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Government Deficits, Consumption and the Price Level

Barbara Annicchiarico* and Giancarlo Marini†

March, 2003

Abstract

This paper investigates the dynamics of the price level in a continuous time monetary version of the Yaari-Blanchard model with capital accumulation. It is shown that there is an interaction between fiscal discipline and price stability even when the government budget is intertemporally balanced. Relevant implications are that high debt and slow adjustment adversely affect both price-stability oriented monetary policies and growth.

JEL Classification: E31, E63. Keywords: Fiscal Deficits, Price Stability, Government Debt.

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1 Introduction

The possible negative consequences of large government debt and deficits on price stability and growth are a central issue in the current macroeconomic policy debate. According to the conventional monetarist view the existence of an independent central bank is a necessary and sufficient condition to ensure price stability. However, it is well known, since Sargent and Wallace (1981), that fiscal policy matters for the stability of prices. Excessive deficits and increasing government debt may have to be money financed by an "accommodating" central bank, eventually.

Limitations on the size of public debt and deficits are imposed on each country in the European Monetary Union (EMU), where a single monetary policy is conducted independently by the European Central Bank (ECB). However, concerns for the negative consequences of loose fiscal policies on growth and price stability are periodically expressed by the ECB and the European Commission.

The President of the ECB, Willem Duisenberg, has stated the importance of fiscal discipline for economic growth and for the effectiveness of a monetary policy oriented to price stability on several occasions . For instance, in a speech delivered in 2002 the President asserts that "The Stability and Growth Pact has been successful in promoting sound public finances in the euro area. As a result, 8 of the 12 euro area countries have reached budget positions which are 'close to balance or in surplus', a development which has helped to support the maintenance of price stability and growth in employment and real GDP"[1]. The European Commission also stresses the importance of maintaining a neutral fiscal stance in the EMU: "If the euro area is to enjoy favourable growth and low inflation, it is essential to eliminate the remaining

[1]Willem F. Duisenberg: Opening remarks of the panel on "Recent developments in the world economy" at the 2nd international conference organised by the Banco de Mexico, Mexico City, 12 November 2002.

imbalances and maintain a neutral fiscal stance"[2].

A possible theoretical rationale for the concern about fiscal discipline may be the so-called fiscal theory of the price level proposed by Woodford (1994, 1995) and Sims (1995) [3]. In particular, the proponents of this theory distinguish between *Ricardian* and *non-Ricardian* fiscal regimes. In the first case the nominal anchor is provided by monetary policy: the price level and the inflation rate are determined by the monetary authority. In the second case fiscal policy provides the nominal anchor and selects the equilibrium price level. Specifically, in a *non-Ricardian* regime, prices adjust to satisfy the intertemporal budget constraint of the government and the monetary authorities alone are unable to control the rate of inflation. Canzoneri et Al. (2001) apply the fiscal theory of the price level to the study of common currency areas and monetary unions, demonstrating that a more severe fiscal discipline is required to guarantee the stability of the monetary system. In particular, they show that the fiscal constraints written into the Maastricht Treaty and in the Stability and Growth Pact are sufficient conditions for a *Ricardian regime.* The fiscal theory of the price level has recently been questioned by Buiter (2002) who argues that the theory is logically inconsistent: in particular, the interpretation of the intertemporal budget constraint of the government as an equilibrium condition and not as a constraint is not justifiable.

The present paper analyzes the effects of fiscal policy on price level dynamics in a continuous time general equilibrium model with capital accumulation, where all agents are requested to respect their budget constraints. Our main aim is to show that fiscal variables do influence nominal prices and capital accumulation in a standard optimizing macroeconomic model. In particular, we study both the implications of a lump-sum transfer to households and of an increase in the level of public expenditure in turn. The fiscal expansion is assumed to be entirely financed by issuance of new bonds and

[2]European Commission (2002), p. 13.

[3]This line of research was started by Leeper (1991) following the seminal contribution of Aiyagari and Gertler (1985).

future lump-sum taxation. The nominal stock of money is kept constant over time and both households and the government must always satisfy their intertemporal budget constraint for any sequence of the price level. It is shown that the impossibility of money financing government deficits is not sufficient to ensure price stability after a fiscal shock, even when the fiscal authorities adopt policies ruling out explosive paths for the government debt.

The scheme of the paper is as follows. Section 2 presents the optimizing general equilibrium monetary model with capital accumulation; Section 3 illustrates the effects of a fiscal expansion on the price level, capital stock and consumption. Section 4 concludes.

2 The Optimising General Equilibrium Model

In order to analyze the effects of a fiscal expansion we consider a continuous time monetary model of a closed economy with capital accumulation, perfect foresight and overlapping generations. The economy is assumed to be populated by three type of agents: consumers, firms and the government[4].

2.1 Consumers

The demand side of the economy is described by a monetary version of the Yaari (1965)-Blanchard (1985) model with real money holdings as an argument of the utility function. Money is assumed to provide transaction services and to yield direct utility to the consumers.

All agents are forward looking and face uncertainty about the duration of their lives, since they have a constant instantaneous probability of death, β. For agents born at time s the probability of being alive at time $t \geq s$ is given by $e^{-\rho(t-s)}$.

The birth and death rate are assumed to be identical, so that there is no population growth. For simplicity the size of total population is normalized

[4]Some implications of fiscal policy for price stability in a fruit-tree economy are analised in Annicchiarico and Marini (2003).

to one and is fully employed by firms.

Consumers have the same preferences, face the same sequences of taxes and labor income and decide on consumption, real money holdings, wealth accumulation and portfolio composition. For simplicity, labor income and taxes are independent of age. There is no bequest motive and newly born agents have no assets. Total wealth is composed of real money balances, physical capital and government bonds.

Each agent of the generation born at time s is assumed to solve the following maximization problem

$$\max_{\{c(s,v),\varphi(s,v)\}} \int_t^{\infty} \log[c(s,v)^{\xi}\varphi(s,t)^{1-\xi}]e^{-(\rho+\beta)(v-t)}dv \tag{1}$$

subject to the instantaneous budget constraint

$$\frac{d}{dt}a(s,t) = [r(t)+\beta]a(s,t) + \omega(s,t)l(s,t) - \tau(s,t) - c(s,t) - i(t)\varphi(s,t) \tag{2}$$

and the transversality's condition

$$\lim_{v\to\infty} a(s,v)e^{-\int_t^v [r(u)+\beta]du} = 0 \tag{3}$$

where $c(s,t)$, $\varphi(s,t), a(s,t)$, $\omega(s,t)$, $l(s,t)$ and $\tau(s,t)$ denote consumption, real money holdings, real wealth, real wage income, labor supply and lump-sum taxation at time t of an agent born at time s, respectively; ρ is the constant rate of time preference, $r(t)$ is the real interest rate, $i(t)$ is the nominal interest rate and $0 < \xi < 1$. Real wealth is held in the form of real money balances, physical capital $k(s,t)$ and real government bonds, $b(s,t)$. The instantaneous budget constraint incorporates the hypothesis that in each period consumers of generation s receive an actuarially fair premium $\beta a(s,t)$ from competitive life insurance companies. At the time of death all their wealth goes to the insurance companies.

By using a two-stage budgeting procedure the consumer's maximization problem yields the following solution for individual consumption

$$c(s,t) = \frac{\beta+\rho}{1+\eta}\left[a(s,t) + h(s,t)\right] \tag{4}$$

and the portfolio balance condition

$$\frac{m(s,t)}{P(t)} = \frac{\eta c(s,t)}{r(t)+\pi(t)} \tag{5}$$

where $\eta \equiv \frac{1-\xi}{\xi}$, $m(s,t)$ is the nominal stock of money $P(t)$ is the price level, $\pi(t) = \frac{\overset{\bullet}{P}(t)}{P(t)}$ is the inflation rate and $h(s,t)$ is human wealth defined as the present discounted value of labor incomes net of taxes

$$h(s,t) = \int_t^{\infty} \left[\omega(s,v)l(s,v) - \tau(s,v)\right] e^{-\int_t^v [r(u)+\beta]du} dv \tag{6}$$

Population aggregates for each economic variable $x(s,t)$ can be easily obtained operating as follows

$$X(t) = \int_{-\infty}^{t} x(s,t)\beta e^{\beta(s-t)} ds \tag{7}$$

where the upper case letter indicates the aggregate value of the generic economic variable x. At time t, in fact, the size of the surviving generation born at time $s \leq t$ is $\beta e^{-\beta(t-s)}$. Integration over all generations gives the following expressions for aggregate consumption, $C(t)$, the portfolio balance condition and human wealth $H(t)$

$$C(t) = \frac{\beta+\rho}{1+\eta}[A(t) + H(t)] \tag{8}$$

$$\frac{M(t)}{P(t)} = \eta \frac{C(t)}{r(t)+\pi(t)} \tag{9}$$

$$H(t) = \int_t^{\infty} [W(v)L(v) - T(v)] e^{-\int_t^v [r(u)+\beta]du} dv \tag{10}$$

where total non-human wealth is defined as

$$A(t) = B(t) + \frac{M(t)}{P(t)} + K(t) \tag{11}$$

The dynamic equations for total consumption, human wealth and non-human wealth are given by

$$\overset{\bullet}{C}(t) = [r(t) - \rho]\, C(t) - \frac{\beta + \rho}{1 + \eta} \beta A(t) \tag{12}$$

$$\overset{\bullet}{H}(t) = [r(t) + \beta]\, H(t) - W(t)L(t) + T(t) \tag{13}$$

$$\overset{\bullet}{A}(t) = r(t)A(t) + W(t)L(t) - C(t) - [r(t) + \pi(t)] \frac{M(t)}{P(t)} - T(t) \tag{14}$$

Equilibrium in the money market is characterized by

$$[\frac{\overset{\bullet}{M}(t)}{P(t)}] = [\mu(t) - \pi(t)]\, \frac{M(t)}{P(t)} \tag{15}$$

where μ is the rate of nominal money growth. Combining (15) with the optimal portfolio balance condition (9) yields

$$\frac{\overset{\bullet}{P}(t)}{P(t)} = \frac{\eta C(t) P(t)}{M(t)} - r(t) \tag{16}$$

2.2 Firms

Competitive firms produce output using a neoclassical production function and rent labor and physical capital from consumers. The representative firm is assumed to choose labor and capital in order to maximize the present discounted value of future profits

$$\max_{\{L(v),K(v)\}} \int_t^{\infty} \{F[K(v), L(v)] - W(v)L(v) - I(v)\} e^{-\int_t^v r(u)du} dv \tag{17}$$

subject to

$$\overset{\bullet}{K}(t) = I(t) \tag{18}$$

where $I(t)$ denotes investment, $F[.]$ is a neoclassical production function, and physical capital is predetermined $K(0) = K_0$. For simplicity the rate of depreciation of physical capital is set equal to zero. The first order conditions for the representative firm maximization problem are $F_K = r$ and $F_L = W$. Labor supply is inelastic and normalized to one, $L(t) = 1$. The equilibrium condition in the goods market is

$$Y(t) = I(t) + C(t) + G(t) \tag{19}$$

where G is real government spending.

2.3 The Government

Public expenditures, transfer payments and interest payments on government bonds are financed by lump-sum taxation, seignorage revenues and issuance of new bonds according to the flow budget constraint

$$\overset{\bullet}{B}(t) = r(t)B(t) - T(t) - \mu(t)\frac{M(t)}{P(t)} + G(t) \tag{20}$$

The public sector must respect the solvency condition

$$\lim_{v \to \infty} B(v) e^{-\int_t^v r(u)du} = 0 \tag{21}$$

Integrating equation (20) forward, given the solvency condition, yields

$$B(t) = \int_t^\infty \left[T(v) - G(v) + \mu(v)\frac{M(v)}{P(v)} \right] e^{-\int_t^v r(u)du} dv \tag{22}$$

which is the intertemporal budget constraint of the public sector.

3 Fiscal Deficits and Price Stability

In this section we analyze the effects of fiscal policy under the assumption that the intertemporal budget constraint of the government always holds for all sequences of price. The government controls lump-sum transfers to households and the level of public expenditure. In order to rule out explosive paths of the public debt, the following taxation rule is adopted

$$T(t) = \alpha(t)B(t) - X \tag{23}$$

where X is a lump-sum transfer constant over time and α is a function of the real interest rate, $\alpha(t) = \alpha[r(t)]$, reflecting the degree of fiscal adjustment and is such that $\alpha' > 1$. The policy rule is such that taxes are raised in order to ensure fiscal solvency, while the rate of money growth is set equal to zero, $\mu(t) = 0$, for simplicity.

Given the policy rule, the basic aggregate model can be described by the following set of differential equations

$$\dot{C}(t) = [r(t) - \rho]\, C(t) - \frac{\beta + \rho}{1 + \eta}\beta[B(t) + K(t) + M_0 Q(t)] \tag{24}$$

$$\frac{\dot{Q}(t)}{Q(t)} = r(t) - \frac{\eta C(t)}{M_0 Q(t)} \tag{25}$$

$$\dot{K}(t) = F[K(t), 1] - C(t) - G \tag{26}$$

$$\dot{B}(t) = r(t)B(t) - \alpha(t)B(t) + X + G \tag{27}$$

with

$$\lim_{v \to \infty} B(v) e^{-\int_t^v r(u)du} = \lim_{v \to \infty} K(v) e^{-\int_t^v r(u)du} = 0 \tag{28}$$

where $Q(t)$ is the inverse of the price level, $Q(t) = 1/P(t)$. The inverse of the price level is the purchasing power of one unit of nominal money. This change of variable allows us to formulate the analysis in terms of real variables. Domestic bonds and physical capital are predetermined, $B(0) = B_0 = 0$, $K(0) = K_0$, while real money, consumption and prices are jump variables, so that $Q(0)$, $C(0) = free$.

Assume that the economy is initially in a steady state equilibrium, with the steady-state values of the macrovariables denoted by C_0, K_0, P_0 and B_0. For simplicity, the nominal stock of money is normalized to one, $M_0 = 1$. At time $t = 0$ there is an unexpected fiscal expansion. We consider the effects of two types of fiscal expansion in turn: an increase in the lump sum transfer X

and an increase in the level of public expenditure G. The fiscal shock creates a sequence of budget deficits and an increase in government debt. At the same time the sequence of current and future lump-sum taxes is determined according to the rule (23), so as to satisfy the solvency constraint. The two fiscal experiments are assumed to be implemented under the hypothesis that the intertemporal budget constraint of the government is always satisfied.

3.1 Long-run effects of a lump-sum transfer

The steady-state effects of a once-for-all lump-sum transfer on consumption, the inverse of the price level, physical capital and real government bonds can be described by the relations

$$\frac{d\overline{C}}{dX} < 0; \frac{d\overline{Q}}{dX} < 0; \frac{d\overline{K}}{dX} < 0; \frac{d\overline{B}}{dX} > 0 \tag{29}$$

The following propositions summarize the results of the fiscal experiment.

Proposition 1 *An increase in lump-sum transfers determines a steady state reduction of both physical capital and consumption, while the price level and the stock of debt increase.*

Proof. See Appendix. ■

On impact, the fiscal expansion increases consumption which determines a decumulation of physical capital during the adjustment process. This can be easily verified considering the consumption function (8) and the definition of human wealth (10). The direct effects of fiscal policy on total consumption can be synthesized by the fiscal index $f(t)$

$$f(t) = \frac{\beta + \rho}{1 + \eta}[B(t) - \int_t^{\infty} T(v)e^{-\int_t^v [r(u)+\beta]du} dv] \tag{30}$$

Before the shock the index is equal to zero. After the unexpected fiscal expansion the index becomes positive, implying a positive effect on current consumption. The terms in brackets present a larger discount factor than the intertemporal budget constraint of the government (22), representing net wealth for consumers. In this model with overlapping generations the positive effect on total wealth produced by an increase in the transfer payments on impact outweighs the negative effects of the future tax increase (see Blanchard, 1985). The fall in capital determines an increase in the interest rate and a fall in the demand for real money balances. Prices increase to restore equilibrium in the money market.

Proposition 2 *The larger α', the smaller the long-run effect of a lump-sum transfer on the price level.*

Proof. See Appendix. ■

The long-run effects are shown to crucially depend on the speed of adjustment of government debt towards the new steady state value. It follows that alternative time paths of fiscal rules ensuring public solvency exert different effects on price level dynamics after a fiscal shock. The more reactive the function $\alpha(.)$ to change in the real interest rate, the greater the burden of fiscal adjustment borne by current generations. The effect on private consumption is smaller after the fiscal experiment and the inflation consequences of fiscal deficits are lower.

Proposition 3 *When $\beta = 0$ the steady-state levels of consumption, the capital stock and the price level are unaffected by a lump-sum transfer.*

Proof. See Appendix. ■

When the instantaneous probability of death is set equal to zero, the model collapses into the infinitely lived representative agent framework. In this case "Ricardian equivalence" holds and the time profile of consumption is not affected by the time profile of taxation. The equilibrium in the money market is not altered by the fiscal experiment and prices do not have to change. In such a case, inflation is a pure monetary phenomenon and fiscal variables do not influence the price level in the absence of money financing.

Heterogeneity is, of course, the key for our result, when the policy experiment is an intertemporal reallocation of debt and taxes in such a way to maintain the intertemporal budget constraint always satisfied. However, as shown below our results still hold even in the case of the representative agent model when the fiscal experiment takes the form of an increase in government purchases.

3.2 Long-run effects of public expenditure

Consider the steady-state effects of an increase in public expenditure financed by the issue of new bonds and future taxation on the relevant variables of the model

$$\frac{dC}{dG} < 0; \frac{dQ}{dG} < 0; \frac{dK}{dG} < 0; \frac{dB}{dG} > 0 \tag{31}$$

The results of the fiscal experiment are summarized by the following propositions.

Proposition 4 *An increase in public expenditure determines a reduction in the long-run level of physical capital and consumption and an increase in the steady state price level and government bonds.*

Proof. See Appendix. ■

The fiscal expansion negatively affects human wealth, the level of physical capital and reduces consumption. Following the increase in public expenditure, the present discounted value of future taxes increases, reducing human wealth. However, in a first stage the reduction in consumption offsets the excess of demand for goods only partially. There is a decumulation of physical capital during the transition. In the long-run consumption falls by a greater amount and physical capital is below its original level: the price level has to increase to absorb the excess supply of money.

Proposition 5 *The steady-state values of consumption and of the price level are affected by an increase in public spending even when $\beta = 0$.*

Proof. See Appendix. ■

An increase in public expenditure determines a fall in private consumption brought about by a decrease in human capital even in the absence of heterogeneity. Equilibrium in the money market requires an increase in the price level to choke off the excess of real money supply and therefore there is non-monetary inflation during the adjustment path.

3.3 Model Stability

The dynamic properties of the economy can be examined linearizing the system of equations (24)-(27) around the new steady state

$$\begin{pmatrix} \overset{\bullet}{C}(t) \\ \overset{\bullet}{Q}(t) \\ \overset{\bullet}{K}(t) \\ \overset{\bullet}{B}(t) \end{pmatrix} = J \begin{pmatrix} C(t) - \overline{C} \\ Q(t) - \overline{Q} \\ K(t) - \overline{K} \\ B(t) - \overline{B} \end{pmatrix} \tag{32}$$

where J is the Jacobian matrix of the system defined as

$$J = \begin{pmatrix} F_K - \rho & -\frac{\beta+\rho}{1+\eta}\beta & F_{KK}\overline{C} - \frac{\beta+\rho}{1+\eta}\beta & -\frac{\beta+\rho}{1+\eta}\beta \\ -\eta & F_K & F_{KK}\overline{Q} & 0 \\ -1 & 0 & F_K & 0 \\ 0 & 0 & (1-\alpha')F_{KK}\overline{B} & F_K - \overline{\alpha} \end{pmatrix}$$

The model presents a unique convergent adjustment path if the Jacobian has two positive and two negative eigenvalues, since public debt and physical capital are predetermined, while prices and consumption are jump variables. The system satisfies the stability property and the model yields a unique convergent path towards the steady state, as shown in the Appendix.

4 Conclusions

The major finding of the paper is that a fiscal expansion entirely financed by future lump-sum taxation determines an increase in the long-run level of prices and a reduction in the steady state level of physical capital. Prices increase along the adjustment path towards the new steady state, even when the government always satisfies the intertemporal budget constraint without resorting to seignorage revenues. In the long run government bonds crowd out physical capital and consumption is reduced. The excess of real money supply requires an increase in the price level to clear the money market.

The inflationary consequences of high public debt and slow adjustment have also been derived. In particular, both the propositions that high debts are inflationary and that a slow speed of adjustment has adverse effects on price stability unambiguously emerge from the model.

Notably, the result that an increase in public expenditure is inflationary is shown to hold even in the representative agent framework.

These results show that the respect of the intertemporal budget constraint without resorting to money finance is not sufficient to guarantee price stability. An independent central bank appears to be only a necessary but not a sufficient condition. Our results provide theoretical foundations to the view that fiscal policy can negatively influence price stability and capital accumulation.

References

AIYAGARI, R. S. and GERTLER, M. (1985), "The Backing of Government Bonds and Monetarism", *Journal of Monetary Economics*, 16, 19-44.

ANNICCHIARICO, B. and MARINI, G. (2003), "Fiscal Policy and Price Stability", (Mimeo: University of Rome Tor Vergata).

BLANCHARD, O. J. (1985), "Debt, Deficits, and Finite Horizons", *Journal of Political Economy*, 93, 223-247.

BUITER, W. H. (2002), "The Fiscal Theory of the Price Level: A Critique", *The Economic Journal*, 112, 459-480.

CANZONERI, M. B. , CUMBY, R. E. and DIBA, B. T. (2001), "Fiscal Discipline and Exchange Rate Systems", *The Economic Journal*, 111, 667-690.

DUISENBERG, W. F. (2002), Opening remarks of the panel on *Recent developments in the world economy* at the 2^{nd} international conference organised by the Banco de Mexico, Mexico City, 12 November 2002.

EUROPEAN COMISSION (2002), "The Euro Area in the World Economy - Developments in the first three Years", *Communication from the Commission no. 332*, Brussels.

LEEPER, E. M. (1991), "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policy", *Journal of Monetary Economics*,.27, 129-147.

MARINI, G. and VAN DER PLOEG, F. (1988), "Monetary and Fiscal Policy in an Optimising Model with Capital Accumulation and Finite Lives", *The Economic Journal*, 98, 772-786.

SARGENT, T. J. and WALLACE, N. (1981), "Some Unpleasant Monetarist Arithmetic", *Quarterly Review of Minneapolis Federal Reserve Bank*, Fall, 1-17.

SIMS, C. (1994), "A Simple Model for the Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy", *Economic Theory*, 4, 381-399.

WOODFORD, M. (1994), "Monetary Policy and Price Level Determinacy in a Cash-in-Advance Economy", *Economic Theory*, 4, 345-380.

WOODFORD, M. (1995), "Price Level Determinacy without Control of a Monetary Aggregate", *Carnegie Rochester Conference Series on Public Policy*, 43, 1-46.

YAARI, M. E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *The Review of Economic Studies*, 32, 137-150.

Appendix

Proof of Proposition 1

Consider the system of equations (24)-(27) when $\dot{C}(t) = \dot{Q}(t) = \dot{K}(t) = \dot{B}(t) = 0$. The equilibrium system can be solved for $\overline{C}$, $\overline{Q}$, $\overline{K}$, and $\overline{B}$. Total differentiation yields the following partial derivatives

$$\frac{d\overline{C}}{dX} = \beta\frac{\beta+\rho}{1+\eta}\frac{F_K^2}{\Delta_\beta} < 0$$

$$\frac{d\overline{Q}}{dX} = \beta\frac{\beta+\rho}{1+\eta}\frac{\eta F_K - F_{KK}\overline{Q}}{\Delta_\beta} < 0$$

$$\frac{d\overline{K}}{dX} = \beta\frac{\beta+\rho}{1+\eta}\frac{F_K}{\Delta_\beta} < 0$$

$$\frac{d\overline{B}}{dX} = \frac{(F_K-\rho)F_K^2 + \frac{\beta+\rho}{1+\eta}\beta F_{KK}\overline{Q} + F_K(F_{KK}\overline{C} - \frac{\beta+\rho}{1+\eta}\beta) - \frac{\beta+\rho}{1+\eta}\beta\eta F_K}{\Delta_\beta} > 0$$

where

$$\Delta_\beta \equiv F_{KK}(\alpha' - 1)\overline{B}\frac{\beta+\rho}{1+\eta}\beta F_K + (\overline{\alpha} - F_K)[(F_K - \rho)F_K^2 + \frac{\beta+\rho}{1+\eta}\beta F_{KK}\overline{Q} + F_K(F_{KK}\overline{C} - (\beta+\rho)\beta)] < 0$$

Proof of Proposition 2

The absolute value of Δ_β is increasing in α'. It follows that the larger α', the smaller the long-run responses of the economy to the lump-sum transfer.

Proof of Proposition 3

When $\beta = 0$ it is straightforward to verify that

$$\frac{d\overline{C}}{dX} = 0$$

$$\frac{d\overline{Q}}{dX} = 0$$

$$\frac{d\overline{K}}{dX} = 0$$

$$\frac{d\overline{B}}{dX} = \frac{1}{\overline{\alpha} - F_K} > 0$$

where in steady state $F_K = \rho$.

Proof of Proposition 4

To analyze the long-run effects of public expenditure we consider the system of equations (24)-(27) when $\overset{\bullet}{C}(t) = \overset{\bullet}{Q}(t) = \overset{\bullet}{K}(t) = \overset{\bullet}{B}(t) = 0$. Total differentiation of the equilibrium system gives the following partial derivatives

$$\frac{dC}{dG} = \beta \frac{\beta+\rho}{1+\eta} \frac{F_K^2}{\Delta_\beta} + \\ - \frac{\beta F_{KK}(\alpha'-1)\overline{B}\frac{\beta+\rho}{1+\eta}F_K + (\overline{\alpha}-F_K)\left[\frac{\beta+\rho}{1+\eta}\beta F_{KK}\overline{Q} + F_K(F_{KK}\overline{C} - \frac{\beta+\rho}{1+\xi}\beta)\right]}{\Delta_\beta} < 0$$

$$\frac{dQ}{dG} = \beta \frac{\beta+\rho}{1+\eta} \frac{\eta F_K - F_{KK}\overline{Q}}{\Delta_\beta} + \\ - \frac{\eta F_{KK}(\alpha'-1)\overline{B}\frac{\beta+\rho}{1+\eta}\beta + (\overline{\alpha}-F_K)\left[\overline{Q}(F_K-\rho)F_{KK} + \eta(F_{KK}\overline{C} - \frac{\beta+\rho}{1+\eta}\beta)\right]}{\Delta_\beta} < 0$$

$$\frac{dK}{dG} = \beta \frac{\beta+\rho}{1+\eta} \frac{F_K}{\Delta_\beta} + \frac{(\overline{\alpha}-F_K)\left[F_K(F_K-\rho) - \frac{\beta+\rho}{1+\eta}\beta\eta\right]}{\Delta_\beta} < 0$$

$$\frac{dB}{dG} = \frac{(F_K-\rho)F_K^2 + \frac{\beta+\rho}{1+\eta}\beta F_{KK}\overline{Q} + F_K(F_{KK}\overline{C} - \frac{\beta+\rho}{1+\eta}\beta) - \frac{\beta+\rho}{1+\eta}\beta\eta F_K}{\Delta_\beta} + \\ - \frac{F_{KK}(\alpha'-1)\overline{B}\left[F_K(F_K-\rho) - \frac{\beta+\rho}{1+\eta}\beta\eta\right]}{\Delta_\beta} > 0$$

Proof of Proposition 5

When $\beta = 0$ the long-run relations are

$$\frac{dC}{dG} = -1 < 0$$

$$\frac{dQ}{dG} = -\frac{\eta}{\rho} < 0$$

$$\frac{dK}{dG} = 0$$

$$\frac{dB}{dG} = \frac{1}{\overline{\alpha}-\rho} > 0$$

Stability properties of the linearized system

The determinant of the Jacobian matrix J of the linearized system (32) is

$$|J| = -\Delta_\beta > 0$$

Hence the system presents an even number of roots with negative real parts.

The characteristic equation of matrix J can be written as

$$\lambda^4 + b_3\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0 = 0$$

where

$b_3 = -tr(J) = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$

$b_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4$

$b_1 = -(\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4)$

$b_0 = \lambda_1\lambda_2\lambda_3\lambda_4 = -\Delta_\beta$

with λ_i, for $i = 1..4$, denoting the roots of the characteristic equation.

It is straightforward to verify that

$b_3 = -tr(J) = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) = -(4F_K - \rho - \overline{\alpha}) < 0$ if $4F_K > \rho + \overline{\alpha}$

so that the trace of matrix J can be assumed to be positive. This result allows us to rule out the hypothesis that all roots have negative real parts.

The coefficient b_2 can be written as

$b_2 = 3F_K(2F_K - \rho - \overline{\alpha}) + F_{KK}\overline{C} - \beta(\rho + \beta) + \overline{\alpha}\rho < 0$

if $\overline{\alpha}\rho + 6F_K < \beta(\rho + \beta) + 3F_K(\rho + \overline{\alpha}) - F_{KK}C$

The sign restrictions imposed appear to be satisfied for plausible parameter values. There are two eigenvalues with negative real parts and two with positive real parts, implying that the system is saddle-path stable.

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