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Sustainable Development, Renewable Resources and Technological Progress

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Abstract

Conflicts between optimality and sustainability are typical in the literature on sustainable development. Using a 'capital-resource' model of optimal growth, Pezzey and Withagen (1998) have recently proved that if natural resources are exhaustible, the time-path of consumption is single-peaked, declining from some point in time onwards. This paper extends the model to include technical progress, resource renewability and population growth. The main result is that, for any constant returns to scale technology, optimal paths can be sustainable only if the social discount rate does not exceed the sum of the rates of resource regeneration and augmentation net of population growth. Capital depreciation is neutral with respect to this necessary condition for sustainability. The development of resource-saving technologies is crucial for sustaining consumption per capita in the long run.

JEL codes : Q20, O11, O30. *Keywords* : Optimal Growth, Renewable Resources, Sustainable Development, Technological Progress.

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1 Introduction

Sustainable Development (SD) has increasingly attracted the interest of economic theorists and policymakers. In line with recent literature, sustainable paths are defined in this paper as those along which consumption per capita never decreases.

Several authors have analysed the role of natural resources in economic growth after the oil shocks in the seventies. The typical approach was that of including 'natural capital' in a well-behaved production function and to solve the standard Ramsey optimal growth problem. However, in such a model there is a trade-off between optimality and sustainability: Dasgupta and Heal (1974) have shown that consumption is bound to decrease if natural resources are exhaustible; Pezzey and Withagen (1998) have recently proved that the optimal consumption path is in fact 'single-peaked' - *i.e.,* it declines monotonically from some point in time onwards - in a model with a non-renewable natural input and a static, constant returns to scale technology.

This paper considers a more general model that includes resource renewability, technical progress, man-made capital depreciation, population growth and extraction costs. The main result is that, for any constant-returns-to-scale technology, the asymptotic rate of growth of consumption per capita is negative when the social discount rate exceeds the rate of resource regeneration net of the rate of population growth, plus the rate of resource-augmenting technical progress. Man-made capital depreciation is neutral with respect to this critical condition, which is necessary for SD along optimal paths. This result is shown to be valid also for the case of Cobb-Douglas technology with decreasing returns to scale and inelastic labour supply.

The plan of the paper is as follows: section 2 describes the capital-resource model of optimal growth. In section 3 we solve the Ramsey problem assuming resource renewability and resource-augmenting technical progress, deriving the sustainability condition under the assumption of constant returns to scale. We also consider a Cobb-Douglas technology with decreasing returns to scale, under Hicks neutral technical progress and inelastic labour supply: it is shown that the sustainability condition also applies in this case, generalising previous results by Stiglitz (1974). Section 4 presents some concluding remarks.

2 The capital-resource growth model

The benchmark 'capital-resource model' is a Ramsey-Cass-Koopmans optimal growth model (Dasgupta and Heal, 1974; Stiglitz, 1974). Aggregate ouput (Y) is produced by means of natural capital (R) and conventional, man-made capital (K). The production function can be specified as

$$Y(t) = F(K(t), R(t)) \tag{1}$$

where $F : \Re^2_+ \to \Re_+$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies $\lim_{R\to 0} \partial F(K,R)/\partial R = \infty$. Man-made capital and natural capital are both essential for production, that is $F(K,0) = F(0,R) = 0$. Per capita variables will be indicated by small letters: assuming that population grows at the exogenous constant rate $n \geq 0$, the dynamics of the stock per capita of man-made capital (k) is given by[1]

$$\dot{k}(t) = y(t) - c(t) - a \cdot r(t) - (\gamma + n) \cdot k(t) \tag{2}$$

where c is consumption per capita, a the marginal cost of extraction (assumed constant), r the amount per capita of resource extracted and used in production, γ the rate of physical depreciation of man-made capital, and $y(t) = Y(t) \exp[-nt]$ is gross output per capita. We assume that the natural resource is renewable: the time-variation of the resource stock per capita s equals

$$\dot{s}(t) = (g - n) \cdot s(t) - r(t) \tag{3}$$

where $g \geq 0$ is the exponential rate of resource regeneration. Optimal paths will be defined according to the standard criterion: taking initial amounts $k(0)$ and $s(0)$ as given, we solve the command optimum problem

$$\max_{\{c(t),r(t)\}_0^\infty} \int_0^\infty u(c(t)) \cdot e^{-\delta t} dt \tag{4}$$

subject to the constraints (1)-(2)-(3) and $s(t) \geq 0$, $k(t) > 0$, $c(t) \geq 0$ for any $t \in [0, \infty)$. The social discount rate $\delta > 0$ is assumed constant. For simplicity, the individual instantaneous utility function is of the isoelastic form:

$$u(c) = \left(c^{1-\eta} - 1\right)(1-\eta)^{-1}, \qquad 0 < \eta < 1. \tag{5}$$

In the following section we analyse the trade-off between optimality and sustainability. Our formal definition of sustainable development is

$$\text{SD} \Leftrightarrow \dot{c}(t) \geq 0 \text{ for each } t \in [0, \infty) \tag{6}$$

[1] A dot over a variable indicates its total time-derivative, *e.g.* $\dot{q}(t) = \frac{dq(t)}{dt}$. Population at time zero is normalized to unity.

Hence, sustainability is characterised by non-decreasing consumption *per capita* over an infinite time-horizon.

3 Is sustainability optimal?

After the oil shocks of the seventies, variants of the capital-resource model have been studied at lenght in the context of exhaustible resources: Dasgupta and Heal (1974) have shown that, without technical progress and resource renewal, aggregate consumption is bound to decrease in the long run;[2] Pezzey and Withagen (1998) have recently proved that consumption declines monotonically from some point in time onwards along the optimal path, in a model with constant returns to scale technology.

These results do not consider resource renewability, man-made capital depreciation, population growth, extraction costs and technical progress. Natural regeneration is a standard assumption in bioeconomic models (see Conrad and Clark, 1987). Resource renewability in optimal growth models has been analysed by Beltratti *et al.* (1998) in the standard framework, and by Krautkraemer and Batina (1999) and Mourmouras (1993) in the context of overlapping generations. In the next subsection we show that for any constant returns to scale technology, SD cannot be achieved along optimal paths if the social discount rate δ exceeds the rate of resource regeneration net of the rate of population growth. Subsection 3.2 extends the model to include technical progress.

3.1 Resource renewability

Assume $g, n, \gamma,$ and a all strictly positive, and that F is homogeneous of degree 1. Under constant returns to scale, we can substitute gross output per capita $F(k(t), r(t))$ with $y(t)$ in eq.(2) and solve problem (4) subject to (2)-(3)-(5) and $s(t) \geq 0$, $k(t) > 0$, $c(t) \geq 0$ for each $t \in [0, \infty)$. Denoting marginal productivities by $F_r = \partial F/\partial r$ and $F_k = \partial F/\partial k$, the first order conditions yield

$$\dot{F}_r = (F_r - a)(F_k - g - \gamma) \tag{7}$$

which is the modified Hotelling rule. Define the capital-resource ratio as $x = \frac{k}{r}$: under constant returns to scale, the output-resource ratio equals $F(x, 1) = \varphi(x)$ and

[2] See also Krautkraemer (1985), who analyses the feasibility of sutained consumption paths in the capital-resource model with CES production function and non-renewable resources.

$$\varphi(x) = F_r + x\varphi'(x), \qquad \varphi'(x) = F_k > 0, \qquad \varphi''(x) < 0 \tag{8}$$

where $\varphi' = \frac{\partial\varphi}{\partial x}$ and $\varphi'' = \frac{\partial^2\varphi}{\partial x^2}$. Since $\dot{\varphi}' = \varphi''\dot{x}$, the Hotelling rule (7) can be rewritten as

$$(\varphi(x) - x\varphi'(x) - a)(\varphi'(x) - g - \gamma) = -x\dot{x}\varphi''(x) \tag{9}$$

The first term in brackets in (9) is the marginal net rent from extraction. Our analysis is focused on paths with strictly positive net rents.[3] In this case, the marginal product of man-made capital converges monotonically to $g+\gamma$: if $\varphi'(x)$ is initially greater than $g+\gamma$, the right hand side of (9) must be positive, implying $\dot{x} > 0$ and $\varphi'(x)$ be decreasing thereafter; conversely, if $\varphi'(x)$ is initially below $g+\gamma$, then $\dot{x} < 0$ and the marginal product of capital will subsequently increase.

The conclusion is that, with renewable resources and depreciating capital, the modified Hotelling rule implies that the marginal product of man-made capital converges to $g+\gamma$, and that the capital-resource ratio x converges to a steady state value x^{ss} along the optimal path.[4] The steady state of x with positive net rents is defined as x^{ss} such that

$$\varphi'(x^{ss}) = g + \gamma \tag{10}$$

As regards consumption, define $z = c/k$. From (5) and the first order conditions of the problem, the consumption-capital ratio z evolves over time according to[5]

$$\dot{z}/z = z + \sigma\varphi' - \frac{\varphi - a}{x} + (\gamma + n)(1-\sigma) - \sigma\delta \tag{11}$$

where $\sigma = \eta^{-1}$ is the elasticiy of marginal utility. Consider the stationary equilibrium of system (9)-(11): setting $\dot{z} = 0$ in (11) and using (10), the simultaneous steady state in z and x implies a consumption-capital ratio equal to

[3]Under constant returns to scale, there is a unique value x^a of the capital-resource ratio which corresponds to zero net rents: in our analysis we consider an optimal path such that $x(t) > x^a$ for any $t \in [0, \infty)$; see Figure 1.

[4]Linearizing (9) around x^{ss} and integrating between time 0 and time t gives $x(t) - x^{ss} = (x(0) - x^{ss}) \exp\left[-\left(\frac{F_r^{ss} - a}{x^{ss}}\right) t\right]$, where $F_r^{ss} = \varphi(x^{ss}) - x^{ss}\varphi'(x^{ss})$ is greater than a by assumption; hence, $\lim_{t\to\infty} x(t) = x^{ss}$ along the optimal path.

[5]The first order conditions for problem (4) with isoelastic utility yield the Keynes-Ramsey rule $\dot{c}\eta = c(\varphi' - \delta - n - \gamma)$. The growth rate of z equals the difference between the optimal growth rate of consumption and the growth rate of man-made capital derived by the aggregate constraint (2).

$$z^{ss}\left(x^{ss}\right)=\sigma\delta-\left(\gamma+n\right)\left(1-\sigma\right)+\frac{\varphi\left(x^{ss}\right)-\sigma x^{ss}\varphi'\left(x^{ss}\right)-a}{x^{ss}} \tag{12}$$

It can be shown that the stationary equilibrium $(x^{ss}, z^{ss}(x^{ss}))$ with positive net rents and positive consumption is a saddle-point.[6] The optimal path satisfying the first order conditions of problem (4) is the one converging towards $(x^{ss}, z^{ss}(x^{ss}))$ along the stable arm of the saddle: Figure 1 shows the phase diagram of system (9)-(11) in the (x, z) space,[7] from which it is clear that explosive paths would violate either the contraint $c(t) \geq 0$ or the constraint $k(t) > 0$.[8] Therefore, $\lim_{t\to\infty} z(t) = z^{ss}(x^{ss})$ along the optimal path with positive net rents. This implies the following

Proposition 1 *A necessary condition for non-declining consumption per capita along the optimal path is that the social discount rate does not exceed the resource regeneration rate, net of the rate of population growth.*

Proof. Denote the asymptotic level of variable q by $q^{\infty} = \lim_{t\to\infty} q(t)$. Recalling definition (6), a necessary condition for SD is that $(\dot{c}/c)^{\infty} \geq 0$. Since $z^{\infty} = z^{ss}(x^{ss})$, the growth rates of consumption and man-made capital must be asymptotically equal, *i.e.* $(\dot{c}/c)^{\infty} = \left(\dot{k}/k\right)^{\infty}$: therefore, sustainability requires $\left(\dot{k}/k\right)^{\infty} \geq 0$, or equivalently

$$\left(\varphi\left(x^{\infty}\right)-a\right)\left(x^{\infty}\right)^{-1}-z^{\infty}-\gamma-n\geq 0 \tag{13}$$

which is the asymptotic growth rate of k from (2). Since $x^{\infty} = x^{ss}$, from (10)-(12) the left hand side of (13) equals $\sigma(g - n - \delta)$, and condition (13) becomes

$$g-n\geq\delta. \qquad \square \tag{14}$$

[6] Linearizing system (9)-(11) around $(x^{ss}, z^{ss}(x^{ss}))$, the characteristic roots are $\theta_1 = -\left(F_r^{ss} - a\right)\cdot\left(x^{ss}\right)^{-1} < 0$ and $\theta_2 = z^{ss}(x^{ss})$. If there exists a steady state equilibrium with positive consumption, then it is a saddle-point $(\theta_2 > 0)$. From (12), such an equilibrium exists if $\frac{\varphi(x^{ss})-\sigma x^{ss}\varphi'(x^{ss})-a}{x^{ss}} > (\gamma+n)(1-\sigma)-\sigma\delta$.

[7] The slope of locus $\dot{z} = 0$ depends on the values of parameters. In Figure 1, the locus is decreasing in the (x, z) space. In the other case, the locus $\dot{z} = 0$ is increasing and the slope of the stable-arm changes accordingly, but the saddle-point property of the equilibrium is unaffected.

[8] From (2), the growth rate of man-made capital may be written as $\frac{\varphi(x)-a}{x} - z - \gamma - n$; given that $\lim_{t\to\infty} x(t) = x^{ss}$, if z diverges to plus infinity as $t \to \infty$ then k will reach zero in finite time.

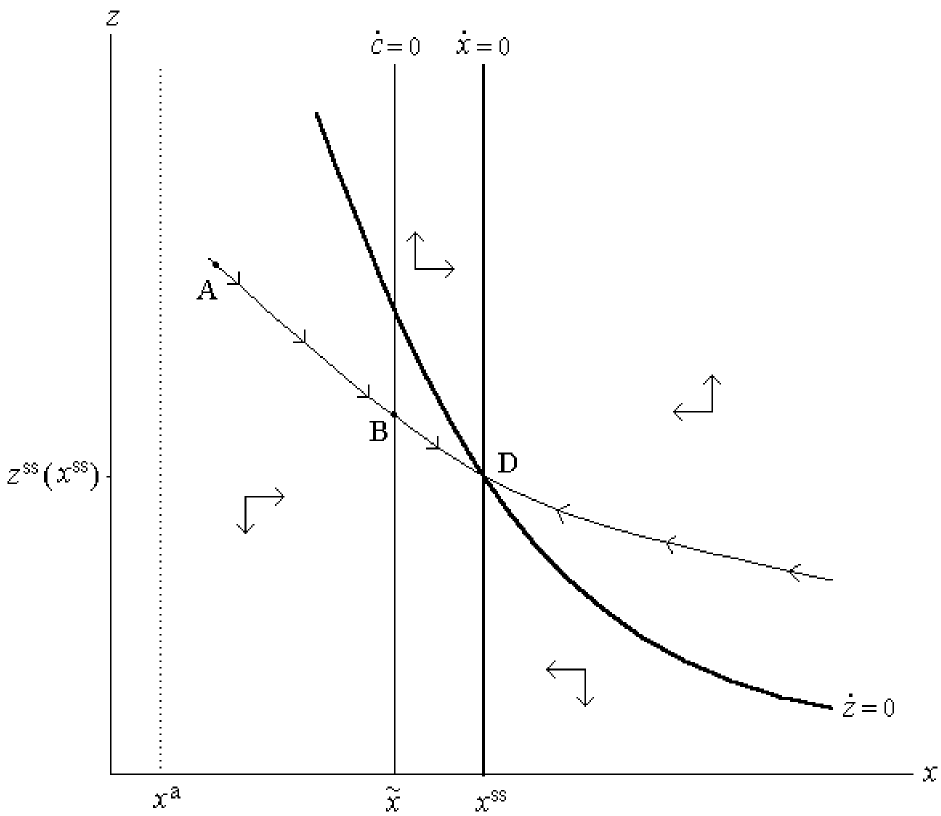

Figure 1. Phase diagram of system (9)-(11) in the unsustainable case $g < \delta+n$. Point D represents the steady state equilibrium $(x^{ss}, z^{ss}(x^{ss}))$. By (15), on the left (right) of the vertical line $x = \tilde{x}$ consumption per capita increases (decreases). The Hotelling rule (9) implies that if the economy starts at point A, the marginal product of capital declines monotonically towards the level corresponding to $x = x^{ss}$: consumption per capita increases until the economy reaches point B, and declines thereafter.

Proposition 1 generalises some of the results by Dasgupta and Heal (1974): without population growth and resource renewability ($g = n = 0$), inequality (14) is clearly not satisfied as long as we assume $\delta > 0$.[9] Condition (14) also implies that *the optimal time-path of consumption per capita is single-peaked when resource renewal is insufficient*: by the Keynes-Ramsey rule, consumption per capita is constant when the marginal product of capital equals $\delta + \gamma + n$; with constant returns to scale, there exists a unique value $\tilde{x}$ of the input ratio such that

$$\varphi'(\tilde{x}) = \delta + \gamma + n \tag{15}$$

Therefore, the locus $\dot{c} = 0$ corresponds to the vertical line $x = \tilde{x}$ in Figure 1. It follows from (10) and (15) that if the sustainability condition (14) is not satisfied, $\tilde{x} < x^{ss}$ and the locus $\dot{c} = 0$ lies on the left of the locus $\dot{z} = 0$, as in Figure 1: given that the initial values $(k(0), s(0))$ are such that we start from point A, the economy converges towards point D, consumption per capita increases until point B is reached but declines thereafter.

It is worth noting that γ and a do not appear in (14): man-made capital depreciation and extraction costs are neutral with respect to this asymptotic condition, which is necessary for sustainability. On the one hand, γ affects the levels of consumption and output, but not the distance between $\tilde{x}$ and x^{ss} (the constant-consumption input ratio and the long run input ratio, respectively). On the other hand, as shown in Figure 1, marginal extraction costs only restrict the space where net rents are positive.[10]

3.2 Resource-augmenting technical progress

The model is now extended to include technological progress. Stiglitz (1974) proved that if the rate of technical progress is sufficiently high, consumption per capita is asymptotically non-decreasing. In the Stiglitz (1974) model, the production function is Cobb-Douglas, utility is logarithmic - *i.e.* a particular case of (5) - and technical progress is *Hicks-neutral*. We consider a generic constant returns to scale technology with *resource-augmenting* progress, finding a necessary condition for SD along optimal paths. Assume that output per capita is now given by

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[9] If social welfare is the unweighted sum of individual utilities, given a constant rate of pure time preference $\tilde{\delta}$, the social discount rate equals $\delta = \tilde{\delta} - n$ and the sustainability condition (14) becomes $g \geq \tilde{\delta}$.

[10] The vertical line $x = x^a$ in Figure 1 is the locus along which net rents are zero: choosing $x^a : \varphi(x^a) - x^a \varphi'(x^a) = a$, net rents are strictly positive in each point on the right of $x = x^a$.

$$y(t) = \Phi(k(t), m(t) \cdot r(t)) \tag{16}$$

where $m(t) = \exp[\mu t]$, $\mu > 0$ is the exogenous rate of resource-augmenting technical progress and Φ is homogeneous of degree one.[11] For simplicity, we assume no extraction costs ($a = 0$). A possible interpretation of (16) is the following: technical progress $m(t)$ is a process that increases the productive services of r (a physical quantity extracted from the resource stock) by virtue of resource-saving technologies that become available over time. We can now prove the following

Proposition 2 *A necessary condition for non-declining consumption per capita along the optimal path is that the social discount rate does not exceed the sum of the rates of resource regeneration and augmentation, net of the rate of population growth.*

Proof. Define the input ratio as $w = k/mr$, and the augmented output-resource ratio as $\phi = \Phi/mr$. Under constant returns to scale, the marginal product of man-made capital is $\phi'(w) = (\partial\Phi/\partial k)$. Solving problem (4) subject to (2)-(3)-(16) and the usual non-negativity constraints, the Hotelling rule reads[12]

$$(\phi(w) - w\phi'(w))(\phi'(w) - g - \gamma - \mu) = -w\dot{w}\phi''(w) \tag{17}$$

As in the previous analysis, we consider strictly positive rents: the first term in brackets in (17) is thus positive, and the Hotelling rule implies that both the marginal product of man-made capital and the input ratio converge to steady state values. The marginal product of capital converges to

$$\phi'(w^{ss}) = g + \gamma + \mu \tag{18}$$

This implies that the asymptotic growth rate of consumption per capita is (by the Keynes-Ramsey rule)

$$\lim_{t\to\infty} \frac{\dot{c}(t)}{c(t)} = \sigma(\phi'(w^{ss}) - \delta - \gamma - n) \tag{19}$$

It follows from (18) that the limit (19) is positive if and only if

$$g - n + \mu \geq \delta. \qquad \square \tag{20}$$

[11] We assume that Φ exhibits the same properties as F (differentiability, concavity, and Inada conditions), as well as essential inputs ($\Phi(k,0) = \Phi(0,mr) = 0$).

[12] Differentiating $(\partial y/\partial r) = m\left(\phi(w) - w\phi'(w)\right)$ with respect to time and substituting the resulting expression in the first order conditions gives eq.(17).

Given our definition of SD, condition (20) is necessary for sustainability. The intuition for this result is the following: SD can be achieved if the rate of improvement in the production possibilities per capita $(g + \mu - n)$ is at least equal to the rate at which future social benefits are discounted (δ).

The dynamics of the z and w can be analysed along the same lines as in the previous subsection. The steady state locus for the consumption-capital ratio now equals

$$z^{ss}(w) = (\phi(w)/w) - \gamma - n - \sigma(\phi'(w) - \delta - \gamma - n) \tag{21}$$

The stationary equilibrium $(w^{ss}, z^{ss}(w^{ss}))$ with positive consumption is again a saddle-point,[13] and the phase diagram of the system is quite similar to that described in Figure 1.

3.3 The Cobb-Douglas case: Hicks-neutral progress and decreasing returns

As shown by Pezzey and Withagen (1998), the single-peakedness result also applies in the case of Cobb-Douglas technology with non-increasing returns to scale. The particular form of the production function allows to consider Hicks-neutral technical progress. Moreover, we assume that population N supplies labour inelastically: aggregate ouput is given by $Y = K^{\alpha_1} R^{\alpha_2} N^{\alpha_3} \exp[\omega t]$, where $\omega > 0$ is the rate of technical progress and $\alpha_1 + \alpha_2 + \alpha_3 \leq 1$. Since $N(t) = \exp[nt]$, output per capita equals

$$y(t) = k(t)^{\alpha_1} r(t)^{\alpha_2} \exp[\Omega t] \tag{22}$$

where $\Omega = \omega - n(1 - \alpha_1 - \alpha_2 - \alpha_3)$. Following Stiglitz (1974) and Pezzey and Withagen (1998), we define the output-capital ratio as $\pi = y/k$. Solving problem (4) subject to (2)-(3)-(22) and the usual non-negativity constraints, the first order conditions yield the Keynes-Ramsey rule:

$$\dot{c} = c\sigma(\alpha_1 \pi - \delta - \gamma - n) \tag{23}$$

Given (22), the first order conditions also imply

$$(\dot{r}/r)(1 - \alpha_2) = \alpha_1 \left(\left(\dot{k}/k \right) - \pi \right) + g + \gamma + \Omega \tag{24}$$

[13] The evolution of the consumption-capital ratio is now described the equation $\dot{z}/z = z + \sigma\left[\phi'(w) - \delta - \gamma - n\right] - [\phi(w)/w] + \gamma + n$. Setting $\dot{z} = 0$ gives the steady state locus (21). Evaluating (21) at $w = w^{ss}$ and imposing $z^{ss}(w^{ss}) > 0$ gives the existence condition $[\phi(w^{ss})/w^{ss}] > \gamma + n + \sigma[g + \mu - \delta - n]$.

Using (22),(23),(24) and the dynamic constraint (2), the growth rates of π and $z = c/k$ along optimal paths are

$$\frac{\dot{\pi}}{\pi} = z\left(\frac{1-\alpha_1-\alpha_2}{1-\alpha_2}\right) + \frac{\alpha_2 g + \gamma(1-\alpha_1) + \omega + n\alpha_3}{1-\alpha_2} - (1-\alpha_1)\pi \tag{25}$$

$$\frac{\dot{z}}{z} = z + \pi\left(\frac{\alpha_1-\eta}{\eta}\right) - \frac{\delta + (\gamma+n)(1-\eta)}{\eta} \tag{26}$$

And the steady state loci are represented by

$$\frac{\dot{\pi}}{\pi} = 0 : \pi = z\left[\frac{1-\alpha_1-\alpha_2}{(1-\alpha_1)(1-\alpha_2)}\right] + \frac{g\alpha_2 + \gamma(1-\alpha_1) + \omega + n\alpha_3}{(1-\alpha_1)(1-\alpha_2)} \tag{27}$$

$$\frac{\dot{z}}{z} = 0 : \pi = \frac{\delta + (\gamma+n)(1-\eta)}{\alpha_1-\eta} - z\left(\frac{\eta}{\alpha_1-\eta}\right) \tag{28}$$

From (27) and (28), in the steady state equilibrium (π_E, z_E) the output-capital ratio is

$$\pi_E = \frac{[\delta+\gamma+n(1-\eta)](1-\alpha_1-\alpha_2) + \eta(g\alpha_2 + \gamma(1-\alpha_1) + \omega + n\alpha_3)}{(\alpha_1-\eta)(1-\alpha_1-\alpha_2) + \eta(1-\alpha_1)(1-\alpha_2)} \tag{29}$$

Provided $z_E > 0$ (positive consumption), the equilibrium (π_E, z_E) is a saddle-point, and the optimal path is such that $z^\infty = z_E$ and $\pi^\infty = \pi_E$.[14] Equation (23) implies that c decreases when $\pi < \frac{\delta+\gamma+n}{\alpha_1}$: since the optimal path converges to E, it follows that consumption per capita does not decline in the long run only if $\pi_E \geq \frac{\delta+\gamma+n}{\alpha_1}$. From (29), this inequality can be rewritten as

$$\frac{\omega - n(1-\alpha_1-\alpha_2-\alpha_3)}{\alpha_2} \geq \delta + n - g \tag{30}$$

[14] As in Pezzey and Withagen (1998), explosive paths are ruled out by recalling that if $z^\infty \neq z_E$ then either consumption or man-made capital will become negative in finite time (see Figures 2a-2b).

Figure 2a

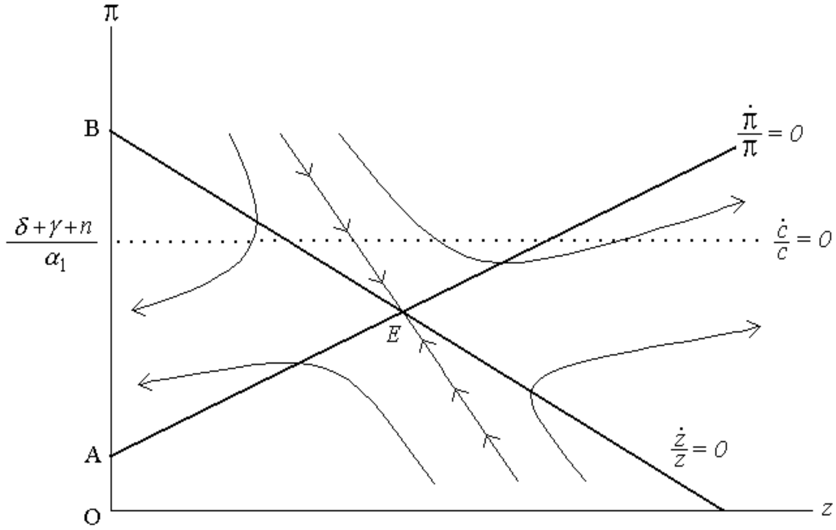

Figure 2b

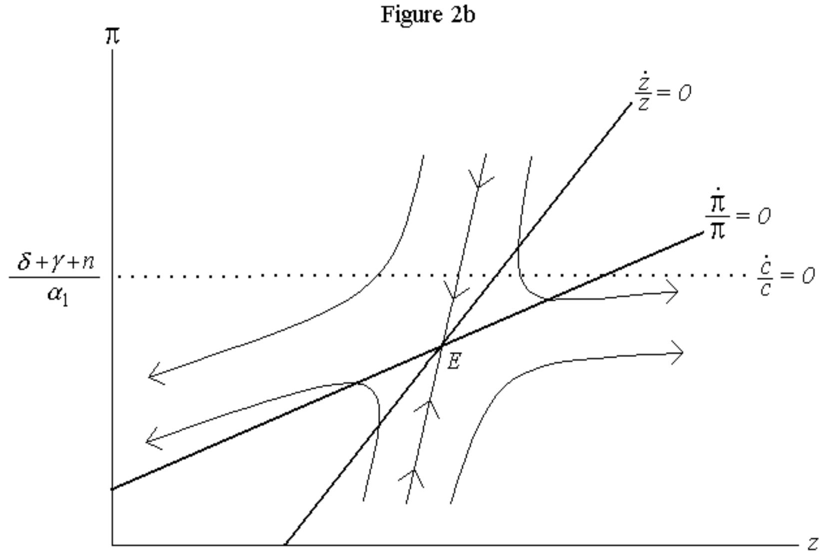

Figure 2. Phase diagrams of system (27)-(28): in Figure 2a, we assume that $\alpha_1 > \eta$ (the slope of the locus $(\dot{\pi}/\pi) = 0$ is negative), whereas Figure 2b depicts the case with $\alpha_1 < \eta$ (the locus $(\dot{\pi}/\pi) = 0$ is increasing). The steady state of consumption per capita is the horizontal line $\pi = \frac{\delta+\gamma+n}{\alpha_1}$, below which $\dot{c} < 0$: in both examples, condition (30) is not satisfied and consumption per capita declines in the long run.

Figures 2a and 2b describe two cases where (30) is not satisfied and consumption per capita falls in the long run. The left hand side of (30) shows that sustainability crucially depends on the net effect of two opposing forces, technical progress and decreasing returns.

Condition (30) can be interpreted along similar lines as condition (20). Indeed, there is a direct link between Hicks-neutral progress in the Cobb-Douglas case, and the resource-augmenting rate μ in our previous analysis: the production function (22) may be rewritten as $y = k^{\alpha_1} \left(r \exp \left[\left(\Omega / \alpha_2 \right) t \right] \right)^{\alpha_2}$, which implies that (Ω/α_2) is the resource-augmenting rate of technical progress adjusted for decreasing returns (coinciding with μ when returns to scale are constant). Hence, we can write our general result as follows:

Proposition 3 *Given either a constant returns to scale technology or a Cobb-Douglas technology with non-increasing returns to scale, and an isoelastic utility function, a necessary condition for SD along the optimal path is that the social discount rate does not exceed the sum of the rates of resource regeneration and augmentation, net of the rate of population growth.*

The implications of Proposition 3 are twofold: on the one hand, it generalises Stiglitz (1974) results with respect to technology and utility specifications, with further extension to resource renewability.[15] On the other hand, the result shows that the trade-off between optimality and sustainability is not removed by the presence of technical progress and resource renewability: high rates of resource regeneration and augmentation allow to sustain consumption, but if these rates are relatively low, consumption is single-peaked by virtue of a positive discount rate.

4 Conclusions

This paper has analysed the capital-resource model of optimal growth in the presence of resource renewability and technical progress. In a context of exhaustible resources and static technology, consumption per capita is bound to decrease in the long run (Dasgupta and Heal, 1974). A constant rate of time preference implies the optimal consumption time path be single-peaked (Pezzey and Withagen, 1998). We have shown that a necessary condition for

[15]Stiglitz (1974) assumes that instantaneous utility is discounted at a social discount rate equal to the rate of pure time preference net of population growth, and then solves the problem in terms of aggregate variables. In our notation we can set the social discount rate $\delta = \tilde{\delta} - n$, where $\tilde{\delta}$ is pure time preference: imposing $g = 0$, condition (30) becomes $\tilde{\delta} \leq \Omega/\alpha_2$, that is analogous to Stiglitz (1974) sustainability condition.

sustaining consumption per capita along optimal paths is that the social discount rate does not exceed the sum of the rates of resource regeneration and augmentation, net of the rate of population growth. This critical condition holds for any constant returns to scale technology and allows to generalise previous results by Dasgupta and Heal (1974), and Stiglitz (1974). We have shown that physical depreciation of man-made capital does not affect the necessary condition for sustainability.

The capital-resource model exhibits an implicit trade-off between optimality and sustainability, deriving from the optimality criterion underlying the 'discounted utilitarian framework' (Heal, 1998; Pezzey and Withagen, 1998). Our results confirm that resource renewability and technical progress cannot eliminate the implicit trade-off between discounted utilitarianism and sustainability: if the rates of resources regeneration and augmentation are relatively low, the time-path of consumption per capita is still single-peaked. On the other hand, the sustainability condition defines the critical level of technical progress required to sustain consumption along optimal paths. Prospects for sustainability crucially depend on the rate of resource-augmentation, which may be interpreted as the effect of resource-saving technologies that become available as new methods of production are developed.

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