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Endogenous Growth, Tax Policy and Human Capital Formation

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Abstract

We study the effects of public investments in education financed through proportional taxes in a model with overlapping generations. Studying-time and educational expenditures increase labor efficiency, generating a tradeoff between human and physical capital at the aggregate level. We describe the reactions of the labor supply to perspective tax rates, and we compare private and public education regimes. Under the optimal policy, public education is plausibly growth-improving, but welfare gains are unevenly distributed among generations due to high taxation in the first period of life. Under an alternative, non-optimal policy that shifts the tax burden on the second period of life, public education is unambiguously growth-improving with respect to the private system, and welfare gains are distributed more equally among generations.

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1 Introduction

There is a considerable amount of recent literature exploring the implications of the life-cycle hypothesis in endogenous growth models. After the pioneering contributions of Romer (1986) and Lucas (1988), the assumption that human capital is the engine of economic growth has been exploited to address policy and distributional issues (*e.g.* Azariadis and Drazen, 1990; Glomm and Ravikumar, 1992; Jappelli and Pagano, 1994; 1999; De Gregorio and Kim, 2000). This paper studies the effects of educational expenditures and taxation on growth, welfare and intergenerational equity. We present a model with two overlapping generations of consumers-workers, where young individuals devote part of their time to study (De Gregorio, 1996; Bouzahzah *et al.*, 2002; Yakita, 2003). Studying-time and educational expenditures increase labor efficiency in the subsequent period of life through a learning technology, which determines long-run growth (Buiter and Kletzer, 1995). Newborn generations inherit a positive fraction of individual knowledge from the current state of the economy. This intergenerational externality makes the market equilibrium sub-optimal, implying an active role for public intervention (Docquier and Michel, 1999).

The effects of fiscal policies are analyzed by comparing two different education systems, a *private school regime*, where young individuals pay their own education costs, and a *public school regime*, where education is financed through proportional taxes on labor earnings. Our first aim is to characterize optimal policies: we solve the standard Ramsey-problem, deriving a simple fiscal rule that decentralizes optimal allocations in the public regime. The optimal scheme involves high public propensity to spend in education, and high tax rates on young generations. The reason is that agents do not internalize the benefits of knowledge transmission: in the private regime human capital formation is below socially optimal levels, while the optimal fiscal policy in the public regime implies higher studying time.

We analyze the reactions of the labor supply to perspective tax rates, showing that optimal policies improve long-run growth for plausible ranges of the values of parameters. However, the benefits of public education are unevenly distributed among generations. Since adults do not save, taxes on young individuals reduce aggregate savings. Consequently, individual utility may be higher in the private education regime in the short run, while late-in-time generations are better off in the public school system, due to welfare gains stemming from growth.

We study alternative tax policies which redistribute the social benefits of public education among generations: we assume that the government, instead of pursuing optimal policies, shifts the tax burden from the first to the second period of life. We consider a particular fiscal rule, which we call *labor-neutral taxation*. Under this policy, tax rates are adjusted so that working time is equal between public and private regimes. We show that labor-neutral taxation in the public regime implies higher growth with respect to the private system. This result hinges on a crowding-in mechanism: public education increases disposable income of young generations, and physical capital is higher; the government taxes adult generations, that would otherwise consume their income, to finance expenditures; public investments increase human capital formation, and the long-run growth rate is always higher with respect to the private regime.

The issue of intergenerational equity is tackled by comparing individual welfare under labor-neutral versus optimal policies. Optimal policies reduce the accumulation of physical capital, whereas labor-neutral policies sustain short-run output through increased savings. Hence, the welfare of early-in-time generations is higher under labor-neutral policies. A numerical example confirms that optimal policies may yield high long-run growth, while intertemporal benefits are distributed more equally when young generations are allowed to postpone the cost of education.

2 The model

In order to compare welfare and growth under alternative school systems, we consider two economies indexed by $i = A$, B, with identical technologies, preferences, population and initial endowments. We assume a constant population of consumers-workers who live for two periods: in period t there are n young and n adult individuals, and each young inherits individual knowledge from the current state of the economy. Knowledge is represented by $\bar{h}^i$, measured in terms of labor-efficiency units. Individuals are endowed with one unit of time: in the first period of life a fraction $(1 - \ell_t^i)$ is devoted to study, and $\ell_t^i \bar{h}_t^i$ labor units are supplied for production. In the second period, individuals only work, and consume all their income. Labor choices of young individuals generate a tradeoff between human and physical capital at the aggregate level: studying in the first period of life implies higher potential labor income when adult; working allows to save in the form of interestbearing physical capital. The level of efficiency achieved at the beginning of the second period of life depends on studying time and school quality E^i, according to the learning technology

$$\bar{h}^i_{t+1} = \bar{h}^i_t \varphi^i_t, \tag{1}$$

$$\varphi^i_t = \Psi \left(1 - \ell^i_t\right)^{\varepsilon} \left(E^i_t\right)^{\eta}, \qquad i = A, B, \tag{2}$$

where $\Psi > 0$ is a proportionality factor, the learning technology exhibits decreasing returns in both arguments ($0 < \varepsilon < 1$, $0 < \eta < 1$) and non-increasing returns to scale ($\varepsilon + \eta \leq 1$).

Equation (1) describes an intergenerational externality *à la* Lucas (1988). Similar learning processes are assumed by Glomm and Ravikumar (1992), Docquier and Michel (1999), Yakita (2003).[1] School quality is indexed by the levels of private and public spending in education, which are perfect substitutes (Buiter and Kletzer, 1995): denoting private spending in education of each young consumer by V^i, and per-young public spending by G^i, school quality equals

$$E^i_t = \frac{V^i_t + G^i_t}{\bar{h}^i_t} = v^i_t + g^i_t, \tag{3}$$

where $v^i_t = V^i_t / \bar{h}^i_t$ is the private spending ratio, and $g^i_t = G^i_t / \bar{h}^i_t$ is the public spending ratio. Hence, public spending is inherently productive.[2] Aggregate human capital H^i_t is the amount of labor supplied by the two generations alive in period t :

$$H^i_t = n\bar{h}^i_t + n\ell^i_t \bar{h}^i_t = \left(1 + \ell^i_t\right) h^i_t, \tag{4}$$

where $h^i_t = n\bar{h}^i_t$ is the aggregate amount of knowledge in each generation. Since agents have identical preferences, total labor supply evolves according to

$$H^i_{t+1} = \left(1 + \ell^i_{t+1}\right) h^i_t \varphi^i_t = \varphi^i_t \left(\frac{1 + \ell^i_{t+1}}{1 + \ell^i_t}\right) H^i_t. \tag{5}$$

[1] Glomm and Ravikumar (1992) compare public and private school systems assuming that school quality is determined by parental bequests, and ruling out physical capital. Buiter and Kletzer (1995) analyze intergenerational transfers when there are explicit borrowing constraints, but there is no tradeoff between studying and working. De Gregorio (1996) studies the effects of borrowing constraints without government intervention. Docquier and Michel (1999) analyze the effects of demographic shocks in a three-period model with retirement and public transfers. Yakita (2003) studies interest and wage taxation without productive expenditures.

[2] The macroeconomic effects of productive public expenditures are analyzed by Barro (1990), Turnovsky (1996), Heijdra and Meijdam (2002).

Aggregate output (Y) is produced by means of human and physical capital (K), which is fully depreciated in the production process. Both inputs are essential, and the production function is $Y = K^{\alpha}H^{1-\alpha}$, with $0 < \alpha < 1$. Setting $k = K/H$, the output-human capital ratio $y = Y/H$ equals

$$y_t^i = \left(k_t^i\right)^{\alpha}. \tag{6}$$

The production sector behaves like a single competitive firm, and profit maximization implies

$$R_t^i = \alpha\left(Y_t^i/K_t^i\right), \tag{7}$$

$$w_t^i = (1-\alpha)\left(k_t^i\right)^{\alpha}, \quad i = A, B, \tag{8}$$

where w is the wage rate, and R is the interest factor. Individual consumption is denoted by c when young, and by d when adult. Preferences are logarithmic and individual lifetime utility U is represented by

$$U_t^i = \log\left(c_t^i\right) + \beta\log\left(d_{t+1}^i\right), \tag{9}$$

where $\beta \in (0,1)$ is the discount factor.

We assume that economy A is a pure private system: education costs are paid by young generations, without any intervention of the government ($v_t^A > 0$, $g_t^A = 0$). Economy B is a pure public school system ($v_t^B = 0$, $g_t^B > 0$), where total spending in education is financed through proportional taxes on labor earnings. The accumulation laws of individual knowledge in the two economies are thus

$$\bar{h}_{t+1}^A = \bar{h}_t^A\varphi_t^A, \quad \text{with} \quad \varphi_t^A = \Psi\left(1-\ell_t^A\right)^{\varepsilon}\left(v_t^A\right)^{\eta}, \tag{10}$$

$$\bar{h}_{t+1}^B = \bar{h}_t^B\varphi_t^B, \quad \text{with} \quad \varphi_t^B = \Psi\left(1-\ell_t^B\right)^{\varepsilon}\left(g_t^B\right)^{\eta}. \tag{11}$$

We begin by considering the temporary equilibrium in the private system.

2.1 Private education regime

In economy A, each consumer faces the following budget constraints:

$$c_t^A = w_t^A\ell_t^A\bar{h}_t^A - v_t^A\bar{h}_t^A - s_t^A, \tag{12}$$

$$d_{t+1}^A = R_{t+1}^A s_t^A + w_{t+1}^A\bar{h}_{t+1}^A, \tag{13}$$

where s represents individual savings. Consumers choose first-period working time, and how to allocate consumption between the two periods in order to maximize lifetime utility: each individual solves

$$\max_{\left\{c_t^A, d_{t+1}^A, \ell_t^A, v_t^A\right\}} \log\left(c_t^A\right) + \beta \log\left(d_{t+1}^A\right)$$

subject to

$$c_t^A + \frac{d_{t+1}^A}{R_{t+1}^A} = \bar{h}_t^A \left[\ell_t^A w_t^A - v_t^A + \frac{w_{t+1}^A}{R_{t+1}^A}\varphi_t^A\right], \tag{14}$$

taking wage rates and the interest factor as given. In this problem, the private spending ratio v^A is a control variable. The solution is given by a two-step procedure: in the first step, individuals maximize lifetime income by choosing ℓ^A and v^A, with first order conditions[3]

$$R_{t+1}^A w_t^A = -\varphi_{\ell_t}^A w_{t+1}^A, \tag{15}$$

$$R_{t+1}^A = \varphi_{v_t}^A w_{t+1}^A. \tag{16}$$

In the second step, consumers maximize U taking lifetime income as given: logarithmic preferences imply

$$d_{t+1}^A = \beta c_t^A R_{t+1}^A. \tag{17}$$

Denoting aggregate savings by $S_t = ns_t$, we can substitute equilibrium prices (7)-(8) and the first order conditions in the budget constraint (14), obtaining

$$S_t^A = \frac{w_t^A h_t^A}{1+\beta}\left[\beta \ell_t^A + \beta v_t^A \left(\varphi_{v_t}^A / \varphi_{\ell_t}^A\right) + \left(\varphi_t^A / \varphi_{\ell_t}^A\right)\right] \equiv K_{t+1}^A, \tag{18}$$

which can be rewritten as

$$k_{t+1}^A = \frac{(1-\alpha)\left[\ell_t^A (1+\beta\eta+\beta\varepsilon) - 1 - \beta\eta\right]}{(1+\beta)\left(1+\ell_{t+1}^A\right)\varphi_t^A} y_t^A. \tag{19}$$

The optimal amount of working time supplied by young generations determines, together with the accumulation rule (19), the temporary equilibrium

[3]The assumed learning technology ensures that it is optimal to spend a positive fraction of time $(1-\ell)$ in studying. The assumption $\varepsilon + \eta \leq 1$ implies the marginal propensity to consume be non-negative.

of the economy. When there is tradeoff between studying and working, it is possible to obtain a *stationary solutions*, where working time jumps at the equilibrium level $\ell_\star$ in period zero and is constant thereafter (De Gregorio, 1996; de la Croix and Michel, 2002). In our model, the assumed learning technology implies a stationary solution:[4] substituting the first order condition (15) in the accumulation rule (19), the dynamics of ℓ_t^A are described by

$$\ell_{t+1}^A = \varepsilon q^A \frac{\ell_t^A}{1-\ell_t^A} - p^A, \tag{20}$$

where

$$q^A = \frac{\beta\,(1-\alpha)}{\alpha\,(1+\beta)} > 0, \quad p^A = 1 + \frac{(1+\beta\eta)\,(1-\alpha)}{\alpha\,(1+\beta)} > 1. \tag{21}$$

We show in the Appendix that equation (20) has a unique steady state solution, with unstable dynamics outside the stationary equilibrium. Therefore,

Lemma 1 *In the private education regime, working time supplied by young generations is equal to the optimal level $\ell_\star^A$ in each period, with*

$$\ell_\star^A = \frac{1}{2}\left(1-\varepsilon q^A - p^A\right) + \frac{1}{2}\sqrt{\left(1-\varepsilon q^A - p^A\right)^2 + 4p^A}. \tag{22}$$

Proof. See Appendix. ‖

Using the accumulation rule (19), we can define a lower bound $\ell_{\min}^A$ which is the minimum amount of ℓ^A consistent with positive savings:

$$\ell_\star^A > \ell_{\min}^A = \frac{1+\beta\eta}{1+\beta\varepsilon+\beta\eta}. \tag{23}$$

We also define the *private propensity to spend in education* as $\rho_t^A = (v_t^A/y_t^A)$. The first order condition (16) can be rewritten as

$$v_t^A = \rho_t^A y_t^A = \frac{\eta}{\varepsilon}\,(1-\alpha)\left(1-\ell_\star^A\right) y_t^A, \tag{24}$$

[4] When the learning technology is of the form (2), the dynamics of ℓ_t^A do not depend on the level of the private spending ratio v_t^A. Lemma 1 below shows that this is a sufficient condition for stationary working time in the private regime.

which implies that the private propensity to spend in education is time-invariant. As a consequence, the accumulation rule (19) may be rewritten as

$$k_{t+1}^{A} = z^{A}\left(y_{t}^{A}\right)^{1-\eta} = z^{A}\left(k_{t}^{A}\right)^{\alpha(1-\eta)}, \tag{25}$$

where the *accumulation rate* z equals

$$z^{A} = \frac{(1-\alpha)\left[\ell_{\star}^{A}\left(1+\beta\eta+\beta\varepsilon\right)-1-\beta\eta\right]}{\varepsilon\left(1+\beta\right)\left(1+\ell_{\star}^{A}\right)\Psi\left(1-\ell_{\star}^{A}\right)^{\varepsilon}\left(\rho^{A}\right)^{\eta}}. \tag{26}$$

Since $\alpha\left(1-\eta\right) < 1$, the physical-human capital ratio will converge to a steady state level in the long run.

2.2 Public education regime

In economy B, the government finances public expenditures through labor income taxation. The individual budget constraints are

$$c_{t}^{B} = w_{t}^{B}\ell_{t}^{B}\bar{h}_{t}^{B}\left(1-x_{t}\right)-s_{t}^{B}, \tag{27}$$

$$d_{t+1}^{B} = R_{t+1}^{B}s_{t}^{B}+w_{t+1}^{B}\bar{h}_{t+1}^{B}\left(1-\theta_{t+1}\right), \tag{28}$$

where x and θ are proportional tax rates levied on labor earnings in the first and in the second period of life, respectively. The government keeps balanced budget in each period:

$$g_{t}^{B}h_{t}^{B} = w_{t}^{B}h_{t}^{B}\left(\theta_{t}+x_{t}\ell_{t}^{B}\right). \tag{29}$$

The *public propensity to spend in education* is $\rho_{t}^{B} = \left(g_{t}^{B}/y_{t}^{B}\right)$. In the public regime, individuals maximize lifetime income by choosing ℓ^{B}, anticipating tax rates and public spending plans with perfect foresight. Each consumer solves

$$\max_{\left\{c_{t}^{B},d_{t+1}^{B},\ell_{t}^{B}\right\}} \log\left(c_{t}^{B}\right)+\beta\log\left(d_{t+1}^{B}\right)$$

subject to

$$c_{t}^{B}+\frac{d_{t+1}^{B}}{R_{t+1}^{B}} = \bar{h}_{t}^{B}\left[\ell_{t}^{B}w_{t}^{B}\left(1-x_{t}\right)+\left(1-\theta_{t+1}\right)\frac{w_{t+1}^{B}}{R_{t+1}^{B}}\varphi_{t}^{B}\right]. \tag{30}$$

Applying the two-step procedure as before, the first order conditions for an interior solution are

$$R_{t+1}^{B} w_{t}^{B} \left(1 - x_{t}\right) = -\varphi_{\ell_t}^{B} w_{t+1}^{B} \left(1 - \theta_{t+1}\right), \tag{31}$$

$$d_{t+1}^{B} = \beta c_{t}^{B} R_{t+1}^{B}. \tag{32}$$

Since there is no public debt, net investments equal aggregate savings ($S_t^B \equiv K_{t+1}^B$), and the accumulation rule of the economy is

$$k_{t+1}^{B} = \frac{\left(1 - \alpha\right) \left[\ell_{t}^{B} \left(1 + \beta\varepsilon\right) - 1\right] \left(1 - x_{t}\right)}{\left(1 + \beta\right) \left(1 + \ell_{t+1}^{B}\right) \varphi_{t}^{B}} y_{t}^{B}. \tag{33}$$

Rewriting the optimality condition (31) as

$$k_{t+1}^{B} = \frac{\alpha \left(1 - x_{t}\right)}{-\varphi_{\ell_t}^{B} \left(1 - \theta_{t+1}\right)} y_{t}^{B}, \tag{34}$$

we can substitute (34) in (33) to obtain

$$\ell_{t+1}^{B} = \frac{\ell_{t}^{B}}{1 - \ell_{t}^{B}} \varepsilon q_{t+1}^{B} - p_{t+1}^{B}, \tag{35}$$

$$q_{t+1}^{B} = \frac{\beta \left(1 - \alpha\right)}{\alpha \left(1 + \beta\right)} \left(1 - \theta_{t+1}\right) > 0, \tag{36}$$

$$p_{t+1}^{B} = 1 + \frac{\left(1 - \alpha\right)}{\alpha \left(1 + \beta\right)} \left(1 - \theta_{t+1}\right) > 1. \tag{37}$$

Expressions (35), (36) and (37) describe the dynamics of the labor supply of young generations in the public regime. Coefficients q^B and p^B display two differences with respect to the private regime. First, the dynamics of ℓ^B depend on the perspective tax rate on adult generations; in particular, if θ_t is kept constant over time by the policymaker, we have a stationary solution also in the public education regime:

Lemma 2 *If the government sets $\theta_t = \theta$ in each period, working time of young generations in the public education regime is equal to the optimal level $\ell_{\star}^{B}$ in each period, where*

$$\ell_{\star}^{B} = \frac{1}{2} \left(1 - \varepsilon q^{B} - p^{B}\right) + \frac{1}{2} \sqrt{\left(1 - \varepsilon q^{B} - p^{B}\right)^{2} + 4 p^{B}}, \tag{38}$$

$$\ell_{\star}^{B} > \ell_{\min}^{B} = \frac{1}{1 + \beta\varepsilon}. \tag{39}$$

Proof. See Appendix. ‖

A second difference between the two regimes is that optimal working time in the public system does not depend on the marginal effect of educational expenditures on learning.[5] The implication is that working time differs between the two regimes when no taxes are levied on adult generations in economy B. This point is clarified in the following proposition, which characterizes the reaction of the labor supply to perspective tax rates:

Proposition 3 *If the government levies labor income taxes on adult generations with constant tax rate, optimal working time in the public school regime depends on θ with the following properties:*

i. $\frac{\partial}{\partial\theta}\ell_{\star}^{B}(\theta) > 0,\ \lim_{\theta\to 1}\ell_{\star}^{B}(\theta) = 1,\ \lim_{\theta\to-\infty}\ell_{\star}^{B}(\theta) = \ell_{\min}^{B};$

ii. there exists a unique $\bar{\theta}$ such that $\ell_{\star}^{B}\left(\bar{\theta}\right) = \ell_{\star}^{A};$

iii. $\bar{\theta} > 0;$

iv. $\ell_{\star}^{B}(0) < \ell_{\star}^{A}.$

Proof. See Appendix. ‖

Figure 1 describes all the above results. Property (i) is intuitive: when the tax rate on second-period labor earnings is lower, young generations study more and devote less time to work; taxing adults heavily forces individuals to work in the first period of life, in order to accumulate savings and rely on capital income in the second period; symmetrically, subsidizing adult generations reduces working time, bringing $\ell_{\star}^{B}$ towards the lower bound.

Properties (ii)-(iii) define a critical tax rate: setting $\theta_t = \bar{\theta}$ in each period, working time is the same in the two regimes. We will refer to $\bar{\theta}$ as to the *labor-neutral* tax rate. By (iii), labor-neutral policies imply a positive tax on adult generations. Property (iv) establishes that setting $\theta_t = 0$ implies lower working time in the public school regime.

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[5]Formally, by (21) and (37), coefficient p^A depends on η, whereas coefficient p^B does not.

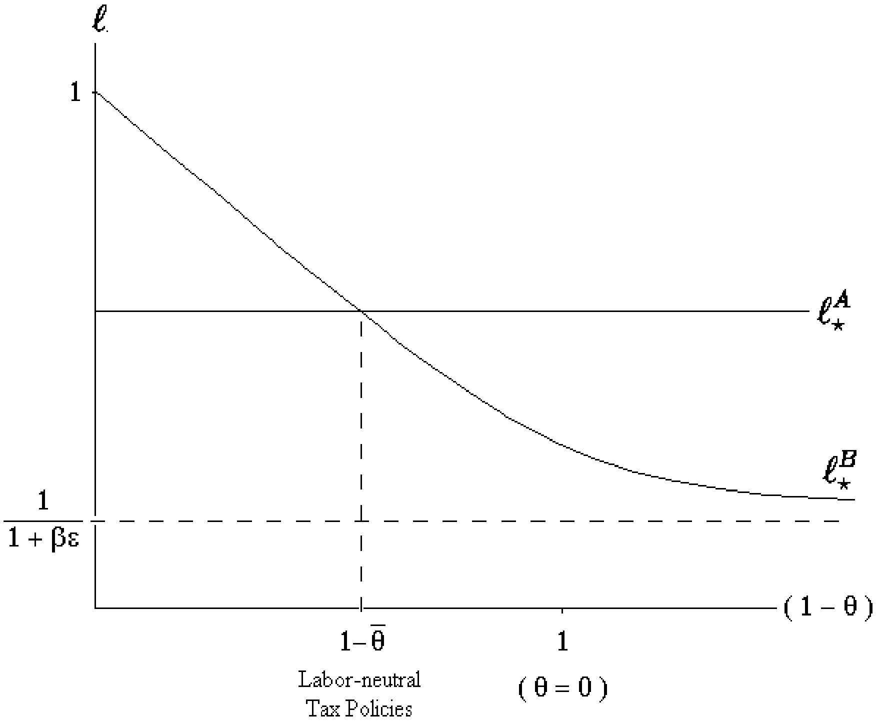

FIGURE 1. Optimal working time as a function of θ in the public regime.

When both tax rates are time-invariant, the economy converges to the balanced growth path: setting $x_t = x$ and $\theta_t = \theta$ in the government budget constraint implies a constant public propensity ρ^B, and the accumulation rule (33) becomes[6]

$$k_{t+1}^B = z^B \left(y_t^B\right)^{1-\eta} = z^B \left(k_t^B\right)^{\alpha(1-\eta)}, \tag{40}$$

$$z^B = \frac{(1-\alpha)\left[(1+\beta\varepsilon)\cdot \ell_\star^B(\theta) - 1\right](1-x)}{\varepsilon(1+\beta)(1+\ell_\star^B(\theta))\,\Psi\,(1-\ell_\star^B(\theta))^\varepsilon (\rho^B)^\eta}. \tag{41}$$

The results of this section can be summarized as follows: from (25) and (40), the physical-human capital ratio and the output-human capital ratio in

[6]When the government adopts an optimal spending plan, assuming constancy of ρ^B, x, and θ is not particularly restrictive. In the next section we show that if θ is constant, the optimal propensity to spend is constant as well, implying also x be time-invariant.

both regimes converge to

$$\lim_{t\to\infty} k_t^i = k_{ss}^i = \left(z^i\right)^{\frac{1}{1-\alpha(1-\eta)}}, \tag{42}$$

$$\lim_{t\to\infty} y_t^i = y_{ss}^i = \left(z^i\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}, \quad i = A, B, \tag{43}$$

and long-run growth rates are determined by the law of motion of human capital:

$$\lim_{t\to\infty}\left(\frac{Y_{t+1}^i}{Y_t^i}\right) = \lim_{t\to\infty}\left(\frac{H_{t+1}^i}{H_t^i}\right) = \Psi\left(1-\ell_\star^i\right)^\varepsilon\left(\rho^i y_{ss}^i\right)^\eta. \tag{44}$$

From (43) and (44), long-run saving rates are constant, and proportional to the respective accumulation rates z^i.

3 Private regime versus optimal policy

In this section we compare growth and welfare in the two regimes, assuming that the policymaker enacts an optimal policy in economy B. Optimal policies are defined according to the standard criterion: the solution of the Ramsey-problem determines the optimal allocation, and an optimal policy is a sequence of fiscal instruments $\{\rho_t^B, x_t, \theta_t\}$ that implements such allocation in the public regime. The Ramsey-problem is solved by a hypothetical central planner, who seeks the sequence of consumption levels, working time and school quality $\{c_t^*, d_t^*, \ell_t^*, E_t^*\}$ that maximizes the discounted sum of lifetime utilities

$$\Upsilon = \sum_{t=0}^{\infty} n\Phi^t\left(\log c_t + \beta\log d_{t+1}\right), \tag{45}$$

where $\Phi \in (0,1)$ is the social discount factor. The social welfare function Υ is maximized subject to the transition law of human knowledge, and the aggregate resource constraint of the economy,

$$h_{t+1} = h_t\Psi\left(1-\ell_t\right)^\varepsilon E_t^\eta, \tag{46}$$

$$K_{t+1} = K_t^\alpha\left[h_t\left(1+\ell_t\right)\right]^{1-\alpha} - nc_t - nd_t - E_t h_t, \tag{47}$$

taking initial endowments (h_0, K_0) as given. Setting the Lagrangean

$$\Pi = \sum_{t=0}^{\infty}\Big\{n\Phi^t\left(\log c_t + \beta\log d_{t+1}\right)$$

$$+\lambda_t^h\left[h_t\Psi\left(1-\ell_t\right)^{\varepsilon}E_t^{\eta}-h_{t+1}\right]$$
$$+\lambda_t^K\left[K_t^{\alpha}h_t^{1-\alpha}\left(1+\ell_t\right)^{1-\alpha}-nc_t-nd_t-E_th_t-K_{t+1}\right]\Big\},$$

we obtain the optimality conditions

$$\lambda_{t+1}^K d_{t+1}^* = \lambda_t^K\beta c_{t+1}^*, \tag{48}$$
$$R_{t+1}^*\lambda_{t+1}^K = \lambda_t^K, \tag{49}$$
$$\lambda_t^h\varphi_t^* = \lambda_{t-1}^h-\lambda_t^K\left[w_t^*\left(1+\ell_t^*\right)-E_t^*\right], \tag{50}$$
$$\lambda_t^K w_t^* = -\varphi_{\ell_t}^*\lambda_t^h, \tag{51}$$
$$\lambda_t^K = \varphi_{E_t}^*\lambda_t^h, \tag{52}$$

where w_t^* and R_t^* indicate optimal marginal productivities of labor and physical capital. The optimal allocation can be implemented in the public regime only if the policymaker sets tax rates x_t and θ_t and the public spending ratio $g_t^B \equiv E_t^B$ in order to satisfy the budget constraint, the aggregate resource constraint, and the optimality conditions of the centralized problem. By (48)-(52), fiscal policy is optimal only if the following relations are satisfied in each period:

$$\rho_t^B = \frac{\eta}{\varepsilon}\left(1-\alpha\right)\left(1-\ell_t^B\right), \tag{53}$$
$$\rho_t^B = \left(1-\alpha\right)\left(\theta_t+x_t\ell_t^B\right), \tag{54}$$
$$\frac{1-\theta_{t+1}}{1-x_t} = 1+\frac{1}{\varepsilon}\left[1-\eta-\left(1-\varepsilon-\eta\right)\cdot\ell_{t+1}^B\right]. \tag{55}$$

Equation (53) is the optimal public propensity to spend in education, which derives from (51)-(52); equation (54) is the government budget constraint; imposing the equality between market factor prices (w_t^B, R_t^B) and optimal marginal productivities (w_t^*, R_t^*) we obtain equation (55) - see Appendix. It derives from (55) that individuals should be taxed more heavily in their first period of life: in particular, when the learning technology is constant-returns-to-scale ($\varepsilon+\eta=1$), optimal taxation requires[7]

$$\frac{1-\theta_{t+1}}{1-x_t} = 2. \tag{56}$$

[7] The properties of optimal policies (see Proposition 4 below) are substantially the same when $\varepsilon+\eta<1$. In the next section we relax the simplifying assumption of constant returns to scale.

From (56), individuals are subsidized in their second period of life whenever the optimal tax rate on first-period income x is below $1/2$, which is always the case for a wide range of plausible values of parameters (see Table 1).

We now define the optimal policy with constant tax rates. A constant tax rate on adult generations implies balanced growth in the long run: when θ is constant, by (53), the optimal propensity ρ^B is time-invariant and, from (54), the tax rate on young generations x must be constant as well. Substituting the optimal tax ratio (56) in the budget constraint, the opportunity set of the policymaker is

$$\rho^B(\theta) = (1-\alpha)\left[x\left(2+\ell_\star^B(\theta)\right)-1\right]. \tag{57}$$

By (57), the optimal policy with constant tax rates and an optimal propensity $\rho^* = \rho^B(\theta^*)$ is feasible and unique.[8] Substituting (53) in (57), optimal tax rates are determined by the system

$$x^* = \frac{1-\eta\ell_\star^B(\theta^*)}{2\varepsilon+\varepsilon\ell_\star^B(\theta^*)} > 0, \tag{58}$$

$$\theta^* = \frac{2\eta-(1+\eta)\,\ell_\star^B(\theta^*)}{2\varepsilon+\varepsilon\ell_\star^B(\theta^*)}, \tag{59}$$

where the sign $x^* > 0$ derives from the fact that $\ell_\star^B < 1 < 1/\eta$. The implications of the optimal policy are summarized in the following

Proposition 4 *Under the optimal policy $\{\rho^*, x^*, \theta^*\}$, working time and the accumulation rate in the public regime are lower than in the private regime, whereas the public propensity to spend in education is higher than private propensity:*

$$\ell_\star^B(\theta^*) < \ell_\star^A, \tag{60}$$

$$\rho^* > \rho^A, \tag{61}$$

$$z^B(\rho^*, x^*, \theta^*) < z^A. \tag{62}$$

[8] By (57), the optimal tax policy is feasible because, for a given level of θ, any value of ρ^B can be sustained by adjusting the tax rate x accordingly. It is unique because for a given level $\theta = \theta^*$, also the optimal propensity $\rho^* = \rho^B(\theta^*)$ is determined - by (53) - and there is only one value $x = x^*$ that satisfies the budget constraint (57).

Proof. We prove inequality (60) by showing that $\theta^* < \bar{\theta}$. The proof is by contradiction: assuming $\theta^* \geq \bar{\theta}$, Lemma 2 and Proposition 3 would imply

$$\ell^A_{\min} < \ell^A_\star \leq \ell^B_\star\left(\theta^*\right). \tag{63}$$

However, if $\theta^* \geq \bar{\theta}$, the optimal tax rate must be strictly positive, and this requires $\ell^B_\star\left(\theta^*\right) < \frac{2\eta}{1+\eta}$. Combining this result with (63), we obtain

$$\ell^A_{\min} < \frac{2\eta}{1+\eta}. \tag{64}$$

Setting $\varepsilon + \eta = 1$ in (23), the lower bound $\ell^A_{\min}$ equals $(1+\beta\eta)(1+\beta)^{-1}$, and inequality (64) reduces to $\beta\eta > 1$, which is absurd. Therefore, $\theta^* < \bar{\theta}$ and $\ell^B_\star\left(\theta^*\right) < \ell^A_\star$. Result (61) is obtained by comparing (24) and (53): the public propensity to spend is higher than ρ^A because studying-time is higher in economy B. Finally, comparing (26) with (41), the accumulation rate z^B is lower than z^A for three reasons: x^* is positive, working-time is higher in the public regime, and the public propensity to spend is higher than in the private system.

Proposition 4 can be interpreted as follows. In the private regime, individual studying-time and expenditures in education are below socially optimal levels, because atomistic agents do not fully internalize the intergenerational benefits stemming from human capital formation. Optimal policies cure this market failure by increasing studying-time and the propensity to spend in education. However, the associated tax burden is supported by young generations, thus the accumulation rate z^B is lower: in the public regime, the saving rate is driven down by high tax rates on young individuals, high studying-time, and high public propensity to spend in education.

In terms of growth rates, the negative effect of taxes on savings is plausibly offset by the benefits of human capital formation, at least in the long run. The asymptotic *growth ratio* equals (see Appendix)

$$\frac{\varphi^A_{ss}}{\varphi^B_{ss}} = \left(\frac{1-\ell^A_\star}{1-\ell^B_\star\left(\theta^*\right)}\right)^{1+\frac{\alpha\eta(1-\eta)}{1-\alpha(1-\eta)}} (2)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}. \tag{65}$$

From (65), whether optimal taxation is growth-improving generally depends on the values of α and η. However, we know from Proposition 4 that studying-time is higher in the public regime, and simulations show that the long-run growth rate is higher in economy B for a wide range of plausible values of parameters. Table 1 shows that fixing $\alpha = 0.3$, the optimal policy implies higher long-run growth with η ranging from 0.3 to 0.7.[9] In particular, adult generations are subsidized as a consequence of high tax rates on young workers: the burden of optimal spending falls on young generations, but public investments raise their private benefits in the second period of life.

η	0.7		0.5		0.3	
Economy	A	B	A	B	A	B
$\ell_\star$	0.927	0.904	0.877	0.846	0.825	0.792
x^*		42.1%		40.5%		39.0%
θ^*		-15.8%		-18.8%		-22.0%
ρ	11.9%	15.6%	8.6%	10.8%	5.2%	6.2%
Asy. Saving Rate	7.3%	4.8%	7.4%	4.6%	7.5%	4.5%
$\varphi^B_{ss}/\varphi^A_{ss}$	1.116		1.106		1.101	

TABLE 1. Private versus public regime under the optimal policy. Reported values are obtained by setting $\alpha = 0.3$ and $\beta = 0.8$, and are directly comparable (all variables are independent of the scale factor Ψ).

As regards individual welfare, reduced savings in the public regime have non-negligible consequences on economic activity in the short run. Lifetime utility in both regimes is calculated on the basis of the consumer's first order conditions and equilibrium factor prices. We show in the Appendix that

[9]In Table 1, the magnitude of the growth gains increases when the learning process tends to be spending-intensive. However, this is not always true: a counter-example with $\alpha = 0.5$ shows that growth gains are higher when the learning process tends to be studying-intensive, being negative when $\eta = 0.7$.

welfare in period t is the sum of three components:

$$U_t^i = \underbrace{\Lambda^i}_{\text{Static}} + \underbrace{(1+\alpha\beta)\log k_{t+1}^i}_{\text{Accumulation}} + \underbrace{(1+\beta)\sum_{j=0}^{t}\log\varphi_j^i}_{\text{Growth}}, \quad i = A, B. \tag{66}$$

The *static term* Λ depends on the optimal levels of working time in the two economies, and the sign of the gap $(\Lambda^B - \Lambda^A)$ is generally ambiguous when optimal policies are enacted in the public regime. In addition to Λ, there are two dynamic elements in (66), the accumulation term and the growth term. The *accumulation term* varies in the short run, since it converges to $(1+\alpha\beta)\log k_{ss}^i$ as the economy approaches balanced growth. The last term in (66), instead, grows indefinitely, implying that individual welfare exhibits a positive time-trend over generations: for t_0 large enough, the *growth term* can be rewritten as

$$(1+\beta)\sum_{j=0}^{t}\log\varphi_j^i \approx (1+\beta)\sum_{j=0}^{t_0}\log\varphi_j^i + (t-t_0)(1+\beta)\log\varphi_{ss}^i, \tag{67}$$

where $t \geq t_0$. Expression (67) shows that welfare gains stemming from growth dominate the static term Λ in the long run.

Hence, when the optimal policy is growth-improving $(\varphi_{ss}^A < \varphi_{ss}^B)$, public education increases individual welfare at least in the long run. However, short-run welfare may be higher in the private regime, because the optimal policy lowers *physical* capital accumulation. This case is depicted in Figure 2: the government follows an optimal policy that improves long-run growth, but early-in-time generations are worse off with respect to a pure private regime, and the transition to positive welfare gains may last several periods (depending on the initial conditions, K_0 and h_0).

4 Labor-neutral taxation

Assume that the policymaker, instead of implementing the optimal scheme, shifts the burden of education costs from the first to the second period of life. Intuitively, this strategy has positive consequences on short-run growth because economic activity is sustained by increased savings. In order to assess the overall effect on growth and welfare, we assume that the government implements labor-neutral taxation - as defined in Proposition 3 - and keeps public spending at the optimal level. Formally, the policymaker sets $\theta_t = \bar{\theta}$ in each period, and the public propensity to spend in education equals

$$\bar{\rho} \equiv \rho^B \left(\bar{\theta}\right) = \frac{\eta}{\varepsilon} (1 - \alpha) \left(1 - \ell_\star^B \left(\bar{\theta}\right)\right). \tag{68}$$

Since $\ell_t^B = \ell_\star^B \left(\bar{\theta}\right)$ in each period, the public propensity to spend is constant over time, and the balanced-budget constraint implies a constant tax rate $\bar{x}$ on young generations.[10] The properties of this labor-neutral policy are summarized in the following

Proposition 5 *If the government follows the labor-neutral strategy* $\{\bar{\rho}, \bar{x}, \bar{\theta}\}$, *young generations are subsidized and the accumulation rate is always higher with respect to the private regime: public education guarantees higher growth and welfare, at least in the long run.*

Proof. By Proposition 3, working time is the same in the two economies ($\ell_\star^A = \ell_\star^B = \ell_\star$) and, by (68), private and public propensities coincide ($\rho^A = \rho^B = \bar{\rho}$). The ratio between the accumulation rates equals, by (26) and (41),

$$\frac{z^A}{z^B} = \frac{\ell_\star (1 + \beta\eta + \beta\varepsilon) - 1 - \beta\eta}{[\ell_\star (1 + \beta\varepsilon) - 1] (1 - \bar{x})}. \tag{69}$$

The consumer's first order conditions (15) and (31) may be rewritten as

$$z^A = \frac{\alpha (1 - \ell_\star)}{\varepsilon \Psi \bar{\rho}^\eta}, \qquad z^B = \frac{\alpha (1 - \ell_\star)}{\varepsilon \Psi \bar{\rho}^\eta} \left(\frac{1 - \bar{x}}{1 - \bar{\theta}}\right). \tag{70}$$

Substituting expressions (70) in (69), the labor-neutral tax rate equals

$$\bar{\theta} = \frac{\beta\eta (1 - \ell_\star)}{\ell_\star (1 + \beta\varepsilon) - 1} > 0. \tag{71}$$

It follows from (70) that $z^B > z^A$ if and only if $\bar{\theta} > \bar{x}$. We now prove that $\bar{\theta}$ is surely greater than $\bar{x}$ by showing that $\bar{x} < 0$: from the budget constraint,

$$\bar{x}\ell_\star = (1 - \ell_\star) \frac{\eta}{\varepsilon} - \bar{\theta}. \tag{72}$$

[10] This fiscal policy rule is feasible and unique by a similar reasoning to that made for the optimal policy.

Substituting the critical rate (71), it is easily proven that $\bar{x} > 0$ only if

$$\ell_\star (1 + \beta\varepsilon) > 1 + \beta\varepsilon, \tag{73}$$

which is absurd since $\ell_\star < 1$. Therefore, young generations are subsidized. Since $\bar{\theta} > 0 > \bar{x}$, it derives from (70) that $z^B > z^A$. By (43), the asymptotic levels of the output-human capital ratio y in the two regimes are

$$y_{ss}^A = \left(z^A\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} < y_{ss}^B = \left(z^B\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}, \tag{74}$$

and, by (42), also $k_{ss}^B > k_{ss}^A$. Expression (74) implies that, in the long run, the spending ratio is higher in the public school system ($\bar{\rho}y_{ss}^B > \bar{\rho}y_{ss}^A$). Therefore, long run growth is higher in the economy with public education:

$$\Psi (1 - \ell_\star)^\varepsilon \left(\bar{\rho}y_{ss}^A\right)^\eta = \varphi_{ss}^A < \varphi_{ss}^B = \Psi (1 - \ell_\star)^\varepsilon \left(\bar{\rho}y_{ss}^B\right)^\eta. \tag{75}$$

As regards welfare, choosing t_0 large enough, the welfare gap $U_t^B - U_t^A$ can be written as

$$\Lambda^B - \Lambda^A + \log\left(\frac{k_{ss}^B}{k_{ss}^A}\right)^{1+\alpha\beta} + \sum_{j=0}^{t_0} \log\left(\frac{\varphi_j^B}{\varphi_j^A}\right)^{1+\beta} + (t - t_0)\log\left(\frac{\varphi_{ss}^B}{\varphi_{ss}^A}\right)^{1+\beta}, \tag{76}$$

where we have substituted (67) in (66). Since $k_{ss}^B > k_{ss}^A$ and $\varphi_{ss}^B > \varphi_{ss}^A$, all terms in (76) except the gap $(\Lambda^B - \Lambda^A)$ are surely positive. Even if $\Lambda^B < \Lambda^A$, the last term grows indefinitely and dominates for t sufficiently large, yielding $U_t^B > U_t^A$ in the long run. Note that we have not assumed $\varepsilon + \eta = 1$. ‖

When labor supply effects are neutralized by tax policy, studying in the public regime implies higher disposable income for young generations, and higher saving rates in the long run.[11] This *crowding-in* effect sustains economic activity in the short run, allowing the government to finance high

[11] The crowding-in effect of fiscal policy builds on the fact that *private* education expenditures would be financed out of disposable income by young generations. Whether this is a realistic assumption is an empirical question: in this regard, some evidence suggests that human capital formation is typically constrained, even in most industrialized countries (see Buiter and Kletzer, 1995; De Gregorio, 1996; and the empirical literature quoted therein).

education expenditures, and hence to obtain high growth through human capital formation.[12]

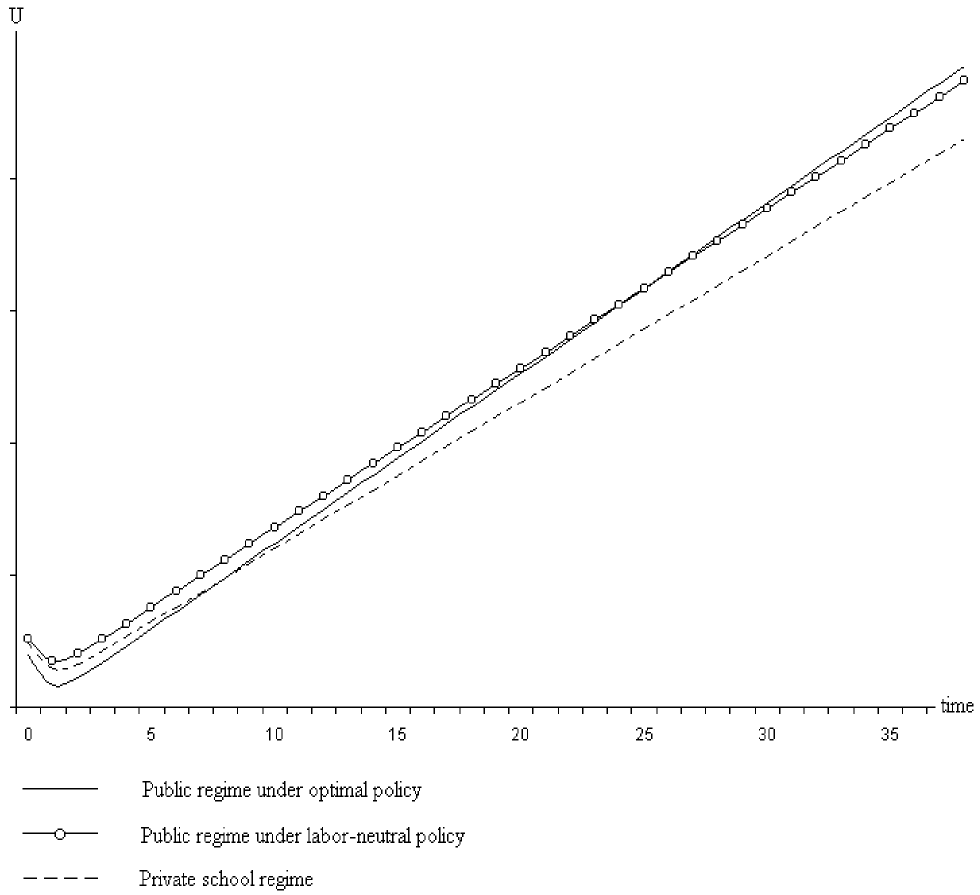

Figure 2. Individual welfare in the public regime under the optimal policy, under labor-neutral policy, and in the private regime. Parameters values are $\alpha = 0.3$, $\beta = 0.8$, $\varepsilon = 0.5$, $\eta = 0.5$, $\Psi = 17.5$, $K_0 = 10$, $h_0 = 1$. Labor-neutral taxation implies $\bar{\theta} = 21.6\%$ and $\bar{x} = -7.4\%$, whereas optimal policy implies $\theta^* = -18.9\%$ and $x^* = 40.5\%$. Asymptotic growth rates equal 2.6% (private regime), 8.4% (labor-neutral policy), and 13.5% (optimal policy).

[12]This mechanism can be exploited by an infinite number of alternative tax policies where the tax burden is supported by adults. The labor-neutral case $\theta = \bar{\theta}$ is only a convenient benchmark to draw analytical comparisons. In general, the *growth-maximizing* policy is neither 'optimal' nor 'labor-neutral': if the government aims exclusively at maximizing long-run growth, the appropriate policy mix depends on whether the learning process is studying-intensive (high ε) or spending intensive (high η).

We now compare individual welfare among the private regime, the public regime under the optimal policy, and the public regime under labor-neutral taxation, in a numerical example. Figure 2 depicts the time paths of lifetime utility in the three cases: the optimal policy brings the highest long-run growth, but low economic activity in the short run; under the labor-neutral policy, short-run productivity is increased through private investments in physical capital, and the welfare gain of public education is distributed more equally among generations;[13] the private regime exhibits low growth in the long run, and occupies a middle position in the short run. These results suggest that optimal policies might not be attractive for a policymaker aiming at preserving the welfare of close-to-the present generations, especially when the net welfare gain of optimal taxation becomes positive only in the very long run (*e.g.* Figure 2).

5 Conclusions

We have analyzed the effects of public investments in education financed through proportional taxes on labor earnings. Human capital formation is the engine of economic growth, and agents face a tradeoff between studying and working in the first period of life. We have shown that public education increases growth when savings and/or human capital formation are stimulated by suitable fiscal policies. In related work, Glomm and Ravikumar (1992) show that public education is growth-*reducing* with respect to a private school system, in a model with parental bequests and no physical capital. Our results hinge on different assumptions: physical capital is essential for production, and private education is self-financed by young generations. In this setting, private education crowds-out savings by reducing disposable income, hence public education may relieve the tradeoff between physical and human capital.

We have studied the effects of alternative tax policies on growth and welfare. Since agents do not internalize the benefits of knowledge transmission, public spending and taxes may be used to replicate optimal allocations: the optimal tax policy increases studying-effort and may yield large welfare gains

[13]Almost by definition, labor-neutral policies imply a deadweight loss for adults in period zero: public education of those who are young in $t = 0$ must be financed by a generation who experienced the previous school system. This 'first generation of adults' is implicitly left aside in our considerations about intergenerational equity.

in the long run. However, early-in-time generations may be worse off with respect to a pure private regime, because high tax rates on young generations reduce savings. In the opposite strategy, the government shifts the burden from the first to the second period of life: short-run activity is sustained by private investments in physical capital and, in the 'labor-neutral case', long-run growth is higher with respect to the private regime. The welfare gains of the public regime are distributed more equally among generations when the government allows young individuals to postpone education costs.

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Appendix

A1. Lemma 1 and Lemma 2: a common proof

The dynamics of optimal working time in the two regimes are described by equations (20) and (35). Since q^B and p^B are constant over time when the policymaker keeps $\theta_t = \theta$ in each period, Lemma 1 and Lemma 2 can be proven by studying the generic dynamic equation

$$\ell_{t+1} = \varepsilon q \frac{\ell_t}{1-\ell_t} - p, \tag{A1}$$

where we have suppressed the superscript $i = A, B$ to simplify notation. Taking the limits on the right hand side of (A1) we obtain $\lim_{\ell_t \to 0} \ell_{t+1} = -p$ and $\lim_{\ell_t \to 1} \ell_{t+1} = +\infty$, that imply the existence of a stationary solution $\ell_{t+1} = \ell_t$. Rewriting (A1) as

$$\ell_{t+1} = -q \left(\frac{\varphi_{\ell_t}}{\varphi_t} \right) \ell_t - p,$$

the derivative

$$\chi(\ell_t) = \frac{\partial \ell_{t+1}}{\partial \ell_t} = -q \left[\frac{\varphi_{\ell_t}}{\varphi_t} + \ell_t \left(\frac{\varphi_{\ell_t \ell_t}}{\varphi_t} \right) - \ell_t \left(\frac{\varphi_{\ell_t}}{\varphi_t} \right)^2 \right]$$

is strictly positive, implying that the stationary equilibrium $\ell_\star$ is unique. Setting $\ell_{t+1} = \ell_t$ in (A1) gives a second-order equation in ℓ with two roots of opposite sign: since $p > 1$, the positive root is $\ell_\star$ as defined by equations (22) and (38). Evaluating χ at $\ell_t = \ell_\star$ gives

$$\chi(\ell_\star) = \frac{p + \ell_\star}{\ell_\star} - q\ell_\star \left[\frac{\varphi_{\ell_t \ell_t}}{\varphi_t} - \left(\frac{\varphi_{\ell_t}}{\varphi_t} \right)^2 \right] > 1 + \frac{p^A}{\ell_\star^A} > 1,$$

which implies that optimal working time displays unstable dynamics outside the stationary equilibrium $\ell_\star$. Consequently, working time jumps at the optimal level $\ell_\star$ in period zero and is constant thereafter. $\|$

A2. Proof of Proposition 3

Since q^B and p^B depend on the tax rate on adult generations, optimal working time in the public regime is a function of θ. To simplify notation, we define $\Gamma = 1 - \theta$ and study the function $\ell_\star^B(\Gamma)$. Setting $\Omega = 1 - \varepsilon q^B - p^B$ we can write, by (38),

$$\ell_\star^B = \frac{1}{2}\Omega + \frac{1}{2}\sqrt{(\Omega)^2 + 4p^B}, \tag{A2}$$

which implies

$$\frac{\partial \ell_\star^B}{\partial \Gamma} = \frac{1}{2} \left(\Omega_\Gamma + \frac{2\Omega_\Gamma \Omega + 4p_\Gamma^B}{2\sqrt{\Omega^2 + 4p^B}} \right), \tag{A3}$$

where $p_\Gamma^B = \partial p^B / \partial \Gamma > 0$ and $\Omega_\Gamma = \partial \Omega / \partial \Gamma < 0$. We now prove property (i) of Proposition 3: the first step is to show that optimal working time $\ell_\star^B$

decreases with Γ. The proof is by contradiction: assume that $\partial\ell_\star^B/\partial\Gamma > 0$: by (A3), this requires (recalling that $\Omega_\Gamma < 0$)

$$\sqrt{(\Omega)^2 + 4p^B} - \Omega < 2\frac{p_\Gamma^B}{\Omega_\Gamma}.$$

Substituting $\ell_\star^B$ in this inequality we obtain $\ell^B < \frac{p_\Gamma^B}{\Omega_\Gamma} + \Omega < 0$, which is absurd. Therefore $\partial\ell_\star^B/\partial\Gamma < 0$, *i.e.* optimal working time $\ell_\star^B$ is increasing in the tax rate θ. The behavior of $\ell_\star^B$ as Γ goes from 0 to $+\infty$ is as follows: on the one hand, from (36), (37) and (A2), $\lim_{\Gamma\to 0}\ell_\star^B = 1$; on the other hand, after applying de l'Hospital's rule, it can be shown that $\lim_{\Gamma\to\infty}\ell_\star^B = \ell_{\min}^B$ (a detailed proof is available from the author). These results imply property (i) in Proposition 3.

Property (ii) derives from property (i): looking at figure 1, since $\ell_\star^B$ decreases monotonically from 1 to $\ell_{\min}^B$ as Γ goes from 0 to $+\infty$, there is a unique intersection between $\ell_\star^B$ and $\ell_\star^A$. The existence of an intersection $\ell_\star^B = \ell_\star^A$ is ensured by the fact that $\ell_\star^A > \ell_{\min}^A > \ell_{\min}^B$.

Now we prove Property (iv): when $\Gamma = 1$ (that is, when $\theta = 0$) working time is lower in the public regime. When $\Gamma = 1$, the coefficients q^A and q^B coincide, and optimal working time in the two economies differs only because p^A is different from p^B: setting $q^A = q^B = q$ in equations (20) and (35),

$$\begin{aligned}
\ell_\star^A &= \frac{1}{2}\left(1 - \varepsilon q - p^A\right) + \frac{1}{2}\sqrt{(1 - \varepsilon q - p^A)^2 + 4p^A}, \\
\ell_\star^B &= \frac{1}{2}\left(1 - \varepsilon q - p^B\right) + \frac{1}{2}\sqrt{(1 - \varepsilon q - p^B)^2 + 4p^B}.
\end{aligned}$$

By (21) and (37), we know that $p^B < p^A$. Hence, if $\partial\ell_\star^i/\partial p^i > 0$, working time in the public regime is unambiguously lower than in the private regime. In fact, little algebra shows that (evaluating $\partial\ell_\star^i/\partial p^i$ and substituting (A1) in the resulting expression)

$$\frac{\partial\ell_\star^i}{\partial p^i} = \frac{1 - \ell_\star^i}{\sqrt{(1 - \varepsilon q - p^i)^2 + 4p^i}} > 0,$$

which implies that $\ell_\star^B < \ell_\star^A$ when $\theta = 0$. This result also implies Property (iii): $\ell_\star^B$ is increasing in θ, hence $\ell_\star^B$ coincides with $\ell_\star^A$ only for a strictly positive value of θ, which is the labor-neutral rate $\bar{\theta}$.

A3. Derivation of the optimality condition (55)

From (50) and (53), the optimal allocation requires

$$\lambda_{t-1}^{h}=\lambda_{t}^{K}\left(1-\alpha\right)y_{t}\left(1+\ell_{t}-\frac{\eta}{\varepsilon}\left(1-\ell_{t}\right)-\frac{\varphi_{t}}{\varphi_{\ell_{t}}}\right),$$

which implies

$$\frac{\lambda_{t-1}^{h}H_{t}}{\lambda_{t-1}^{K}K_{t}}=\frac{1-\alpha}{\alpha}\left(1+\ell_{t}-\frac{\eta}{\varepsilon}\left(1-\ell_{t}\right)-\frac{\varphi_{t}}{\varphi_{\ell_{t}}}\right). \tag{A4}$$

Substituting the optimality condition (51) in (31) we obtain

$$\frac{1-\theta_{t+1}}{1-x_{t}}=\frac{\alpha}{1-\alpha}\cdot\frac{\lambda_{t}^{h}H_{t+1}}{\lambda_{t}^{K}K_{t+1}}. \tag{A5}$$

Posticipating (A4) and substituting the resulting equation in (A5) gives the optimality condition (55) in the text.

A4. Derivation of equation (65)

We now derive the asymptotic growth ratio $(\varphi_{ss}^{A}/\varphi_{ss}^{B})$ when the optimal policy is enacted in economy B. In general, by (43) and (44), the ratio $(\varphi_{ss}^{A}/\varphi_{ss}^{B})$ equals

$$\frac{\varphi_{ss}^{A}}{\varphi_{ss}^{B}}=\frac{\Psi\left(1-\ell_{\star}^{A}\right)^{\varepsilon}\left(\rho^{A}y_{ss}^{A}\right)^{\eta}}{\Psi\left(1-\ell_{\star}^{B}\right)^{\varepsilon}\left(\rho_{t}^{B}y_{ss}^{B}\right)^{\eta}}=\frac{\left(1-\ell_{\star}^{A}\right)^{\varepsilon}\left(\rho^{A}\right)^{\eta}\left(z^{B}\right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}}{\left(1-\ell_{\star}^{B}\right)^{\varepsilon}\left(\rho_{t}^{B}\right)^{\eta}\left(z^{B}\right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}}. \tag{A6}$$

From (31), the accumulation rate in the public regime can be expressed as

$$z^{B}=\frac{\alpha\left(1-\ell_{\star}^{B}\right)}{\varepsilon\Psi\left(\rho_{t}^{B}\right)^{\eta}}\left(\frac{1-x_{t}}{1-\theta_{t+1}}\right),$$

whereas, in the private regime,

$$z^{A}=\frac{\alpha\left(1-\ell_{\star}^{A}\right)}{\varepsilon\Psi\left(\rho^{A}\right)^{\eta}}.$$

Substituting these expressions for z^{A} and z^{B} in (A6), and using (24), (53), and (56), we obtain expression (65) in the text.

A5. Individual welfare

The optimal levels of consumption in the first period of life in the two economies are $c_t^A = \zeta^A Y_t$ and $c_t^B = \zeta^B Y_t$, where the marginal propensities to consume equal

$$\zeta^A = \frac{\delta(1-\alpha)}{1+\beta}\left(\frac{1-\eta-(1-\varepsilon\eta)\,\ell^A}{n\varepsilon(1+\delta\ell^A)}\right), \tag{A7}$$

$$\zeta^B = \frac{\delta(1-\alpha)}{\varepsilon(1+\beta)}\left[\frac{1-\ell_t^B(1-\varepsilon)}{1+\delta\ell}\right](1-x). \tag{A8}$$

Equations (A7) and (A8) derive from substituting the consumer's first order conditions in individual budget constraints (12)-(14) and (27)-(30), respectively. By (13) and (28), the optimal levels of second-period consumption are

$$d_{t+1}^i = \alpha\beta\left(\frac{Y_t^i}{K_{t+1}^i}\right)\zeta^i Y_{t+1}^i, \qquad i = A, B. \tag{A9}$$

Setting the aggregate saving rate $K_{t+1}^i/Y_t^i = \tilde{z}^i$, it derives from (25) and (40) that $\tilde{z}^i = z^i\Psi(1-\ell_\star^i)^\varepsilon(\rho^i)^\eta$. Using this result, substituting (A7), (A8), and (A9) in (9), lifetime utilities U_t^i along the optimal path may be written as

$$U_t^i = \log\zeta^i + \beta\log\alpha\beta\left(\zeta^i/\tilde{z}^i\right) + \log Y_t^i + \beta\log\left(\tilde{z}^i Y_t^i\right)^\alpha\left(H_{t+1}^i\right)^{1-\alpha}, \tag{A10}$$

where the last term derives from $Y_{t+1} = K_{t+1}^\alpha H_{t+1}^{1-\alpha}$. Substituting, $\log Y_t^i = \log k_{t+1}^i + \log H_{t+1}^i - \log\tilde{z}^i$ in eq.(A10) yields

$$U_t^i = (1+\beta)\log\left(\frac{\zeta^i}{\tilde{z}^i}\right) + \beta\log\alpha\beta + (1+\alpha\beta)\log k_{t+1}^i + (1+\beta)\log\left(H_{t+1}^i\right). \tag{A11}$$

From (5), the dynamic equation of aggregate human capital implies

$$H_{t+1}^i = H_0^i\prod_{j=0}^{t}\varphi_j^i, \tag{A12}$$

where $H_0^i = (1+\ell_\star^i)\,h_0$, where $h_0 = h_0^A \equiv h_0^B$ by assumption. Substituting (A12) in (A11) and rearranging terms we obtain equation (66) in the text, where the static terms in the two regimes equal

$$\Lambda^A = \beta\log\alpha\beta(1-\alpha) + \log\left[\frac{1-\eta-\ell_\star^A(1-\varepsilon-\eta)}{\ell_\star^A(1+\beta\eta+\beta\varepsilon)-1-\beta\eta}\right]^{1+\beta}\left(H_0^A\right)^{1+\beta},$$

$$\Lambda^B = \beta \log \alpha\beta (1-\alpha) + \log \left[\frac{1-(1-\varepsilon)\,\ell_\star^B}{\ell_\star^B (1+\beta\varepsilon)-1} \right]^{1+\beta} \left(H_0^B \right)^{1+\beta}.$$