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Genuine Dissaving and Optimal Growth

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Abstract

Measuring sustainability is a major goal for applied research on economic development. This paper studies the theoretical foundations of empirical methods used in recent literature to test sustainability, with particular focus on the theory of green accounting and the genuine saving indicator. The rationale for this empirical method is that non-negative genuine savings at any point in time imply non-decreasing utility at that time. This 'instantaneous' condition is necessary but not sufficient for sustainability: an additional 'asymptotic' condition is also necessary for non-decreasing long-run utility, i.e. the consumption rate of return must be at least equal to the social discount rate. As a consequence, the asymptotic condition should be incorporated in empirical analysis, in order to test sustainability in a forward-looking manner. This paper presents an empirical test of the asymptotic condition using time-series data for the United States.

JEL codes : Q01, O47, D90. *Keywords* : Genuine Saving, Green Accounting, Optimal Growth, Sustainable Development.

1 Sustainability and genuine savings

Recent empirical studies on sustainable development are based on the *genuine saving indicator*. Genuine savings are defined as the difference between aggregate investment in produced assets and the value of net depletion of natural resources. The link between this measure of net investments and sustainability has been clarified by the literature on green national accounting: under competitive conditions, positive genuine savings at some point in time correspond to positive variations in utility (welfare) at that time. Building on this point, empirical studies check whether the genuine savings measure is non-negative.

However, positive genuine savings at a given point in time do not imply sustainability, because sustainable development also requires non-declining welfare in the long run: even if genuine savings are positive in the present, they might become negative in the future. This point can be addressed by means of the capital-resource model pioneered by Dasgupta and Heal (1974) and Stiglitz (1974). In our version of the model, which includes technological progress and renewable resources, we show that sustainability requires the consumption rate of return be at least equal to the social discount rate. This 'asymptotic condition' must be fulfilled in order to obtain non-declining welfare in the long run: if it is not satisfied, genuine savings will eventually turn negative in the future, even if they are positive in the present.

In this paper we show that, given a planning horizon going from "time zero" to infinity, sustainability requires that (i) genuine savings are positive at time zero, and (ii) the long-run condition is satisfied: consequently, the asymptotic constraint (ii) should be incorporated in empirical methods for testing sustainable development. Using time-series data, we check whether the long-run condition is fulfilled in the United States, by estimating the relevant parameters of an aggregate production function where man-made capital, petroleum products and labor services are essential inputs. The long-run condition appears to be satisfied, because the time-trend in aggregate output is sufficiently steep: this result suggests that long-run genuine savings will be positive in the US economy in the future.

2 Instantaneous and asymptotic conditions

Assume that instantaneous social welfare is represented by the well-behaved[1] function $u(c)$, where c is aggregate consumption. Considering an infinite time-horizon, sustainability is defined as follows:

$$\text{SD} \quad \Leftrightarrow \quad \frac{du\,(c\,(t))}{dt} \geq 0 \text{ for each } t \in [0, \infty)\,. \tag{1}$$

Starting from (1), we can define two distinct types of *necessary* conditions for SD, which we label as instantaneous and asymptotic sustainability conditions (ISC and ASC, respectively). On the one hand, sustainability requires the instantaneous variation in social welfare at a given point in time be non-negative; formally, if we are at time $\bar{t}$, the instantaneous condition is

$$\left.\frac{du\,(c\,(t))}{dt}\right|_{t=\bar{t}} \geq 0. \tag{2}$$

On the other hand, a necessary condition for SD is also that the variation in social welfare is non-negative as times goes to infinity:

$$\lim_{z\to\infty} \left.\frac{du\,(c\,(t))}{dt}\right|_{t=z} \geq 0. \tag{3}$$

It must be stressed that ISC (2) and ASC (3) impose different constraints on the economy. In particular, *satisfying ISC does not generally imply ASC be fulfilled.*

3 The genuine savings criterion

Our analysis is focused on the genuine saving method proposed by Pearce and Atkinson (1993) and Pearce *et al.* (1996). Aggregate genuine savings (θ) are usually defined as the difference between conventional savings and the value of net resource depletion. Unlike the original formulations, our definition of θ will consider the presence of disembodied technological progress by applying recent results of the literature on green accounting (Asheim and Weitzman, 2001). We assume a closed economy producing a single good, which can be

[1]Specifically, we assume that $u(.)$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $\lim_{c\to 0} \partial u/\partial c = \infty$.

either consumed or accumulated in the form of man-made capital. Genuine savings at time t equal

$$\theta(t) = \dot{k}(t) + (p_r(t) - a)\,\dot{s}(t) + \pi(t), \tag{4}$$

where k and s are the stocks of man-made and natural capital, p_r the gross marginal rent on natural capital, and a the marginal cost of extraction (assumed constant). Expression (4) is an *augmented* version of the traditional genuine saving measure, since it also includes the *time premium* π. This term accounts for the exogenous shift occurring in the set of production possibilities at each point in time: the presence of disembodied technological progress increases future consumption possibilities, and the value of current net investments must be augmented accordingly (Asheim, 1997; Weitzman, 1997). The role of the time-premium in determining the properties of the genuine savings measure θ will be further clarified in the next subsection.

The *genuine saving criterion* of sustainability consists of checking that the non-negativity condition

$$\theta(\bar{t}) \geq 0 \tag{5}$$

is satisfied at a given point in time $\bar{t}$. This method for measuring SD is widely used and employed for a large number of countries (see *e.g.* Neumayer, 1999; Vincent *et al.* 1997; World Bank, 1999). The rationale for the criterion (5) is the 'weak sustainability approach', postulating that SD requires to preserve the total aggregate stock of both forms of capital: from (4), if genuine savings are non-negative, the value of the total aggregate stock is not declining.

3.1 Genuine savings and instantaneous sustainability

The link between SD and the genuine savings criterion has been investigated by the theory of green national accounting. In this literature, the genuine savings measure is implemented in standard models with utility-maximizing consumers and profit-maximizing firms (see *e.g.* Hartwick, 1990). In particular, Neumayer (1999) and Hamilton and Clemens (1999) consider the capital-resource model of optimal growth as the theoretical foundation of the genuine saving method. In this paper, gross output is represented by the well-behaved function $f(k, r, t)$, where r is the flow of extracted resources used in production. We assume that both k and r are essential inputs, man-made capital depreciates at the constant rate γ, and the natural resource is renewable, with constant rate of regeneration g.[2] The evolution of the stocks is described by the differential equations

$$\begin{aligned} \dot{s}(t) &= g \cdot s(t) - r(t), & (6) \\ \dot{k}(t) &= f(k, r, t) - c(t) - a \cdot r(t) - \gamma \cdot k(t), & (7) \end{aligned}$$

and an *optimal path* is defined as a sequence $\{c(t), r(t), s(t), k(t)\}_0^\infty$ that solves the following social-planning problem:

$$\begin{aligned} & \max_{\{c(t),r(t)\}_0^\infty} \int_0^\infty u(c(t)) \exp[-\delta t]\, dt \\ & \text{subject to (6), (7), and to} \\ s(t) &\geq 0, k(t) > 0, \\ c(t) &\geq 0, r(t) > 0 \text{ for each } t \in [0, \infty), \qquad (8) \end{aligned}$$

where $\delta > 0$ is the social discount rate, and initial amounts $s(0)$ and $k(0)$ are taken as given.

Defining the *consumption rate of return* as

$$\rho(t) = -\frac{\frac{d}{dt}(u_c \exp[-\delta t])}{u_c \exp[-\delta t]}, \qquad (9)$$

the first order conditions for an interior solution of problem (8) imply

$$\begin{aligned} \rho &= f_k - \gamma & (10) \\ \dot{f}_r &= (f_r - a)(\rho - g), & (11) \end{aligned}$$

where $u_c = \partial u / \partial c$, $f_k = \partial f / \partial k$ and $f_r = \partial f / \partial r$. Equation (10) says that, along the optimal path, the consumption rate of return must equal the marginal net rent from man-made capital, whilst (11) is a modified Hotelling rule.

We now prove that the genuine saving criterion (5) is a test of the instantaneous sustainability condition (2). If there is no disembodied technical progress we have $\pi = 0$, and θ coincides with the traditional, non-augmented measure: in this case, it is well known that $\theta(\bar{t}) = 0$ implies $\dot{c}(\bar{t}) = 0$, and

[2] By essential inputs we mean $f(k, 0, t) = f(0, r, t) = 0$. We assume that f is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $\lim_{r \to 0} \partial f(k, r, t) / \partial r = \infty$.

viceversa. The proof is a corollary of the so-called *Hartwick rule*: consumption at time $\bar{t}$ is constant if all net rents from natural resources are invested in the accumulation of man-made capital (Hartwick, 1977; Dixit *et al.* 1980; Asheim and Withagen, 1998), *i.e.* a situation with zero non-augmented genuine savings at time $\bar{t}$ (for a detailed discussion see Asheim *et al.* 2003).

A positive rate of technological progress modifies this result. In our model, consumption can be sustained by investing *less* than the total amount of net rents received. This is because for given amounts of inputs, production possibilities are improved over time by virtue of technological progress. Time has therefore an intrinsic value, which is represented by the time premium π: in our model - following Asheim (1997) and Asheim and Weitzman (2001) - the time premium evolves according to the differential equation

$$\dot{\pi}(t) = \rho(t)\,\pi(t) - f_t(t), \tag{12}$$

where the partial time-derivative $f_t = \partial f(k,r,t)/\partial t$ is the instantaneous exogenous shift in production possibilities. On the basis of (12), we now prove the following

Proposition 1 *Non-negative genuine savings at time $\bar{t}$ imply that consumption is non-decreasing at time $\bar{t}$, and viceversa, along the optimal path with positive net rents from both types of capital:*

$$\dot{c}(\bar{t}) \gtreqless 0 \quad \Longleftrightarrow \quad \theta(\bar{t}) \gtreqless 0. \tag{13}$$

Proof. Choose a constant ξ such that gross investments in man-made capital at time $\bar{t}$ equal

$$\dot{k}(\bar{t}) = \xi - (f_r(\bar{t}) - a)\,\dot{s}(\bar{t}) - \pi(\bar{t}). \tag{14}$$

Since $p_r = f_r$ along the optimal path, it follows from (4) that

$$\xi \gtreqless 0 \Longleftrightarrow \theta(\bar{t}) \gtreqless 0. \tag{15}$$

Differentiating (7) with respect to time gives

$$\dot{c}(\bar{t}) = f_t(\bar{t}) + (f_r(\bar{t}) - a)\,\dot{r} + (f_k(\bar{t}) - \gamma)\,\dot{k}(\bar{t}) - \ddot{k}(\bar{t}), \tag{16}$$

where $\ddot{k} = d^2k/dt^2$. Differentiating (14) with respect to time and substituting the resulting expression in (16) yields

$$\dot{c}(\bar{t}) = f_t(\bar{t}) + (f_k(\bar{t}) - \gamma)\,\dot{k}(\bar{t}) + \dot{s}(\bar{t})\left(\dot{f}_r(\bar{t}) + g\,(f_r(\bar{t}) - a)\right) + \dot{\pi}(\bar{t}). \tag{17}$$

Substituting (11) in (17), and using (12), we obtain

$$\dot{c}\left(\bar{t}\right)=\rho\left(\bar{t}\right)\left(\dot{k}\left(\bar{t}\right)+\dot{s}\left(\bar{t}\right)\left(f_r\left(\bar{t}\right)-a\right)+\pi\left(\bar{t}\right)\right)=\xi\rho\left(\bar{t}\right), \tag{18}$$

where $\rho>0$ by virtue of our assumptions. By (18), the sign of $\dot{c}$ is the same as that of ξ: by (15), this implies (13). ||

It derives from (13) that non-negative genuine savings at some time $\bar{t}$ imply that utility is non-decreasing at time $\bar{t}$, and viceversa: Proposition 1 establishes that the genuine savings criterion (5) is equivalent to test the instantaneous condition (2).

The genuine saving approach exclusively refers to welfare improvements at the instant in which $\theta\left(\bar{t}\right)$ is computed, but there is no guarantee that observed paths are compatible with non-declining utility in the future. Previous literature pointed out that, in the non-augmented case ($\pi=0$), positive genuine savings in the present do not imply sustainability (Asheim, 1994; Vellinga and Withagen, 1996; Asheim *et al.* 2003). Proposition 1 implies that the same problem holds when a positive rate of technical progress is taken into account ($\pi>0$).

4 Asymptotic sustainability

We now discuss necessary conditions to obtain non-declining welfare in the long run. A general expression of the asymptotic condition (3) derives immediately from the first order conditions of the social problem: along the optimal path, the instantaneous variation of consumption is governed by the Keynes-Ramsey rule

$$\dot{c}/c=\sigma\left(c\right)\left(\rho-\delta\right), \tag{19}$$

where $\sigma\left(c\right)$ is the elasticity of marginal utility. Consequently,

Proposition 2 *Sustainability in the capital-resource model requires that the consumption rate of return does not fall below the social discount rate:*

$$\lim_{t\to\infty}\rho\left(t\right)\geq\delta. \tag{20}$$

Proof. Since $\sigma>0$, it follows from (19) that utility is asymptotically positive and non-declining only if (20) is satisfied. ||

Expression (20) is a general form of the asymptotic condition (3). It follows from the distinction between ISC and ASC that this long-run constraint is crucial to assess prospects for sustainability: testing whether the economy is placed along a sustainable path requires to ascertain that (20) is fulfilled. In this regard, an explicit form of the asymptotic condition is needed: we now show that (20) reduces to a specific constraint on exogenous parameters, and provide a qualitative description of the dynamics of genuine savings, under the assumption of resource-saving technological progress.

4.1 Explicit dynamics of genuine savings

In general, explicit forms of (20) can be obtained by imposing restrictions on production and utility functions. In section 5 we will use a modified version of Stiglitz (1974) asymptotic constraint, in order to test the long-run condition empirically. While Stiglitz (1974) condition assumes Cobb-Douglas technology and logarithmic utility, this section studies explicit dynamics of genuine savings in a more general setting. Specifically, we assume that

H1. Aggregate output $f(k, r, t)$ can be represented by the production function $\Phi(k(t), r(t) \cdot e^{\nu t})$, with Φ homogeneous of degree 1.

H2. The instantaneous utility function is isoelastic ($\sigma(c) = \sigma$) and extraction costs are zero ($a = 0$).

Assumption *H2* allows to characterize consumption dynamics along the optimal path and is typical in the literature on capital-resource models (Dasgupta and Heal, 1974; Stiglitz, 1974), whereas assumption *H1* is quite general, since it recalls the hypothesis of constant returns to scale without particular functional forms. The only peculiarity regards technological progress: *H1* assumes resource-augmenting progress, *i.e.* technological progress that increases the productive services of the natural input $r(t)$. The augmenting factor $e^{\nu t}$ is thought of as resulting from the development of resource-saving techniques that become available over time. The main insight for the purposes of our analysis is provided by the following result:

Proposition 3 *Under assumptions H1-H2, $\rho(t)$ converges to $g + \nu$.*

Proof. From (9), $\rho(t)$ evolves according to $f_k(t)$. Under constant returns to scale, f_k solely depends on the input ratio $x = k \exp[-\nu t]/r$, and the

Hotelling rule (11) may be rewritten as (see Appendix)

$$(\phi(x) - x\phi_x(x))(\rho(x) - g - \nu) = -x\dot{x}\phi_{xx}(x), \tag{21}$$

where $\phi(x) = \Phi(x, 1)$ is the production function in intensive form, $\phi_x(x)$ equals f_k, and $\rho(x) = \phi_x(x) - \gamma$. From (21), the steady state input ratio x^{ss} is dynamically stable[3]. Hence, $\lim_{t\to\infty} x(t) = x^{ss}$ and

$$\lim_{t\to\infty} \rho(t) = \rho(x^{ss}) = g + \nu. \qquad \| \tag{22}$$

Comparing Propositions 2 and 3, it follows that testing asymptotic sustainability is equivalent to check that the following inequality is satisfied:

$$g + \nu \geq \delta. \tag{23}$$

Hence, the asymptotic criterion (3) reduces to a general sustainability constraint in the capital-resource model. The economic meaning of condition (23) is as follows (for a detailed discussion see Valente, 2004). Along the optimal path natural inputs are progressively substituted by man-made capital, so that marginal productivity of capital declines over time, driving down the consumption rate of return: whether this process is compatible with sustainability depends on the level of the discount rate, relative to resources renewal and augmentation rates. Previous results of the literature on consumption single-peakedness (Dasgupta and Heal, 1974; Pezzey and Withagen, 1998) can be seen as particular cases of (23), which is necessary for sustainability also with positive extraction costs (Valente, 2004).[4]

In the present context, expression (23) can be reinterpreted from an empirical standpoint: *if genuine savings are positive in the present, they will eventually turn negative in the future if (23) is not satisfied.* In order to describe the underlying dynamics, define $\tilde{x}$ as the value of the input ratio which corresponds to constant consumption (utility), *i.e.*

$$\tilde{x} \quad \Leftrightarrow \quad \rho(\tilde{x}) = \delta. \tag{24}$$

[3] The first term in brackets of (21) is positive (it equals $\Phi_r \exp[-\nu t] > 0$), while ϕ_{xx} is negative. Consequently, $\dot{x} \gtreqless 0 \quad \Leftrightarrow \quad \rho(x) \gtreqless g + \nu$. Since ρ is a decreasing function of x, the input ratio converges to x^{ss}.

[4] Dasgupta and Heal (1974) proved that consumption is bound to decrease with exhaustible resources and no technical progress: indeed, condition (23) cannot be satisfied when $\nu = g = 0$. The general case with positive regeneration and augmentation rates is discussed in Valente (2004), where condition (23) is also necessary for non-declining utility with positive extraction costs.

Figure 1 describes the dynamics of the input ratio and the consumption-capital ratio in a phase diagram: we show in the Appendix that the equilibrium is a saddle-point and that the optimal path converges to the steady state along the stable arm of the saddle. Since genuine savings have the same sign of the variation in consumption, we have $\theta = 0$ along the horizontal line $x = \tilde{x}$. That is, genuine savings are negative in each point lying above $\tilde{x}$. From (24), if the asymptotic condition (23) is not satisfied, the steady state locus $x = x^{ss}$ lies above $\tilde{x}$, as in Figure 1. If the economy starts at point A, the economy converges to x^{ss} along the stable arm and θ is strictly positive when point B is reached; however, consumption will be declining from point D onwards. Hence, if the asymptotic condition is not satisfied, the economy will experience *genuine dissaving* after time t_D, despite the fact that genuine savings were positive at time t_B. The time paths of consumption and genuine savings along this optimal trajectory are depicted in Figure 1, below the phase diagram.

The argument that asymptotic conditions should be incorporated in empirical testing can be further reinforced: while ISC and ASC are only necessary conditions - that is, satisfying each of them does not ensure sustainable development - satisfying both at time zero defines a sufficient condition for sustainable development.[5] More precisely,

Proposition 4 *Assume H1 and H2, and suppose that condition (23) is satisfied. Then, if $\theta \geq 0$ at time zero, genuine savings are always non-negative along the optimal path.*

Proof. Both the consumption-capital ratio c/k and the input ratio x converge to the steady state equilibrium along the optimal saddle-path (see Appendix). Consequently, if $x(0) < x^{ss}$ $(x(0) > x^{ss})$, the marginal product of capital decreases (increases) monotonically along the stable arm of the saddle. This is shown graphically in Figure 2. Assume that the asymptotic condition (23) is satisfied: in this case, the long-run equilibrium x^{ss} is placed below $\tilde{x}$, in the $\theta \geq 0$ zone. If at time 0 we observe $\theta(0) \geq 0$, the economy is certainly below point F. Since x approaches x^{ss} monotonically, once the economy is in the $\theta \geq 0$ zone, it will remain inside of it. Hence, satisfying both ASC and ISC at time zero implies sustainability in the capital-resource model. $\|$

[5] Whether genuine savings are positive at time zero depends on the levels of initial endowments, technological parameters, and the social discount rate. An explicit condition for non-declining consumption per capita at time zero in the Cobb-Douglas case is derived in Valente (2004).

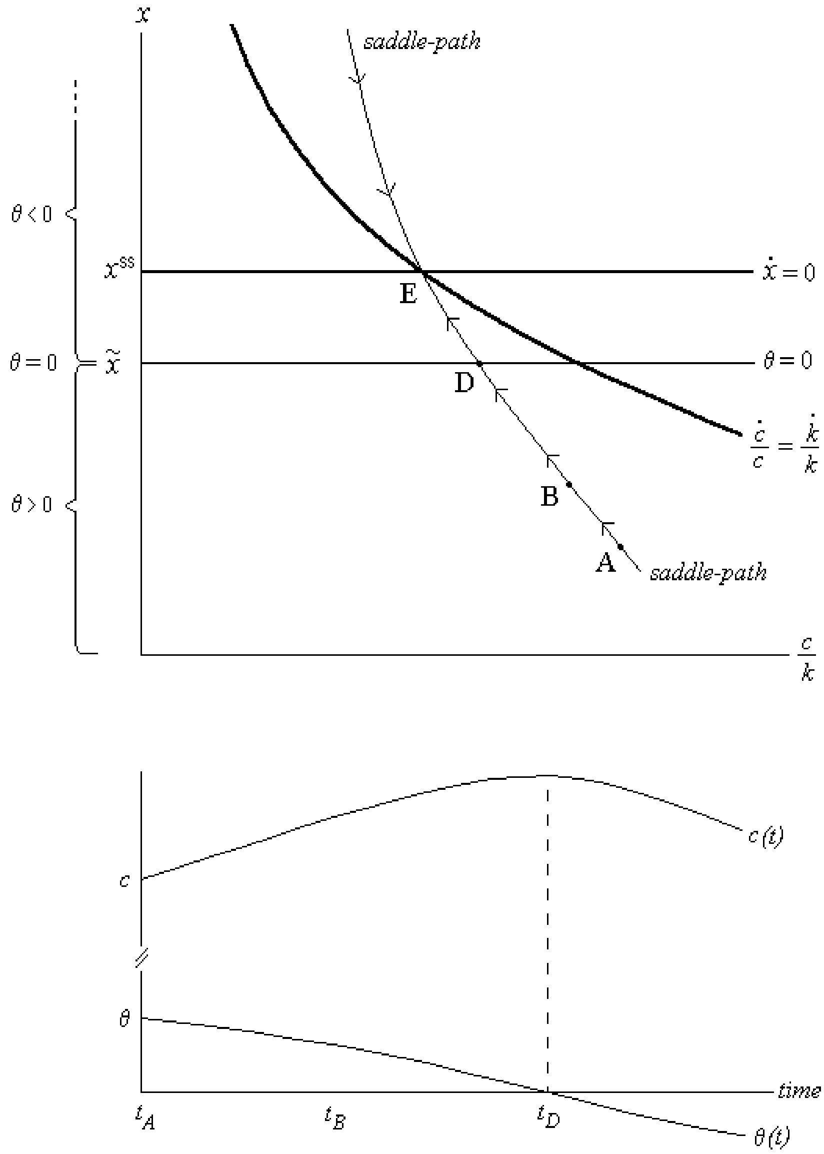

Figure 1: The dynamics of the input ratio x and the consumption-capital ratio along the optimal path. When the asymptotic condition (23) is violated, the equilibrium E lies above the line $x = \tilde{x}$, which corresponds to zero genuine savings. Since the economy converges to E, genuine savings become negative after passing through point D: utility declines in the long run.

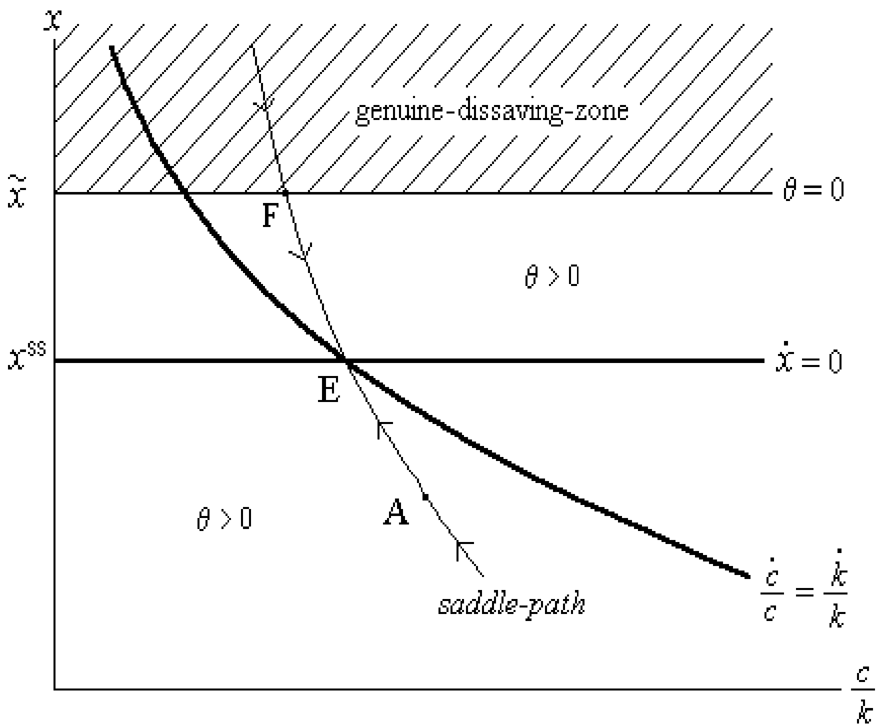

Figure 2: Phase diagram when ASC is satisfied: if genuine savings at time zero are non-negative, the economy starts below point F and the joint dynamics of c/k and x will keep the economy outside the genuine-dissaving-zone (shaded area).

5 Testing asymptotic sustainability

Our results emphasize the role of technological progress in determining long-run sustainability, and imply that the asymptotic condition should be incorporated in empirical methods for testing sustainability in a forward-looking manner. In this section, we suggest a simple procedure to estimate the crucial variable, *i.e.* the resource-augmenting rate, and we apply this method on macroeconomic data of the United States.

The critical discount rate. Assume that aggregate output is represented by the Cobb-Douglas form (Stiglitz, 1974)

$$y(k, r, n, t) = \Psi k(t)^{\alpha_1} r(t)^{\alpha_2} n(t)^{\alpha_3} \exp[\omega t], \tag{25}$$

where ω is the *Hicks-neutral* rate of exogenous technical progress, n is labor, and $\Psi > 0$ is a proportionality factor. We assume decreasing returns in each argument ($0 < \alpha_i < 1$), and constant returns to scale ($\sum_i \alpha_i = 1$). Labor services are supplied inelastically in each point in time, and the amount of labor units grows exponentially at the exogenous rate μ. Technology (25) may be rewritten as

$$y(k, r, n, t) = A \cdot k(t)^{\alpha_1} \cdot \left\{ r(t) \exp\left[\frac{\omega + \mu\alpha_3}{\alpha_2} \cdot t \right] \right\}^{\alpha_2}, \tag{26}$$

where $A = \Psi n(0)^{\alpha_3}$. Expression (26) is similar to the resource-augmenting specification *H1* assumed in the previous section.[6] Following Valente (2004), it can be shown that consumption is asymptotically non-decreasing if and only if

$$\frac{\omega + \mu\alpha_3}{\alpha_2} \geq \delta - g. \tag{27}$$

The asymptotic constraint (27) is a variant of condition (23), the only difference being the presence of the labor share α_3 multiplied by μ: the long-run condition is affected by the assumption of growing labor force, and prospects for sustainability also depend on the rate of population growth.[7] On the basis of condition (27), we can build an indicator of asymptotic sustainability as follows. Standard econometric techniques allow to estimate the input shares $\alpha_1, \alpha_2, \alpha_3$ and the Hicks-neutral rate of technical progress ω. Given the estimates $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\omega})$, the observed average resource renewal rate $\hat{g}$, and the average growth rate of labor units $\hat{\mu}$, we obtain from (27) the maximum level

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[6] The only difference is that, in the Cobb-Douglas form (25), we are relaxing the assumption of constant returns to scale in physical and natural capital: since $\alpha_3 > 0$, the sum $\alpha_1 + \alpha_2$ is strictly less than one.

[7] It is worth noting that the population growth rate appearing in (27) was absent in the sustainability condition derived in Valente (2004) for the Cobb-Douglas case. The reason is that Valente (2004) considers a Command Optimum problem where the objective function is the sum of individual utilities from consumption per capita. Here we are instead assuming a social welfare function where instantaneous welfare depends on aggregate consumption. In view of empirical results presented below, using either condition does not alter our conclusions.

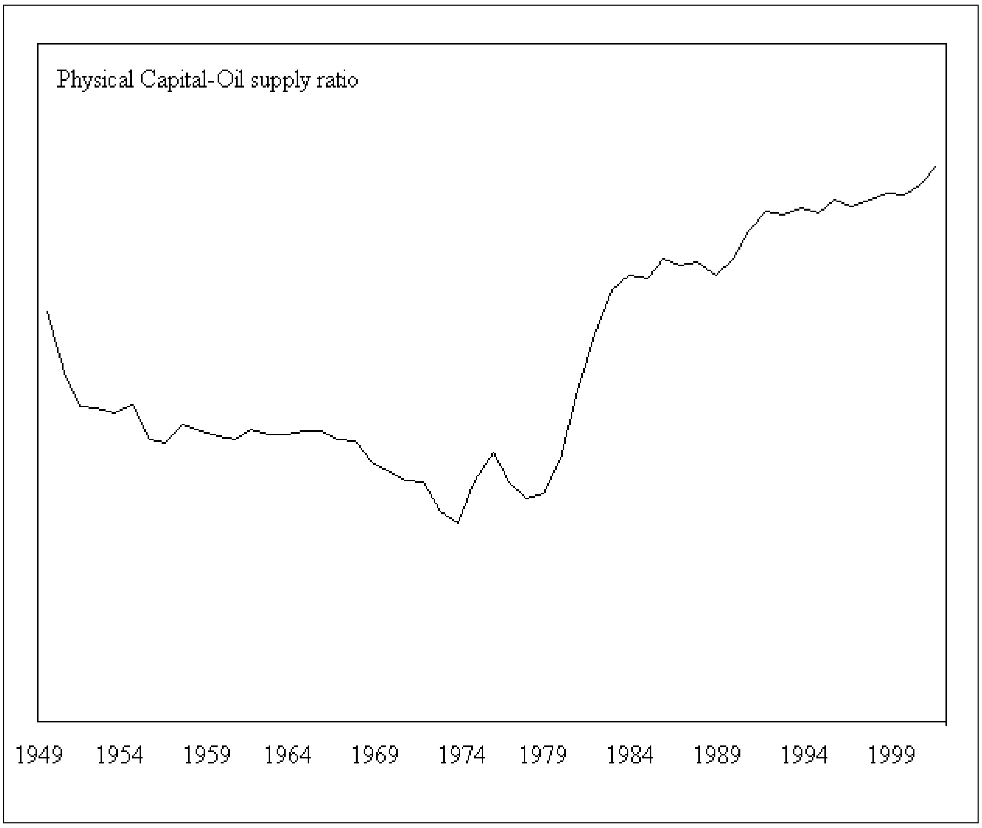

Figure 3: The dynamics of the ratio between real net fixed assets (1996 chained dollars) and an petroleum products supplied (thousand barrels) in the United States (1949-2001). *Sources:* BEA (2003), EIA (2002).

of the time-preference rate which is compatible with sustainability:

$$\delta^{\star} = \frac{\hat{\omega} + \hat{\mu}\hat{\alpha}_3}{\hat{\alpha}_2} + \hat{g}. \tag{28}$$

In general, if $\delta^{\star}$ is very close to zero, we conclude that the effect of technological progress is too weak and that more effort should be devoted in implementing resource-saving techniques. The threshold value $\delta^{\star}$ can also be compared with the values assigned to δ in empirical analyses on sustainable development and genuine savings (*e.g.* Hamilton, 2000; World Bank, 1999). In this case, the sustainability gap $(\delta^{\star} - \delta)$ is more restrictive when natural capital is to a large extent non-renewable. We address this point by assuming that the relevant natural resource in the aggregate production function is oil ($\hat{g} = 0$) while testing asymptotic sustainability for the U.S. economy.

Asymptotic sustainability in the United States. For the resource input-flow r, we use time-series data on petroleum products supplied during the period 1949-2001 (EIA, 2002). As regards other variables, we use time-series of real gross domestic product for y, real net stock of fixed assets for k, and hours worked for n, provided by the Bureau of Economic Analysis (BEA, 2003). Figure 3 depicts the observed time-path of the capital-resource ratio: after the oil shocks of the seventies, the dynamics of (k/r) increase monotonically. This is consistent with the predictions of the model: the natural input is progressively substituted by physical capital as the resource stock is depleted. In terms of phase diagrams, the US economy may be placed below the long-run equilibrium, *e.g.* points A in Figures 1 and 2. Whether the long-run equilibrium implies unsustainability (as in Figure 1) or sustainability (as in Figure 2) it depends on the level of the resource-augmenting progress, which we now estimate.

The discrete-time version of technology (25) is $y_t = \Psi\left(1+\omega\right)^t k_t^{\alpha_1} r_t^{\alpha_2} n_t^{\alpha_3}$. Setting $\alpha_1 + \alpha_2 + \alpha_3 = 1$, the logarithm of the output-labor ratio ($\tilde{y} = y/n$) can be expressed as

$$\log \tilde{y}_t = \log \Psi + t \log\left(1+\omega\right) + \alpha_1 \log \tilde{k}_t + \alpha_2 \log \tilde{r}_t, \tag{29}$$

where $\tilde{k} = k/n$ and $\tilde{r} = r/n$. We have performed unit root tests on the time series of $\log \tilde{k}$ and $\log \tilde{r}$: both the Phillips-Perron and the Augmented Dickey-Fuller procedures suggest that $\{\tilde{k}_t\}$ and $\{\tilde{r}_t\}$ are integrated of order 1. Therefore, we include backward and forward first differences of $\log \tilde{k}$ and $\log \tilde{r}$ in the stochastic equation

$$\begin{aligned}\log \tilde{y}_t &= \beta_0 + t \cdot \beta_1 + \beta_2 \log \tilde{k}_t + \beta_3 \log \tilde{r}_t \\ &\quad + \beta_4 \log\left(\tilde{k}_t/\tilde{k}_{t-1}\right) + \beta_5 \log\left(\tilde{r}_t/\tilde{r}_{t-1}\right) \\ &\quad + \beta_6 \log\left(\tilde{k}_{t+1}/\tilde{k}_t\right) + \beta_7 \log\left(\tilde{r}_{t+1}/\tilde{r}_t\right) + \varepsilon_t, \end{aligned} \tag{30}$$

where ε_t is an error term with zero mean. Since $\{\tilde{k}_t\}$ and $\{\tilde{r}_t\}$ are $I\left(1\right)$, we can obtain efficient estimates of coefficients $(\beta_0, ..., \beta_7)$ in equation (30) using linear dynamic least squares.[8] Estimating the regression equation (30) yields

[8]See Sims *et al.* (1990).

the following results:

$$\hat{\alpha}_1 = 0.290, \qquad \hat{\alpha}_2 = 0.156,$$
$$\hat{\alpha}_3 = 0.554, \qquad \hat{\omega} = 1.25\%.$$

These values are statistically significant and economically plausible - see the Appendix for detailed econometric results. The average growth rate of n in the last ten observations (1991-2001) is $\hat{\mu} = 1.46\%$. Using these estimated values, we obtain from (28) the threshold time-preference rate

$$\delta^{\star} = 13.2\%. \tag{31}$$

It is implausible that the 'true' time-preference rate exceeds the threshold value (31): empirical work by Pearce and Ulph (1999) - based on data for the United Kingdom - suggests that δ is between zero and 1.7%, and the time-preference rate assumed in most analyses on SD is largely below the threshold (*e.g.* Hamilton, 2000). Result (31) imply that the asymptotic sustainability condition is safely fulfilled in the United States: even setting a high time-preference rate, *e.g.* $\delta = 8\%$, would imply a positive sustainability gap ($\delta^{\star} - \delta = 5.2\%$). This result suggests that long-run genuine savings will remain positive, as long as the average time-trend displayed by aggregate output is maintained.

6 Conclusions

In this paper we have investigated the theoretical foundations of the genuine savings criterion for testing sustainable development. Under competitive conditions, positive genuine savings imply positive variations in welfare. However, positive genuine savings at a given point in time do not imply sustainability, because sustainable development also requires non-declining welfare in the long run. This second requirement can be met only if an additional 'asymptotic condition' is satisfied: as suggested by the literature on consumption single-peakedness (Dasgupta and Heal, 1974; Stiglitz, 1974; Pezzey and Withagen, 1998; Valente, 2004), low rates of natural regeneration and insufficient resource-saving innovation are sources of unsustainability, and may imply genuine savings be negative in the long run. Consequently, sustainability tests should incorporate the asymptotic condition.

We have employed the capital-resource model of optimal growth to derive explicit long-run conditions, showing that the asymptotic criterion may be used in conjunction with the genuine saving indicator to test sustainability in a forward looking manner. In the method presented, we define an upper-bound for the time-preference rate. Applying this method on time-series data for the United States, we have shown that the asymptotic condition is likely to be met because the time-trend in aggregate output is sufficiently steep, suggesting that long-run genuine savings will be positive in the US economy in the future.

Appendix

The modified Hotelling rule. The production function assumed in *H1* can be rewritten as $f(k,r,t) \equiv \Phi(k(t), m(t)r(t))$, where $\dot{m} = m\nu$. Since Φ displays constant returns to scale, $\Phi = \Phi_k k + \Phi_{mr} mr$, and the augmented output-resource ratio is $\Phi/mr = (\Phi_k k/mr) + \Phi_{mr}$. Defining the input ratio $x = k/mr$ and $\phi(x) = \Phi/mr$, we can write

$$\phi(x) = \Phi_{mr} + x\phi_x(x), \tag{32}$$

where $\phi_x(x) = \partial\phi/\partial x = \Phi_k$. The partial derivative Φ_r equals $\Phi_{mr} m$. Hence, from (32) we obtain

$$f_r \equiv \Phi_r = m(\phi(x) - x\phi_x(x)), \tag{33}$$

which implies that the time variation of f_r is

$$\dot{f}_r = -x\phi_{xx}(x)\dot{x} \cdot m + f_r \cdot \nu. \tag{34}$$

Setting the right hand side of (34) equal to the right hand side of the Hotelling rule (11) we obtain

$$-x\phi_{xx}(x)\dot{x} \cdot m = f_r \cdot (\rho - g - \nu). \tag{35}$$

Substituting f_r with the right hand side of (33), m cancels out and the Hotelling rule may be written as eq.(21) in the text.

Optimal dynamics. Define the consumption capital ratio as $z = c/k$: using the Keynes-Ramsey rule and the aggregate constraint (7), the growth rate of z equals

$$(\dot{c}/c) - \left(\dot{k}/k\right) = \sigma(\rho - \delta) - (\Phi/k) + z + \gamma. \tag{36}$$

Since $\Phi/k = \phi/x$ and $\rho = \phi_x(x) - \gamma$, we obtain

$$\dot{z}/z = z + \sigma\phi_x(x) - (\phi(x)/x) + \gamma(1-\sigma) - \sigma\delta. \tag{37}$$

Setting $\dot{z} = 0$ in (37) gives

$$z^{ss}(x) = \sigma(\delta - \phi_x(x)) - \gamma(1-\sigma) + (\phi(x)/x). \tag{38}$$

The simultaneous steady state equilibrium $(x^{ss}, z^{ss}(x^{ss}))$ is locally a saddle-point, by virtue of the linearized system[9]

$$\begin{pmatrix} \frac{d}{dt}(x - x^{ss}) \\ \\ \frac{d}{dt}(z - z^{ss}) \end{pmatrix} = \begin{bmatrix} -f_r^{ss}/x & 0 \\ \\ z^{ss}\left((f_r^{ss}/x^2) + \sigma\phi_{xx}(x^{ss})\right) & z^{ss} \end{bmatrix} \begin{pmatrix} (x - x^{ss}) \\ \\ (z - z^{ss}) \end{pmatrix} \tag{39}$$

which exhibits two real roots: $\lambda_1 = -f_r^{ss}/x < 0$, and $\lambda_2 = z^{ss}(x^{ss})$. Hence, given a steady state equilibrium with positive consumption $z^{ss}(x^{ss}) > 0$, the equilibrium is a saddle-point $(\lambda_2 > 0)$, and $\lim_{t\to\infty} z(x) = z^{ss}(x^{ss})$ along the optimal path.[10]

Estimation results. Yearly time-series data for k (real net stock of fixed assets excluding consumers durable goods), y (real gross domestic product), and n (hours worked by full-time and part-time employees) are taken from BEA (2003) - National Income and Product Accounts. Time-series for r (thousands barrels of petroleum products supplied) are taken from EIA (2002). Estimating equation (30) with dynamic linear least squares we have obtained the results reported in Table 1. Estimated coefficients $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ are all significant. Estimated values used in the text are $\hat{\alpha}_1 = \hat{\beta}_2$, $\hat{\alpha}_2 = \hat{\beta}_3$, $\hat{\alpha}_3 = 1 - \hat{\alpha}_1 - \hat{\alpha}_2$, and, by (29), $\hat{\omega} = \exp\left[\hat{\beta}_1\right] - 1$.

[9] The steady state locus $\dot{z} = 0$ may be increasing or decreasing in the phase plane (x, z), depending on the values of the parameters, but this does not affect the stability properties of the steady state equilibrium (for expositional clarity, only the case of a positive-sloped locus $\dot{z} = 0$ has been considered in Figures 1 and 2, where the locus $\dot{z} = 0$ is indicated as $\dot{c}/c = \dot{k}/k$).

[10] Explosive paths in the consumption-capital ratio can ruled out as follows: if z diverges to $-\infty$, consumption will become negative, which is not allowed along the optimal path; if z diverges to plus infinity, the budget constraint (7) implies that the stock of man-made capital will become negative in finite time, violating the non-negativity constraint.

Coefficient		std. error	t-statistic
β_0	1.1572	0.2563	4.514
β_1	0.0124	0.0009	12.708
β_2	0.2901	0.0806	3.600
β_3	0.1558	0.0251	6.200
β_4	-0.3342	0.1054	-3.171
β_5	0.0808	0.0957	0.843
β_6	0.0148	0.1315	0.113
β_7	0.1558	0.0873	1.784
Observations:	51	Sample:	1949-2001
Adj. R^2:	0.997	S.E. reg.:	0.0123
DW-test:	0.564	F-stat.:	2926.451

Table 1: Estimation results of equation (30). The third and fourth columns report the standard error and the probability of the t-statistic test associated to estimated coefficients: the relevant coefficients ($\beta_0, ..., \beta_3$) are all significant; 'S.E. reg.' is the standard error of the regression, 'DW-test' is the Durbin-Watson test for autocorrelation.

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