\delta, \theta - (1 - \lambda)), 0). \tag{25}$$

The results from such a comparison are shown in figure 3, where the relative measure of welfare loss implied by asymmetric information, $w = 1 - W/W_{FI}$, is plotted against θ for the two cases: a. $\Delta GFT > 0$, b. $\Delta GFT < 0$.[14]

The magnitude of the inefficiencies generated by asymmetric information is affected by the price setting sellers convention in a non trivial way. Comparing the MSPS with the MA we find that the two models generate the same outcome if competition among sellers is strong, i.e. if $\theta < 1 - \lambda$. If competition among sellers is weak, the two models lead to quite different implications. In the MSPS, trade occurs in a perfectly separating equilibrium, where prices are perfect signals of quality. In the MA, since the auctioneer calls only one price, both qualities are traded if and only if a pooling equilibrium exists. This difference between the two models leads to profound differences with regard to welfare implications.

Generally, no pooling equilibrium in the MA is possible if $1 - \lambda \geq \theta$. For $1 - \lambda < \theta$, a necessary condition is:

$$\lambda u(h) + (1 - \lambda) u(l) \geq v(h) \tag{26}$$

This condition is also sufficient for $\theta \geq 1$. For $1 - \lambda < \theta < 1$, a pooling equilibrium would always be characterized by a price $p^* = v(h)$ and a quality of trade such that, while all low quality sellers sell their goods, only a fraction $[\theta - (1 - \lambda)]/\lambda$ of high quality sellers chooses to sell.[15] Correspondingly, a buyer would obtain a low quality good with probability $(1-\lambda)/\theta$ and a high quality good with probability $[\theta - (1-\lambda)]/\theta$. Therefore, given $1 - \lambda < \theta < 1$, the necessary and sufficient condition for a pooling equilibrium is

$$u(l)\frac{1 - \lambda}{\theta} + u(h)\frac{\theta - (1 - \lambda)}{\theta} \geq v(h) \tag{27}$$

[13] Note that the quantity of trade could never be inefficiently high due to the assumption that the gains from trade for each of the two qualities are both positive.

[14] In figure 3.b, we assume $\lambda > 1/(2 - \delta)$.

[15] Market clearing occurs if and only if all low quality sellers are able to sell. Hence, the fraction of high quality goods traded must be $\theta - (1 - \lambda)$.

where the LHS equals the willingness to pay of an individual buyer, which is an increasing function of θ. Conditions and (26) and (27) require that buyers' maximum willingness to pay is sufficiently high to sustain a pooling equilibrium. These conditions are clearly not necessary in the MSPS. Hence, in a way, it is less likely that high quality sellers are completely driven out of the market as a consequence of adverse selection in the MSPS. This suggests that the MSPS might generate a higher level of welfare than the MA. Indeed, if $u(l)(1-\lambda)+\lambda u(h) < v(h)$ the welfare generated by the MA as a function of θ would be

$$W_W = GFT_l \min(\theta, 1-\lambda). \tag{28}$$

Figure 4, case a, plots the relative welfare loss function $w_w = 1 - W/W_W$ against θ. The level of welfare generated by the MSPS is always greater than that of the MA when demand is sufficiently strong. Otherwise, the two models yield the same welfare. Differently, when buyers maximum willingness to pay is high enough, i.e. $u(l)(1-\lambda)+\lambda u(h) \geq v(h)$, the MA could lead to a higher level of welfare compared to the MSPS. In particular, the MA would guarantee more trade whenever market conditions are such that high quality sellers are rationed in the MSPS. Given (27) and $u(l) < v(h)$, the existence of a pooling equilibrium requires that demand be sufficiently strong, i.e.

$$\theta \geq \underline{\theta} \equiv (1-\lambda)\frac{u(h)-u(l)}{u(h)-v(h)}. \tag{29}$$

Accordingly, the welfare generated by the MA is

$$W_W = \begin{cases} GFT_l \min(\theta, 1-\lambda) & \text{if } \theta < \underline{\theta} \\ GFT_l \min(1,\theta) - \Delta GFT \min(\theta-(1-\lambda), \lambda) & \text{if } \theta \geq \underline{\theta} \end{cases} \tag{30}$$

Figure 4, case b, plots the function $w_w = 1 - W/W_W$, which measures the welfare loss of the MSPS compared to the MA. Independently of whether ΔGFT is positive or negative, the two models yield the same level of welfare if demand is sufficiently low ($\theta < 1-\lambda$). For intermediate values of demand, i.e. $\theta \in [1-\lambda, \underline{\theta}]$, the MSPS guarantees higher welfare than the MA. Given these market conditions, high quality sellers are able to sell with positive probability under the MSPS while they would be driven out of the market in the MA. If demand is stronger, so that $\theta > \underline{\theta}$, then which model yields the highest level of welfare depends on whether high quality sellers in the MSPS are rationed or not. In particular, both models yield the same welfare if θ is such that no rationing takes place in the MSPS, while the MA outperforms the MSPS if demand is sufficiently strong.

The ambiguous welfare effects of the MSPS compared to MA suggest that, in general, the MSPS fails to achieve the second best. The rationale lies in the instability of pooling equilibria established with lemma 2. As argued above, pooling equilibria may improve welfare if gains from trading quality h are higher or demand is particularly strong. However, in the MSPS, high quality sellers would always deviate such equilibria as buyers tend to associate higher quality with prices higher than those announced by the crowd. The ability of high quality sellers to communicate the higher quality of the goods they own by means of prices could result in an aggregate welfare loss. It then follows that suppressing the information conveyed by individual prices might actually improve welfare.

Muthoo and Mutuswami (2005) use mechanism design to characterize the maximal welfare attainable in a lemon market abstracting from the specific trading rule. They study how this varies with the degree of competition. We have focused on a specific price convention that is frequently observed in real world markets (e.g. online markets). Our aim is to understand how, within these real world markets, the role of prices as a communication device for high quality sellers changes with the competitive pressure faced by the sellers. Why such price convention seems to characterize many of the internet markets, even though it does not always attain the second best, is certainly an interesting question which deserves further scrutiny. We take the view put forward by Bester (1993). A posted price convention tends to endogenously emerge in markets with asymmetric information, as opposed to bargaining for instance, when the search costs faced by the buyers become small.

8 Conclusions

This paper tackled the issue of strategic pricing in a competitive market for lemons where the potential gains from trade are always positive. Sellers' pricing decisions are affected by two types of considerations. On the one hand, sellers want to maximize the chance to find a buyer. On the other hand, they may want to use prices to conceal/reveal their true quality. Thus, in markets for lemons, pricing decisions retain a double function. Sellers may lower prices to undercut competitors or increase them to signal high quality. We argue that these two roles of prices may be at odds with each other. When competition among sellers is strong, the use of prices to compete with other sellers prevails. Announcing a price that is higher than the price at which trade occurs in the market does not help to be recognized as a high quality seller by the buyers. The reason is that profits of all sellers are driven to zero by competition. Hence, all sellers, irrespectively of their quality, have no opportunity cost of deviating and announcing a higher price.

By contrast, when competition is weak, announcing a price higher than the lowest price at which trade occurs conveys relevant information to the buyers. The rationale is that in this case at least some of the sellers make positive profits. Thus, announcing a higher price (which harms the seller by reducing the likelihood of making a sale) is relatively more costly for low quality sellers.

The model generates various predictions, some of which are empirically relevant.

First, the degree of price dispersion is inversely related to the degree of competition among sellers, as measured by the sellers to buyers ratio. We should observe concentration of trade around few low prices when competition is strong, whereas trade should spread upon a distribution of relatively dispersed prices when competition is weak. Second, the average quality traded in the market should increase (although in a nonlinear fashion) as competition decreases. This is because weak competition allows high quality sellers (who would be driven out of the market in the presence of fierce competition) to sell their goods. Finally, when competition is weak, we identify the presence of a quality paradox. The more the best quality is appreciated by the potential buyers, the lower the amount that is traded in equilibrium.

A Appendix

Omitted Proofs

Proof of lemma 1

Clearly, whenever $J^l > J^h$, $R_1(l|p) \subseteq R_2(h|p)$ follows. Also, $J^l > J^h$ is necessary as $J^l \leq J^h$ would imply $R_2(h|p) \subseteq R_1(l|p)$. Therefore, in order to prove the result it is sufficient to show that, for any p such that $R_2(h|p)$ is non-empty and $J^l > J^h$, there always exists a profile α^1 such that $J(p, \alpha^0, \alpha^1) \in (J^h, J^l)$ and $\alpha^1 \in MBRP(p^0)$, where p^0 is a sellers' action profile which contains the deviation p. The result is proved by contradiction. Suppose that there is no $\alpha^1 \in MBRP(p^0)$ such that $J(p, \alpha^0, \alpha^1) \in (J^h, J^l)$. If $J(p, \alpha^0, \alpha^1) \leq J^h \ \forall \alpha^1 \in MBRP(p^0)$ then $R_2(h|p)$ is empty by definition. Hence, there must be some $\hat{\alpha}^1 \in MBRP(p^0)$ such that $J(p, \alpha^0, \hat{\alpha}^1) \geq J^l$. Note that, since $J^l > J^h$, $J(p, \alpha^0, \hat{\alpha}^1)$ must be strictly greater than zero. Hence, there is at least one buyer, say b, who selects p with positive probability. Let $\tilde{\alpha}^1$ be a profile constructed from $\hat{\alpha}^1$ as follows. Impose that all buyers different from b select p with probability zero. Note that, when beliefs are given by $\sigma(h|p) = 0$, all mixed best responses select any $p > v(h)$ with probability zero. As for buyer b, replace its strategy $\hat{\alpha}^1_b$ with a strategy $\tilde{\alpha}^1_b$ in which p is selected with probability j, where j can be any number in $[0, 1]$. Note that if selecting p with positive probability is comprised in a best response for some beliefs, then one can always find beliefs such that selecting p with any probability $j \in [0, 1]$ is comprised in a best response. Clearly, there exist $\tilde{\alpha}^1$ which satisfy the above restrictions that are in $MBRP(p^0)$. This implies that for any $j \in [0, 1]$, there exists a profile $\alpha^1 \in MBRP(p^0)$ such that $J(p, \alpha^0, \alpha^1) = j$. Since one can always choose a $j \in (J^h, J^l)$, the hypothesis that there there is no $\alpha^1 \in MBRP(p^0)$ such that $J(p, \alpha^0, \alpha^1) \in (J^h, J^l)$ is contradicted. Therefore, $R_1(l|p) \subset R_2(h|p)$. Finally, $J^l \equiv \frac{\pi^*_s(l)}{p - v(l)}$ and $J^h \equiv \frac{\pi^*_s(h)}{p - v(h)}$ are derived from equations (8) and (10). □

Proof of lemma 2

We first consider pooling equilibria (PE), then Hybrid Equilibria (HE). In any PE of type I there is a single equilibrium price p^* at which both high and low qualities are traded. Hence, all sellers have the same probability to sell, $J^*(p^*)$. Equilibrium profits are $\pi^*_s(q) = J^*(p^*)[p^* - v(q)]$, with $q \in \{l, h\}$. Buyers' payoff is $K^*(p^*)\{\lambda[u(h) - p^*] + (1 - \lambda)[u(l) - p^*]\}$. Since buyers' payoff must be non-negative a necessary condition for a PE is $p^* \leq \lambda u(h) + (1 - \lambda)u(l)$.

Conditions (13) and (14) become

$$\frac{p^* - v(l)}{p - v(l)} \leq \frac{p^* - v(h)}{p - v(h)} \Rightarrow v(h)(p - p^*) \leq v(l)(p - p^*) \qquad \text{(A.1)}$$

$$u(h) - p > K^*(p^*)\{\lambda[u(h) - p^*] + (1 - \lambda)[u(l) - p^*]\} \tag{A.2}$$

Accordingly, a PE is robust if (A.1) holds for any p that satisfies (A.2).

We note that p^* must always be strictly lower than $u(h)$, otherwise buyers would make a loss. Condition (A.1) is never verified for $p > p^*$ so that, for deviations above the pooling equilibrium price, beliefs are such that the seller who deviated is of high quality. As for condition (A.2), buyers prefer to buy at p as long as p is lower than $p^* + \eta$, where:

$$\eta \equiv (1 - \lambda)K^*(p^*)[u(h) - u(l)] + (1 - K^*(p^*))[u(h) - p^*] > 0. \tag{A.3}$$

Since $\eta > 0$, there always exist deviations $p \in (p^*, p^* + \eta)$ which cause the equilibrium to unravel.

The same argument given for pooling equilibria applies to hybrid equilibria (HE). In any HE, there is always a type $q \in \{l, h\}$ who announces at least two different prices with positive probability. This implies that, given a seller's type q, the expected payoff, $\pi^*(q)$, must be the same at all prices announced by type q.

By definition, in any HE there is always a price p^* that is announced by both types of sellers. Note also that, in a HE of type I, $J^*(p^*) > 0$ must hold, since a necessary condition for quality h to be traded is that type l sellers make positive profits. Since $\pi^*(q)$ is the same at all prices announced by type q, in order to asses the robustness, one can just focus on the incentives to deviate from p^*. The equivalent of (A.2) for HE is

$$u(h) - p > K^*(p^*)\{\sigma^*(h|p^*)[u(h) - p^*] + (1 - \sigma^*(h|p^*))[u(l) - p^*]\}, \tag{A.4}$$

while condition (A.1) stays unchanged. Since pooling occurs at p^*, $\sigma^*(h|p^*)$ must be strictly lower than one. But then, any $p \in (p^*, p^* + \tilde{\eta})$, with

$$\tilde{\eta} \equiv (1 - \sigma^*(h|p^*))K^*(p^*)[u(h) - u(l)] + (1 - K^*(p^*))[u(h) - p^*] > 0 \tag{A.5}$$

would cause the equilibrium to unravel. □

Proof of lemma 3

Consider a deviation $p > v(h)$. Rewrite condition (19) as:

$$\begin{aligned} p[J^*(p_l)[p_l - v(l)] - J^*(p_h)[p_h - v(h)]] \leq \\ v(h)J^*(p_l)[p_l - v(l)] - v(l)J^*(p_h)[p_h - v(h)] \end{aligned} \tag{A.6}$$

and the ICC of low quality sellers as a strict equality:

$$J^*(p_l)[p_l - v(l)] = J^*(p_h)[p_h - v(l)] + \xi \tag{A.7}$$

where $\xi \geq 0$. Inspection of (A.6) and (A.7) reveals that they cannot hold simultaneously when $p > p_h$. Hence, high quality sellers could always signal themselves by deviating to p. If $J^*(p_h) > 0$, condition (20) can be rewritten as:

$$u(h) - p > K^*(p_h)[u(h) - p_h] \tag{A.8}$$

This implies that a necessary condition for a SE to be robust is $K^*(p_h) = 1$ (Notice that $K^*(p_h) = 1$ trivially holds whenever $J^*(p_h) = 0$). Otherwise, there would always be a deviation $p = p_h + \epsilon$, with $\epsilon > 0$ small enough, such that buyers would strictly prefer to buy at p since the probability to buy would be one. Given $K^*(p_h) = 1$, condition (20) reduces to $p < p_h$: buyers could only be interested in buying if a seller deviates to a price lower than p_h. We now distinguish the case in which $p_h \leq u(h)$ from the case in which $p_h > u(h)$. Consider the first case. From low quality sellers ICC, the LHS of (A.6) is nondecreasing in p. Thus, it is sufficient to check whether (A.6) holds for $p = p_h$:

$$\begin{aligned} p_h[J^*(p_l)[p_l - v(l)] - J^*(p_h)[p_h - v(h)]] \leq \\ v(h)J^*(p_l)[p_l - v(l)] - v(l)J^*(p_h)[p_h - v(h)] \end{aligned} \tag{A.9}$$

Inspection of (A.9) and (A.7) shows that $\xi = 0$ unless $p_h = v(h)$. Consider now the case $p_h > u(h)$. It is clear that $J^*(p_h)$ must be zero in this case. Otherwise, the buyers' participation constraint would be violated. Condition (19) becomes $J^*(p_l)[p_l - v(l)] = 0$, while low quality sellers ICC is $J^*(p_l)[p_l - v(l)] = \xi$; $\xi = 0$ follows. □

Proof of lemma 4

Assume $1 - \lambda \geq \theta$ and consider a SE of type I, i.e. both $J^*(p_h)$ and $J^*(p_l)$ are greater than zero. If $1 - \lambda \geq \theta$ there is at least a low quality seller for every buyer. Hence, $J^*(p_h) > 0$ implies $J^*(p_l) < 1$. Undercutting implies $p_l = v(l)$. If p_l were greater than $v(l)$, each individual seller would have incentive to deviate and announce $p_l - \epsilon$ where $\epsilon > 0$ is sufficiently small. Such a small reduction in the price would result in a positive change in the level of profits, since the probability to sell jumps from $J^*(p_l)$ to 1. The reason is that, upon observing the deviation, there would always be at least a buyer who would be willing to buy at $p_l - \epsilon$ since her off-equilibrium beliefs cannot assign a quality lower than l to the seller who is deviating. The ICC of type l sellers then implies $J^*(p_h) = 0$, i.e. the high quality is never traded in equilibrium, so there is no SE of type I.

Let us now consider the case $1 - \lambda < \theta$. We first show that in any robust SE with $p_q < u(q)$ $\forall q \in \{h, l\}$, $J^*(p_h) > 0$ must hold [*Step 1*]; i.e. SE with $p_q < u(q)$ are of type I. Then, using lemma 3, we prove that $J^*(p_h) > 0$ also holds in SE where, for some $q \in \{l, h\}$, $p_q = u(q)$, i.e. these SE are of type I [*Step 2*].

Step 1. If $p_q < u(q)$, buyers make a positive surplus and therefore no buyer ever chooses not to buy. As explained in the proof of lemma 3, robustness requires that $K^*(p_h)$=1. Ex post, the amount of quality h goods sold must be equal to the amount bought. When buyers make positive surplus, the amount of quality h goods being bought is given by the total number of buyers minus the number of buyers willing to buy at p_l. Then, $K^*(p_l)$ and $J^*(p_h)$ must satisfy:

$$\lambda J^*(p_h) = \theta - \frac{(1-\lambda)}{K^*(p_l)}. \tag{A.10}$$

Where all quantities are relative to the total number of goods (sellers), S, available. We note that $p_l < u(l)$ implies $K^*(p_l) = 1$. If $K^*(p_l)$ were less than 1, low quality sellers would increase their profits by announcing a price $p = p_l + \epsilon,\ \epsilon > 0$ but sufficiently small. Unless $p_l = u(l)$, this deviation would induce a buyer to prefer p irrespectively of her off-equilibrium beliefs, since she would obtain the good at p with probability $1 > K^*(p_l)$. Substituting for $K^*(p_l) = 1$ into (A.10) yields

$$J^*(p_h) = \frac{\theta - (1-\lambda)}{\lambda} \tag{A.11}$$

which is strictly greater than zero for $\theta > (1-\lambda)$.

Step 2. Assume first $p_l = u(l)$. Then, if $p_h > v(h)$, the ICC of low quality sellers must hold with equality. But then $J^*(p_h) > 0$ must be satisfied since $J^*(p_l) > 0$ in any equilibrium. Buyers' ICC implies that in this case p_h equals $u(h)$. On the other hand, $p_h = v(h)$ can never be a SE if $p_l = u(l)$ as buyers would always prefer p_h. Assume now $p_h = u(h)$. If $J^*(p_h)$ were zero, lemma 3 would ensure either $p_l = v(l)$ or $J^*(p_l) = 0$. Clearly, the second never holds. As for $p_l = v(l)$, it is never an equilibrium if low quality sellers are the short side of the market $(1-\lambda < \theta)$. In this case any type l seller could deviate and announce a price slightly higher than p_l and still be able to make a sale with probability one. Therefore, $J^*(p_h) > 0$ must hold. □

Proof of proposition 1

Lemmata 2 and 4 imply that $1-\lambda < \theta$ is necessary for a robust type I equilibrium. From lemma 2, robust equilibria of type I are separating. From lemma 4, there is no SE of type I if $1-\lambda \geq \theta$. The following characterization of D1-robust equilibria of type I shows that $1-\lambda < \theta$ is also sufficient. When $1-\lambda < \theta$, the following statements must be true in any robust SE of type I:

Statement 1. $J^*(p_l) = 1$ *and* $K^*(p_h) = 1$ *always hold.* Any situation in which $J^*(p_l) < 1$ and $p_l > v(l)$ cannot be an equilibrium. The reason is that type-l sellers have an incentive to undercut their competitors. A full account of this undercutting argument has been provided in the proof of lemma 4. On the other hand, $p_l = v(l)$

would violate would violate the ICC of type l. $K^*(p_h) = 1$ has already been established in the proof of lemma 3.

Statement 2. $p_l = u(l)$ and $p_h = u(h)$ hold whenever $K^*(p_l) < 1$. If $K^*(p_l) < 1$ and $p_l < u(l)$, a type l seller could profit from announcing $p = p_l + \epsilon$. For $\epsilon > 0$ but sufficiently small, buying quality l at p with probability 1 is better than buying quality l at p_l with probability $K^*(p_l)$. Therefore, there is always a buyer who would profit from buying at p (note that buyers' off-equilibrium beliefs cannot assign a quality lower than l to the seller who deviates). Given $p_l = u(l)$, $p_h = u(h)$ follows from buyers' ICC.

Statement 3. $K^*(p_l)$ and $J^*(p_h)$ always satisfy:

$$J^*(p_h) \leq \frac{\theta K^*(p_l) - (1-\lambda)}{\lambda K^*(p_l)}. \tag{A.12}$$

which holds with strict equality if buyers make positive surplus. The result follows from *statement 1* and the observation that, ex post, the fraction of quality h goods sold ($\lambda J^*(p_h)$) cannot exceed the maximum fraction that can be bought (see equation (A.10) in the proof of lemma 4). If buyers make positive surplus, then no buyer plays the strategy of not buying and (A.12) holds with equality.

Endowed with these results, we turn to the particular cases:

Case 1: $\theta > \hat{\theta}$. Assume $K^*(p_l) = 1$. Then if buyers make positive surplus, *statement 3* implies

$$J^*(p_h) = \frac{\theta - (1-\lambda)}{\lambda}, \tag{A.13}$$

However, from *statement 1*, buyers ICC, and type l sellers ICC one obtains:

$$J^*(p_h) \leq \frac{p_l - v(l)}{u(h) - v(l) - (u(l) - p_l)}, \tag{A.14}$$

which implies $J^*(p_h) \leq \delta$ for $p_l \leq u(l)$. By combining equation (A.13) with $J^*(p_h) \leq \delta$, one would obtain $\theta - (1-\lambda) \leq \lambda\delta$ or $\theta \leq \hat{\theta}$, which would be a contradiction. Thus, buyers make zero surplus. *Statement 2* ensures that buyers make zero surplus also when $K^*(p_l) < 1$. It follows that $p_l = u(l)$ and $p_h = u(h)$ must hold and lemma 3 implies $J^*(p_h) = \delta$.

Case 2: $\theta_\gamma < \theta \leq \hat{\theta}$. Assume $K^*(p_l) < 1$. Then $p_q = u(q)\ \forall q$ follows from *statement 2*. Lemma 3 implies $J^*(p_h) = \delta$. But then *statement 3* requires:

$$(\theta - \lambda\delta)K^*(p_l) \geq 1 - \lambda \tag{A.15}$$

which, for $\theta \leq \hat{\theta}$, would imply $K^*(p_l) = 1$ thus contradicting the premise. Therefore, $K^*(p_l) = 1$. By the same token, $p_q < u(q)$, $\forall q$ unless θ is exactly equal to $\hat{\theta}$. The argument goes as follows. If buyers make positive surplus, *statement 3* implies equation

(A.13). Consider first the case $\theta = \hat{\theta}$. Equation (A.13) yields $J^*(p_h) = \delta$. From the ICC of buyers and type l sellers, $p_h \geq u(h)$ would follow. Thus, buyers must make zero surplus ($p_q = u(q)$). This in turn requires $J^*(p_h) = \delta$ from lemma 3 and type l ICC. Consider now the case $\theta < \hat{\theta}$. If $p_q = u(q)$ lemma 3, buyers' ICC, and $K^*(p_l) = 1$ ensure that $J^*(p_h) = \delta$. However, it is easy to check that this would violate *statement 3* given $\theta < \hat{\theta}$. Thus, $p_q < u(q)$, $\forall q$. Then (A.13) applies. If $p_h > v(h)$, then prices can be found by replacing (A.13) into the following equations:

$$p_h = \frac{u(h) - [u(l) - v(l) + v(l)J^*(p_h)]}{1 - J^*(p_h)} \tag{A.16}$$

$$p_l = \frac{v(l) + J^*(p_h)[u(h) - v(l) - u(l)]}{1 - J^*(p_h)}. \tag{A.17}$$

These follow from *statement 1*, lemma 3 (type l ICC with equality), and the ICC of buyers when $K^*(p_l) = 1$. Simple algebra shows that they do not violate any participation constraint when $J^*(p_h)$ is given by (A.13) and $\theta_\gamma < \theta \leq \hat{\theta}$. What remains to show is that $p_h > v(h)$ for $\theta > \theta_\gamma$. Assume $p_h = v(h)$, then, from buyers' ICC $p_l = u(l) - GFT_h$. It is then immediate to check that type l sellers ICC and (A.13) would imply $\theta \leq \theta_\gamma$ which is a contradiction.

Case 3: $\theta \leq \theta_\gamma$. The same arguments as in case 2 can be used to claim that $K^*(p_l) = 1$ and to rule out $p_q = u(q)$. The only difference here is that $p_h = v(h)$ must hold. To see this, assume $p_h > v(h)$. Buyers' ICC implies $p_l > u(l) - GFT_h$. At the same time equation (A.13) and lemma 3 imply that type l ICC can be written as:

$$p_l = v(l) + \frac{\theta - (1 - \lambda)}{\lambda}[v(h) - v(l)] \tag{A.18}$$

Substituting this expression for p_l into $p_l > u(l) - GFT_h$ and solving for θ yields $\theta > \theta_\gamma$ which is a contradiction. Therefore, $p_h = v(h)$, which implies $p_l = u(l) - [u(h) - v(h)]$. Finally, since buyers make positive surplus *statement 3* implies that $J^*(p_h)$ is given by (A.13).

To complete the characterization, note that, by construction, the equilibrium outcome is sustained by robust off-equilibrium beliefs. Therefore, $1 - \lambda < \theta$ is sufficient for a D1-robust equilibrium. □

Proof of proposition 2

We start by showing that all trade occurs at a unique price p^* in all type II equilibria. Then, we characterize the equilibrium outcomes of all type II equilibria in terms of the price at which trade occurs and analyze beliefs that support the existence of these equilibria. Next, we show that equilibria of type II always fail D1 if $1 - \lambda < \theta$ and that there is an equilibrium outcome that passes D1 if $1-\lambda > \theta$. The special case $1-\lambda = \theta$ is then considered. Finally, we characterize the amount of trade.

a. Uniqueness of price. Suppose trade occurs at more than one price. We prove the result for the case of two different equilibrium prices p' and p'' with $p'' > p'$. The same argument applies to any number of prices higher than one. Clearly, p', p'' could be a pair of equilibrium prices if and only if profits for type l sellers were the same at the two prices, which would imply that $J^*(p'') < J^*(p')$. Otherwise, announcing p' would be dominated by p''. It follows that $J^*(p'')$ must be lower than 1. But then, any seller announcing p'' would profit from deviating to $p''-\epsilon$, where ϵ is greater than zero but small enough, and attract buyers willing to buy at p''. Notice that following such a deviation, at least one buyer, say $b \in \mathcal{B}$, will always be induced to buy at $p''-\epsilon$ because: *a.* her beliefs are that sellers announcing p'' are of quality l and her off-equilibrium beliefs cannot assign a quality lower than l to the seller who is announcing $p''-\epsilon$; *b.* if no other buyer is willing to buy at $p''-\epsilon$, buyer b would be able to buy with probability one at $p''-\epsilon$. Since there is always at least one buyer willing to buy at $p''-\epsilon$, a deviation from p'' to $p''-\epsilon$ causes a discrete positive jump of the probability to sell from $J^*(p'')$ to 1. Therefore, the only possible equilibrium outcome implies a single price p^*.

b. Equilibrium price. First note that in every type II equilibrium p^* necessarily lies in the interval $[v(l), u(l)]$. If $(1-\lambda) > \theta$, sellers of type l are the long side of the market, and $J^*(p^*) \leq \theta/(1-\lambda)$. In this case, any $p^* > v(l)$ is never an equilibrium. A seller would have an incentive to deviate and announce $p^*-\epsilon$ where $\epsilon > 0$ is sufficiently small, as this would result in a positive jump in expected profits since to the probability of selling jumps from $J^*(p^*) \leq \theta/(1-\lambda)$ to 1. Again, note that this deviation is profitable whatever buyers' beliefs are. Undercutting leads to $p^* = v(l)$. Clearly, type h sellers must announce a price such that buyers prefer to buy at p^*.

Consider now the case $(1-\lambda) < \theta$. We start by showing that $p^* = u(l)$ and $J^*(p^*) = 1$. If $p^* < u(l)$, buyers make positive surplus and, therefore, are all willing to buy at p^*. Given $1-\lambda < \theta$, there is excess demand at p^*. A type l seller would then profit from announcing $p = p^* + \epsilon$, with $\epsilon > 0$ but sufficiently small. This would always induce a buyer to select p (independently of her off-equilibrium beliefs) since she would be able to buy at p with probability 1. Hence, $p^* = u(l)$. At this price, type l sellers must have no incentive to undercut. This implies $J^*(p^*) = 1$. Note that type h sellers must announce a price $p_h \geq u(h)$. The reason is that $K^*(p^*)[u(l)-p^*] \geq K^*(p_h)[u(h)-p_h]$ should hold in any equilibrium of type II. As long as type h sellers announce p_h, $K^*(p_h) > 0$. Given $K^*(p^*) \leq 1$, $p_h < u(h)$ would imply $p^* < u(l)$ which cannot be an equilibrium. Thus, if $1-\lambda < \theta$, $p^* = u(l)$ must hold and high quality sellers never announce a price less than $u(h)$.

Finally, in both cases, there must be a class of off-equilibrium beliefs such that none has any incentive to deviate. It is easy to verify that such beliefs exist. For instance, beliefs assigning $\sigma^*(h|p) = 0$ to any seller announcing an off-equilibrium price p sustain the equilibrium. The next step shows that beliefs which sustain equilibria of type II are robust if and only if $1 - \lambda \geq \theta$.

c. Robustness. It is immediate to show that type II equilibria satisfy D1 when $1 - \lambda > \theta$. Inspection of condition (13) yields $0 \geq 0$ for any p. By contrast, type II equilibria where $1 - \lambda < \theta$ always fail D1. In equilibrium, low quality sellers make profits $\pi^*(l) = u(l) - v(l)$ while high quality sellers make zero profits. Condition (13) becomes, for any $p > v(h)$:

$$\frac{u(l) - v(l)}{p - v(l)} \leq 0, \tag{A.19}$$

which is not met by any $p > v(h) > v(l)$. Since buyers make zero surplus, condition (14) becomes $u(h) - p > 0$. Any deviation $p \in (v(h), u(h))$ would therefore cause the equilibrium to unravel.

d. Case $1 - \lambda = \theta$. Note that also in this case the equilibrium is characterized by a unique price $p^* \in [v(l), u(l)]$. For a deviation $p > v(h)$, condition (13) becomes:

$$\frac{p^* - v(l)}{p - v(l)} \leq 0. \tag{A.20}$$

Condition (14) is:

$$u(h) - p > u(l) - p^*. \tag{A.21}$$

In this special case, it is necessary to distinguish between $\gamma \leq 0$ and $\gamma > 0$, as in proposition 1. Consider first the case $\gamma \leq 0$ (which implies $u(l) - v(l) \leq u(h) - v(h)$). The only D1 robust equilibrium of type II is such that $p^* = v(l)$. Condition (A.20) does not hold whenever $p^* > v(l)$. Given $\gamma \leq 0$, for any $p^* > v(l)$ it is possible to find a $p > v(h)$ such that (A.21) is satisfied. This implies that buyers prefer to buy at p. Hence, $p^* = v(l)$. Consider now the case $\gamma > 0$. Condition (A.20) is still violated whenever $p^* > v(l)$. However, whether there is any $p > v(h)$ such that (A.21) holds now depends on p^*. A $p > v(h)$ such that (A.21) holds exists only if $p^* > u(l) - [u(h) - v(h)]$. Hence, robustness to D1 requires $p^* \leq u(l) - [u(h) - v(h)]$. In principle, any $p^* \in [v(l), u(l) - [u(h) - v(h)]$ can be an equilibrium price when $\gamma > 0$. The reason why D1 does not permit to select a price in this case is that, when $\gamma > 0$, a discontinuity arises at $\theta = 1 - \lambda$. To see this, consider the limit of a robust type I equilibrium for $\theta \to 1 - \lambda$. When $\gamma > 0$, the right limit of the expression for p_l given in proposition 1 selects $p^* = u(l) - [u(h) - v(h)]$. On the other hand, the left limit for $\theta \to 1 - \lambda$ of the price of a type II robust equilibrium selects $p^* = v(l)$.

e. Fraction of quality l traded. Given the above discussion, it is immediate to check that $K^*(p^*) = 1$ whenever $1 - \lambda \geq \theta$. Therefore, each buyer is able to obtain a unit of a quality l good. It follows that the fraction of quality l goods traded is $\frac{\theta}{1-\lambda}$. □

Proof of proposition 3

Case 1. $1 - \lambda > \theta$. From Proposition 2, there exist D1-robust equilibria of type II. In all of these equilibria, the price, p^*, at which trade occurs and the fraction of quality traded, $f(l)$, are uniquely determined by the model's exogenous parameters. Moreover, according to lemma 4 no equilibrium of type I exists. Hence, we conclude that for $1 - \lambda > \theta$ the equilibrium outcome is unique.
Case 2. $1 - \lambda < \theta$. From Proposition 2 there is no D1-robust equilibria of type II. On the other hand, according to proposition 1 there exist D1-robust equilibria of type I. Again, in all these equilibria, the prices at which trade occurs, p_q, and the fractions of high and low quality traded ($f(h)$ and $f(l)$ respectively) are uniquely determined by parameters. Thus, the equilibrium outcome is unique.
For completeness, we discuss the special case $1 - \lambda = \theta$. The proof of proposition 2 implies that, if $\gamma \leq 0$, the results under case 1 also apply to $1 - \lambda = \theta$. If $\gamma > 0$, a discontinuity arises at $1-\lambda = \theta$. While traded quantities are still uniquely determined, the price p^* experiences a jump from $v(l)$ to $u(l) - [u(h) - v(h)]$. $\square$

Robustness of type II equilibria to weaker refinements

In this section we analyze the robustness of type II equilibria to weaker refinements such as Divinity [Banks and Sobel (1987)]and Sequential Perfection [Grossman and Perry (1986)].

According to Divinity, off-equilibrium beliefs should satisfy the following restriction: whenever $R_1(l|p) \subset R_2(h|p)$, $\sigma(h|p) \geq \lambda$ must hold. Thus, if, for some deviation p, high quality sellers have incentive to deviate in any situation in which low quality sellers have a weak incentive to deviate, the posterior probability that the seller is of high quality should not decrease after observing p.

Using lemma (1), an equilibrium fails Divinity if, for some $p > v(h)$, the condition $J^l > J^h$ holds and there is some buyer who prefers to buy at p rather than at p^*, given her refined beliefs which assign probability greater than λ to type h. As already mentioned (see lemma 1), $J^l > J^h$ implies$\frac{\pi_s^*(l)}{p-v(l)} > \frac{\pi_s^*(h)}{p-v(h)}$. As for the buyers, they prefer to buy at p if:

$$\lambda(u(h) - p) + (1 - \lambda)(u(l) - p) >$$
$$K^*(p^*)\{\sigma^*(h|p^*)[u(h) - p^*] + (1 - \sigma^*(h|p^*))[u(l) - p^*]\}. \tag{A.22}$$

The following result illustrates the effect of imposing Divinity on type II equilibria:

Lemma 5. *a. Given $1 - \lambda \geq \theta$, there exist type II equilibria that are divine; b. Given $1 - \lambda < \theta$, there exist divine equilibria of type II if and only if $\lambda \leq \hat{\lambda} \equiv \frac{v(h)-u(l)}{u(h)-u(l)}$.*

Proof. Case a. Consider first the case in which $1 - \lambda \geq \theta$. The result here directly follows from the fact that divinity is weaker than D1. Hence, the first statement of proposition 2 holds.

Case b. Consider type II equilibria where $1-\lambda < \theta$ so that low quality sellers are the short side of the market. In equilibrium, high quality sellers make zero profits, $\pi_s^*(h) = 0$, while low quality sellers make profits $\pi_s^*(l) = u(l) - v(l)$. Thus, for $J(p, \alpha^0, \alpha^1) > 0$ and $p > v(h)$, condition $J^l > J^h$ becomes:

$$\frac{u(l) - v(l)}{p - v(l)} > 0, \tag{A.23}$$

which implies that, for any $p > v(h)$, high quality sellers need a lower probability J^h to be willing to deviate than low quality sellers. Also, buyers make zero surplus, such that, for a given deviation p, condition (A.22) becomes

$$\lambda(u(h) - p) + (1 - \lambda)(u(l) - p) > 0 \tag{A.24}$$

Since p must be greater than $v(h)$, the necessary and sufficient condition is

$$\lambda(u(h) - v(h)) + (1 - \lambda)(u(l) - v(h)) > 0, \tag{A.25}$$

Solving for λ yields the threshold:

$$\hat{\lambda} \equiv \frac{v(h) - u(l)}{u(h) - u(l)}. \tag{A.26}$$

So if $\lambda > \hat{\lambda}$ a type II outcome does not pass Divinity.

□

Since Divinity is weaker than D1, type II equilibria where low quality sellers are the long side of the market are always robust to Divinity. By converse, unlike D1, Divinity eliminates type II equilibria where low quality sellers are the short side of the market only when λ exceeds the critical value $\hat{\lambda}$. In other words, the fraction of high quality sellers in the population must be large. Consider now both conditions simultaneously: $\lambda \geq 1 - \theta$ (type l are the short side), and $\lambda \leq \hat{\lambda}$ (robustness). If $1 - \theta > \hat{\lambda}$, for any prior λ implying that type l are the short side, there is no equilibrium of type II that passes Divinity. To understand this implication, note that $\hat{\lambda}$ is increasing in $v(h) - u(l)$ and decreasing in $u(h) - u(l)$. Hence, if $v(h) - u(l)$ is small (announcing a price close to buyers' reservation price for the low quality is feasible for the high quality), or if the gap in terms of buyers' utility between the two qualities, $u(h) - u(l)$, is large, there is no divine equilibrium in which only the low quality is traded. This means that whenever high quality sellers are keen to sell their goods and buyers attach great value to quality, equilibria in which high quality sellers do not trade are not robust.

This discussion however does not apply, so that a type II divine equilibrium always exists, if the low quality sellers are relatively more numerous than buyers. In this case competition among the low quality sellers becomes so fierce that eventually drives the high quality out of the market.

An alternative approach is to define as reasonable beliefs those beliefs derived from updating rules that are *credible* in the sense put forward by Grossman and Perry (1986). Accordingly, we shall require that the beliefs following the observation of an off-equilibrium price satisfy the following restriction. Whenever there exists a set of qualities $Q \subseteq l, h$, such that if

1. Each type $q \in Q$ weakly benefits from the deviation if it is thought that a type $q \in Q$ deviated;
2. Types $q \notin Q$ weakly lose from the deviation if it is thought that a type $q \in Q$ has deviated;

then buyers should believe that a seller of quality $q \in Q$ has deviated.

Accordingly, we can then refine the concept of PBE by adding the credibility requirement for the beliefs updating rule to definition (1). Formally,

Definition 3. *A PBE is said to satisfy the sequential perfection restriction (SPR) if the belief function σ satisfies the following property. Let $Q \subseteq \{l, h\}$ be any set of qualities. For all Q such that, for some p that is announced with probability zero in equilibrium,*

$$\pi_s(p, \alpha^0_{-s}, \alpha^1 | q, \sigma) \geq \pi^*_s(q) \ \forall q \in Q \tag{A.27}$$

$$\pi_s(p, \alpha^0_{-s}, \alpha^1 | q, \sigma) \leq \pi^*_s(q) \ \forall q \notin Q \tag{A.28}$$

σ must satisfy

$$\sigma = \sigma(q|p) = \Pr(q)/\Pr(Q), \tag{A.29}$$

where $\Pr(.)$ *is the prior and* $\pi^*_s(q)$ *is the equilibrium payoff for a seller of type q.*

The effect of imposing such a restriction on type II equilibria is made clear by the following result:

Lemma 6. *a. Given $1 - \lambda \geq \theta$, there exist equilibria of type II that satisfy the SPR; b. Given $1 - \lambda < \theta$, there exist equilibria of type II that satisfy the SPR if and only if $\lambda \leq \hat{\lambda} \equiv \frac{v(h)-u(l)}{u(h)-u(l)}$.*

Proof. Case a. The equilibrium price is $p^* = v(l)$. Equilibrium profits are $\pi^*_s(h) = \pi^*_s(l) = 0$. Set $Q = \{h\}$ and $\sigma(h|p) = 1$. Then, provided that buyers have any incentive to buy, all sellers would profit from deviating to any $p \geq v(h)$, while high quality sellers would never deviate to any $p < v(h)$. Thus, $\sigma(h|p) = 1$ would necessarily be disconfirmed. Set now $Q = \{l\}$ and $\sigma(l|p) = 1$. In this case, for any $p > v(l)$, buyers' expected profits would be higher at p^* than at p (note that, in a large market where all low quality sellers announce the same price p^*, the deviation of a single seller does not affect the probability to obtain the good at p^*). Then, the best response is not to buy. Yet, nor type l neither type h are actually made worse or better off by announcing p. But then, since type l weakly benefits and type h weakly loses, $\sigma(l|p) = 1$ is credible.

Case b. The equilibrium price is $p^* = u(l)$. In equilibrium, low quality sellers make profits $\pi_s^*(l) = u(l) - v(l)$, while high quality sellers make zero profits and buyers zero surplus. Set $Q = \{h\}$, so that $\sigma(h|p) = 1$. For this to be the case, $p \geq v(h)$ must hold. Then, provided that $p < u(h)$ buyers best reply is to buy at p. But then, both types of sellers would profit from deviating so that $\sigma(h|p) = 1$ would be invalidated. Hence, $\sigma(h|p) = 1$ is not credible. Set $Q = \{l\}$ and $\sigma(l|p) = 1$. For any $p > u(l)$ the best reply is not to buy so that $\sigma(l|p) = 1$ is again disconfirmed and therefore is not credible. Finally, set $Q = \{h, l\}$. Then assume a deviation $p > v(h)$ (but close enough to $v(h)$). Then, $\Pr(Q) = 1$, $\Pr(h) = \lambda$, and $\Pr(l) = 1 - \lambda$. If $\lambda u(h) + (1 - \lambda)u(l) \geq p$, the best response is to buy following the deviation. All sellers have an incentive to deviate such that beliefs are confirmed. Thus, the equilibrium where $p^* = u(l)$ will be deviated whenever $\lambda u(h) + (1 - \lambda)u(l) > v(h)$. This implies that the equilibrium fails the SPR if $\lambda > \hat{\lambda}$, where $\hat{\lambda}$ is the same as in lemma (5). □

The result shows that the SPR selects the same set of type II equilibria as Divinity. Again, when low quality sellers are the long side, type II equilibria are robust. When they are the short side, the equilibrium satisfies the requirement of sequential perfection only if the prior probability to sample a high quality is greater than $\hat{\lambda}$. In this situation, all sellers have incentive to deviate to some $p \geq v(h)$ so that the only credible beliefs are $\sigma(l|p) = 1 - \lambda$ and $\sigma(h|p) = \lambda$. In other words, the credible beliefs upon observing a deviation are equal to the priors. So long as $\lambda > \hat{\lambda}$, buyers are willing to buy at p. Hence, the equilibrium will be deviated.

Extension to any finite number of qualities

This section generalizes the results concerning the set of equilibria robust to D1 to the case of a finite number $(N+1)$ of qualities. We show that, as in the case of two qualities, D1 guarantees separation at all prices at which trade occurs. The comparative statics for the general case are also consistent with the results obtained in the two qualities case. When θ is so low that buyers are relatively more numerous than sellers of the lowest quality, no quality other than the lowest is traded. Increases in the value of θ allow higher qualities to be traded until, for θ sufficiently large, all qualities are traded.

Qualities are indexed by $q = 0, ..., N$. Each seller's quality is drawn from a distribution $\lambda : \{0, 1, ..., N\} \to [0, 1]$, where λ_q, $\sum_{q=0}^{N} \lambda_q = 1$, denotes the probability associated with quality q. Buyers' posterior beliefs are denoted with $\sigma(q|p_s^0)$, $\sum_{q=0}^{N} \sigma(q|p_s^0) =$

1. We maintain the convention that agents who choose not to trade announce $\overline{p} > u(N)$ (sellers) or accept to buy at $p = 0$ (buyers).

Let us concentrate first on pooling and hybrid equilibria in which two or more types of sellers trade at the same price. In order to assess the robustness of these equilibria, we need to understand how the equivalents of conditions (13) and (14) look like. In any of these equilibria there always exists a price p^* at which a non-singleton non-empty set of qualities, $M \subseteq \{0, ..., N\}$, is traded. Let q_M be the highest quality in M. Take any quality $q \in M,\ q \neq q_M$. Given pooling at p^*, condition (13) becomes:

$$\frac{p^* - v(q)}{p - v(q)} \leq \frac{p^* - v(q_M)}{p - v(q_M)}, \tag{A.30}$$

which, given $v(q) < v(q_M)$, is violated for any $p > p^*$. Thus, robust beliefs should assign probability 0 to a deviation $p > p^*$ by any quality in M except for q_M. As for qualities $q \notin M$, the following applies. Sellers of qualities $q < q_M$ who do not trade at p^* make at least the same profits they would make at p^* by charging a different price (since they could always announce p^*). Since $v(q) < v(q_M)$, they should be assigned probability zero. Sellers of qualities $q > q_M$ should also be assigned probability zero so long as $p < v(q_M + 1)$. Thus, deviations $p^* < p < v(q_M + 1)$ are attributed to sellers of quality q_M with probability 1. As for buyers, the equivalent of condition (14) is:

$$u(q_M) - p > K^*(p^*) \sum_{q \in M} \sigma^*(q|p^*)[u(q) - p^*], \tag{A.31}$$

which is always true for p close enough to p^*. Thus deviating to a price slightly higher than p^* would allow sellers of type q_M to reveal their type and induce buyers to buy. Therefore, neither pooling nor hybrid equilibria in which two or more types trade at the same price survive D1.

The set of robust equilibria therefore includes only separating equilibria and hybrid equilibria with the necessary condition that each price at which trade occurs is announced by only one type of seller (i.e. pooling only occurs at $\overline{p}$). We now focus on these equilibria. If $\theta \leq \lambda_0$, the discussion made in the previous sections leads to the immediate conclusion that only quality 0 is traded.

By converse, when $\theta > \lambda_0$, sellers of quality 0 make positive profits and, therefore, higher qualities must be traded. In order to characterize these equilibria, we analyze the properties of the ICC. We start by showing that when the "adjacent upward" ICC is satisfied, all the ICC with respect to all higher qualities are satisfied. The relevant ICC for sellers is:

$$J(p_{q-s})[p_{q-s} - v(q-s)] \geq J(p_q)[p_q - v(q-s)] \tag{A.32}$$

for all q and $s = 0, ...q$. The usual undercutting argument implies that $J(p_0) = 1$ holds. This, together with (A.32) yields $J(p_q) < J(p_{q-1}) < ... < J(p_1) < 1$, whenever $p_q > p_{q-1} > ... > p_0$. From equation (A.32):

$$J(p_{q-1})[p_{q-1} - v(q-1)] \geq J(p_q)[p_q - v(q-1)] \tag{A.33}$$

and

$$J(p_q)[p_q - v(q)] \geq J(p_{q+1})[p_{q+1} - v(q)]. \tag{A.34}$$

Then, by using $v(q) > v(q-1)$ and $J(p_q) > J(p_{q+1})$, it follows that

$$J(p_{q-1})[p_{q-1} - v(q-1)] \geq J(p_{q+1})[p_{q+1} - v(q-1)] \tag{A.35}$$

always holds. Applying the same reasoning to qualities higher than $q+1$ shows that when the "adjacent upward" ICC are satisfied, all the ICC with respect to all higher qualities are satisfied.

The next step is to show, by applying D1, that the "adjacent upward" ICC of a given quality must hold with equality whenever the "adjacent upward" quality is traded. As explained in the proof of lemma 4, in any robust equilibrium in which buyers make a positive surplus, $K^*(p_q)$ must be 1 for any quality q that is traded in equilibrium. Then, buyers' surplus is constant across the traded quantities and their ICC becomes:

$$u(q) - p_q = k \ \forall q = 0, 1, ..., N, \tag{A.36}$$

where k is a constant parameter.

For $q > 0$, buyers may only be attracted by deviations $p < p_q$, since they are already able to buy quality q with probability 1 at p_q. This implies that one can restrict attention to deviations $p < p_q$. Note that D1 requires that type q should be assigned probability zero of deviating to price p, if there exists q' such that:

$$\frac{J(p_q)[p_q - v(q)]}{p - v(q)} > \frac{J(p_{q'})[p_{q'} - v(q')]}{p - v(q')} \tag{A.37}$$

This is the equivalent of condition (13). The next lemma generalizes lemma 3.

Lemma 7. *For all $q = 1, ..., N$, in any robust equilibrium in which quality q is traded, the "adjacent upward" ICC of sellers of quality $q-1$ holds with equality unless $p_q = v(q)$.*

Proof. Consider a deviation $p \in (v(q), p_q)$. Note that buyers are willing to buy at p if they think that the deviation comes from type q, since $p < p_q$. We argue that whenever it is possible to delete type $q-1$ from the deviation it is also possible to delete all types $q-s, \ s \geq 2$. To show this point, assume that type $q-1$ can be eliminated:

$$\frac{J(p_{q-1})[p_{q-1} - v(q-1)]}{p - v(q-1)} > \frac{J(p_q)[p_q - v(q)]}{p - v(q)}. \tag{A.38}$$

Consider now type $q-s$. From the incentive compatibility condition:

$$\frac{J(p_{q-s})[p_{q-s}-v(q-s)]}{p-v(q-s)} \geq \frac{J(p_{q-1})[p_{q-1}-v(q-s)]}{p-v(q-s)}. \tag{A.39}$$

But then, for any $p > v(q) > p_{q-1}$, the following relationship

$$\frac{J(p_{q-s})[p_{q-s}-v(q-s)]}{p-v(q-s)} > \frac{J(p_{q-1})[p_{q-1}-v(q-1)]}{p-v(q-1)} \tag{A.40}$$

holds, which implies that type $q-s$ can be deleted, whenever type $q-1$ can be deleted. Since $p < p_q < u(q) < v(q+1)$, sellers of type higher than q are never willing to deviate to p. This implies that if type $q-1$ can be deleted, beliefs should be that the deviation comes from q. As in the two-quality case, we show that whenever $p_q > v(q)$, a viable deviation $p \in (v(q), p_q)$, for which type $q-1$ can be deleted, exists as long as the incentive compatibility condition of type $q-1$ holds with inequality. Suppose then that the ICC holds with strict inequality. Assume that the deviation consists in a price $p = p_q - \epsilon$, $\epsilon > 0$ which is a small undercutting of price p_q. We want to show that there exists $\epsilon > 0$ such that:

$$\frac{J(p_{q-1})[p_{q-1}-v(q-1)]}{p_q-\epsilon-v(q-1)} > \frac{J(p_q)[p_q-v(q)]}{p_q-\epsilon-v(q)}. \tag{A.41}$$

Condition (A.41) can be rewritten as:

$$\frac{J(p_{q-1})[p_{q-1}-v(q-1)]}{J(p_q)[p_q-v(q-1)]} > \frac{p_q-v(q)}{p_q-v(q-1)}\frac{p_q-\epsilon-v(q-1)}{p_q-\epsilon-v(q)}. \tag{A.42}$$

For $p_q > v(q)$, the LHS (which does not depend on ϵ) is strictly greater than 1 whenever the ICC of type $q-1$ holds with strict inequality. On the other hand, the RHS goes to 1 as ϵ becomes small. Thus, there always exists ϵ such that type $q-1$ can be deleted unless either the ICC holds with equality or $p_q = v(q)$. In fact, in the case $p_q = v(q)$, undercutting is never profitable for type q. Hence, either the ICC holds with equality or $p_q = v(q)$. □

We are now ready to characterize the robust equilibria. We distinguish between the case in which all qualities are traded and the case in which a subset of qualities is traded.

a) All $N+1$ qualities traded:
If all qualities are traded, sellers' ICC ensure that $p_q > v(q)$ for all qualities except, possibly, quality N. Thus, for $q < N$, sellers' ICC and lemma 7 imply:

$$J(p_q) = J(p_{q-1})\frac{u(q-1)-v(q-1)-k}{u(q)-v(q-1)-k}. \tag{A.43}$$

Unless quality N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - v(q)$, $p_N > v(N)$ must hold and (A.43) holds with equality for quality N as well. In the special case in which N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - v(q)$, $J(p_N)$ is between 0 and the value implied by (A.43).

Using the initial condition $J(p_0) = 1$, equation (A.43) yields:

$$J(p_q) = \prod_{i=0}^{q-1} \frac{u(q-i-1) - v(q-i-1) - k}{u(q-i) - v(q-i-1) - k}. \tag{A.44}$$

Since $k \geq 0$, the maximum value of $J(p_q)$ is achieved when $k = 0$. Thus, the probability to trade at p_q, for $q > 0$, is bounded above by:

$$\overline{J}_q = \prod_{i=0}^{q-1} \delta_{q-i}. \tag{A.45}$$

where:

$$\delta_q \equiv \frac{u(q-1) - v(q-1)}{u(q) - v(q-1)}. \tag{A.46}$$

Note also that $k = 0$ implies $p_q = u(q)\ \forall q = 0, ..., N$. Therefore, buyers make zero surplus. Assume $k > 0$, so that buyers make positive surplus at every price. Then, all buyers want to trade. This implies that in equilibrium:

$$\sum_{q=0}^{N} \lambda_q J(p_q) = \theta, \tag{A.47}$$

where $J(p_q)$ is given by (A.44) for $q < N$. $J(p_N)$ is also given by (A.44) unless N is the only quality in $\arg\min_{q\in\{1,...,N\}} u(q) - v(q)$, in which case $J(p_N)$ is between zero and the value implied by (A.44). For the case in which all qualities are traded, finding an equilibrium outcome is equivalent to finding a value k^* for which (A.47) holds. From (A.44), the LHS of equation (A.47) is monotonically decreasing in k, for $k \in [0, \hat{k}]$, where $\hat{k} \equiv \min_{q\in\{0,...,N\}} u(q) - v(q)$. We note that simultaneous satisfaction of all the participation constraints requires that k always lie in the interval $[0, \hat{k}]$. Given monotonicity, there always exists at most one value k^* in the above interval. This also implies that if there exists an equilibrium, its outcome must be unique. Regarding the existence, consider the following. From (A.44), the LHS of expression (A.47) reaches its maximum in the relevant interval when $k = 0$ and its minimum when $k = \hat{k}$. Therefore, necessary and sufficient conditions for an interior solution are

$$\theta < \lambda_0 + \sum_{q=1}^{N} \lambda_q \prod_{i=0}^{q-1} \delta_{q-i} \tag{A.48}$$

and

$$\theta > \lambda_0 + \sum_{q=1}^{N-1} \lambda_q \prod_{i=0}^{q-1} \frac{u(q-i-1) - v(q-i-1) - \hat{k}}{u(q-i) - v(q-i-1) - \hat{k}} + \lambda_N J_N^{min}. \tag{A.49}$$

where J_N^{min} is given by

$$J_N^{min} = \begin{cases} \prod_{i=0}^{N-1} \frac{u(N-i-1)-v(N-i-1)-\hat{k}}{u(N-i)-v(N-i-1)-\hat{k}} & \text{if } \arg\min_{q\in\{1,...,N\}} u(q)-v(q) \neq \{N\} \\ 0 & \text{if } \arg\min_{q\in\{1,...,N\}} u(q)-v(q) = \{N\} \end{cases} \tag{A.50}$$

Otherwise, if one of the above conditions is not satisfied, the robust equilibrium takes a different form. If condition (A.48) is not satisfied, then k must be equal to zero and the equilibrium is characterized by $p_q = u(q)$ for all $q = 0, 1, ..., N$ and probabilities $\overline{J}_q$ given by (A.45). As $\overline{J}_q > 0$ $\forall q = 0, ..., N$, all qualities are traded also in this case. Thus, a sufficient condition for all qualities being traded is that θ is high enough to ensure that (A.49) holds. Below, we show that this condition is also necessary. In order to gather intuitions on condition (A.49), notice that it is always satisfied when the gains from trade are nondecreasing in the quality since, in this case, $\hat{k} = u(0) - v(0)$ and the RHS is equal to λ_0. Therefore, as long as $\lambda_0 < \theta$, an internal solution in which all qualities are traded must exist.

b) More than one and less than $N+1$ qualities are traded:
Assume now that θ is relatively small so that condition (A.49) is not satisfied. If the value of the RHS of equation (A.49) for $k = \hat{k}$ is greater than or equal to θ, then some qualities are not traded. To see this, note that k cannot exceed $\hat{k}$. Let $\underline{J}_q$ be the probability to sell at p_q when k equals $\hat{k}$. Let also $\hat{q}$ be the lowest quality in $\arg\min_{q\in\{0,...,N\}} u(q) - v(q)$. Then, by definition, $u(\hat{q}) - v(\hat{q}) - \hat{k} = 0$ so that, from equation (A.44), $\underline{J}_q$ is zero for all $q > \hat{q}$. Thus, when $k = \hat{k}$, all qualities above the quality which provides the lowest gain from trade are not traded. Buyers' surplus k should increase, however it fails to increase because, by increasing, it would violate the participation constraint of sellers of type $\hat{q}$. Hence, the incentive compatibility for type $\hat{q}$ requires that no higher quality is traded. Thus, (A.49) is a necessary condition for all qualities being traded.

What are the characteristics of the equilibrium when (A.49) is violated? Note that if (A.49) is not satisfied we have:

$$\sum_{q=0}^{\hat{q}} \lambda_q \underline{J}_q = \sum_{q=0}^{N} \lambda_q \underline{J}_q > \theta, \tag{A.51}$$

where the equality comes from the fact that all qualities higher than $\hat{q}$ are not traded. This suggests that even if only $\hat{q}+1$ qualities are traded out of $N+1$, the quantity sold is still higher than the quantity bought. Then, quality $\hat{q}$ cannot be traded in equilibrium. Let us assume that quality $\hat{q}-1$ is traded. It follows that its price, $p_{\hat{q}-1}$, must be compatible with D1. In other words, there must be no incentive to deviate for sellers of type $\hat{q}$ or above. Therefore, $p_{\hat{q}-1}$ must be such that $k_1 \equiv u(\hat{q}-1) - p_{\hat{q}-1} \geq u(\hat{q}) - v(\hat{q})$. If so, there is no price sellers of type $\hat{q}$ could possibly announce to attract buyers and still make no loss. Of course, buyers incentive compatibility implies $u(q) - p_q = k_1$ for all qualities that are traded, i.e. $q = 0, ..., \hat{q}-1$. Let now $\tilde{q}$ be the lowest quality in $\arg\min_{q\in\{0,...,\hat{q}-1\}} u(q) - v(q)$, i.e. the lowest quality among those which give minimum gain from trade when attention is restricted to qualities lower than $\hat{q}$. It is clear that k_1 should now satisfy $u(\hat{q}) - v(\hat{q}) \leq k_1 \leq u(\tilde{q}) - v(\tilde{q})$. Therefore, all that remains to be checked is whether there exists k_1^* such that:

$$\sum_{q=0}^{\hat{q}-1} \lambda_q \prod_{i=0}^{q-1} \frac{u(q-i-1) - v(q-i-1) - k_1^*}{u(q-i) - v(q-i-1) - k_1^*} = \theta. \tag{A.52}$$

If it does, then the equilibrium is such that qualities $q = 0, ..., \hat{q}-1$ are traded. If it does not, then all the process starts again by choosing $\tilde{q}$ as the first quality that is not traded. It should be noted that, since we are assuming $\theta > \lambda_0$, the process eventually leads to an equilibrium in which more than one quality is traded. In fact, as long as $\theta > \lambda_0$, qualities 0 and 1 are always traded. By iterating this process, one can show that the number of qualities traded in equilibrium decreases with θ. Thus, price dispersion increases as θ increases.

The extension to $N+1$ qualities generalizes the result of an inverse relationship between price dispersion and the degree of competition (as measured by θ) derived for the case of two qualities. When competition among sellers is so strong that $\theta \leq \lambda_0$, only the lowest quality is traded and there is no price dispersion. When competition is weak (θ satisfies condition (A.49)) all qualities are traded and price dispersion is maximized. For intermediate values of θ such that $\theta > \lambda_0$ while (A.49) is not satisfied, the number of qualities which are traded and the degree of price dispersion (weakly) increase with θ.

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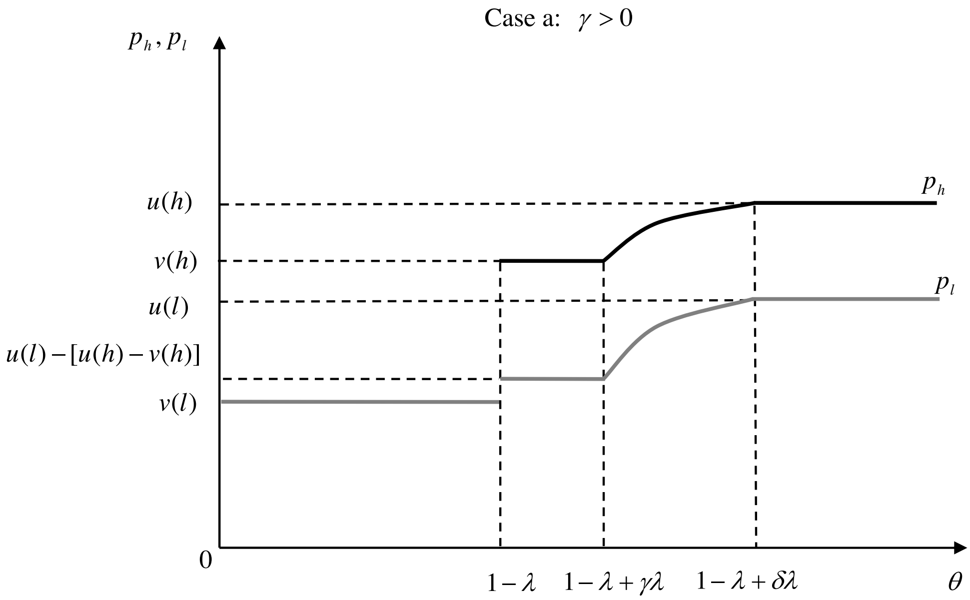
Figure 1: Equilibrium prices as a function of θ



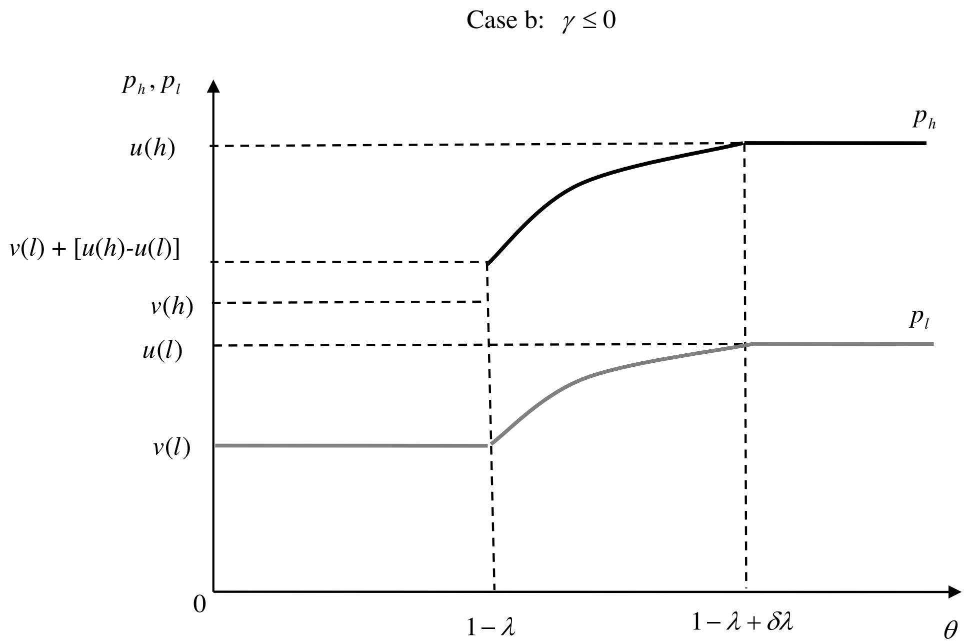


Figure 2: Quality of trade as a function of θ

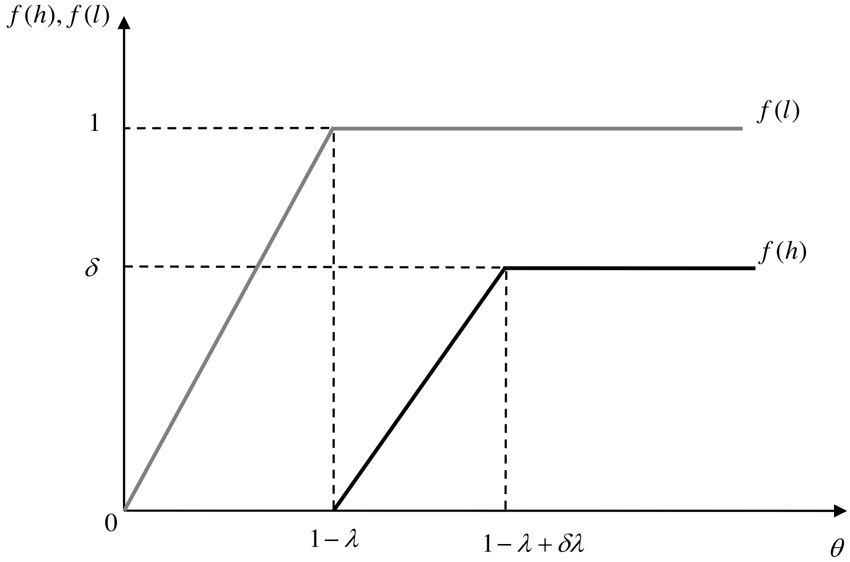


Figure 3: Welfare comparison between MPC and MSPS

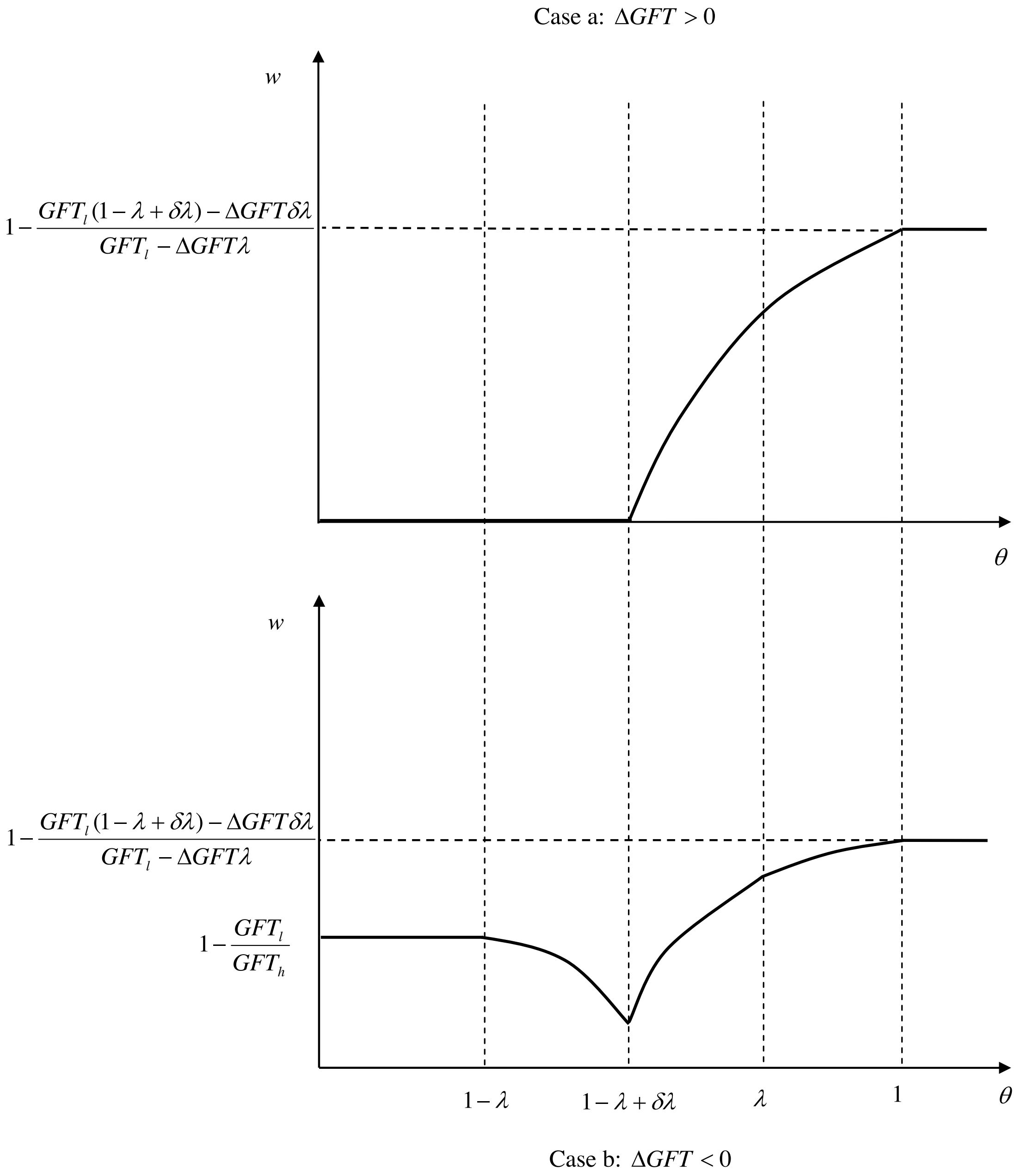


Figure 4: Welfare comparison between MA and MSPS