

Do we need More Time for Leisure?*

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Abstract

"We need more time: more time for leisure" Linton Kwesi Jonhson used to dub. Indeed, the analysis of an OLG economy with endogenous labor supply gives some rational to the dub poet's claims. In our setting, the golden rule is defined as the pair of capital-labour ratio and individual labour supply which maximises the steady state utility of each generation. When, other things equal, agents are motivated to work more the higher the level of wages, individual labor supply will be increasing (decreasing) in capital labor ratio according to whether the elasticity of wages per unit of labour is bigger (smaller) than the relative change of the value of the fraction of labour income saved. Hence, if the economy is dynamically efficient, agents tend to work more than in the Golden Age if the propensity to save evaluated at the golden rule is, other things equal, relatively high. Conversely, under dynamic inefficiency, they work too much if and only if the propensity to save is relatively low. For given values of the parameters determining the propensity to save, individuals in dynamic efficient (inefficient) economies work more than in the Golden Age as long as the labour share of income is sufficiently high (low). These findings appear to be of some interest with reference to the 35 hours working-week debate in Europe.

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1. Introduction

Since the works by Phelps (1961,1962, and 1965) dynamic efficiency is a key issue in long run growth analyses. As it has been shown by Samuelson (1958) and Diamond (1965) infinite-horizon competitive economies with finite-lived agents can reach a steady state in which the marginal rate of return to physical capital is lower than the growth rate of population. Such a situation is described as one of dynamic inefficiency to the extent that a Pareto improvement can be implemented through an intergenerational redistribution scheme from young to old individuals opportunely tuned so to achieve the *Golden Rule*, i.e. the constant level of physical capital which maximises per capita consumption of each generation. The empirical evidence produced by Abel et al. (1989) suggests that economies such as the US and the OECD nations are dynamically efficient (see also Feldstein and Summers (1977)). This offers a theoretical justification of why several economists have argued in favour of policies aimed at increasing the propensity to save. Indeed, as shown by Jappelli and Pagano (1999), while forcing higher savings by repressing consumption credit would -in an Overlapping generation (OLG) model- be detrimental for the welfare of the very first generation, all subsequent generations would experience a welfare gain if the economy is dynamically efficient to start with. Hence, their statement that “[...] the Pareto criterion is extremely demanding and need not forestall all policies intervention. Even there is not obvious candidate for a social welfare function when households are not altruistically linked, any benevolent planner will try to trade-off the interests of current and future generations by weighting their utility appropriately [...]”. Other authors, like for instance Ibbotson (1987) and Mishkin (1984) find that the mean value of riskless interest rates in the US and other countries as well was well below the economy’s average growth rate over the sixty years’ period from the *20s* to the *80s*. Accordingly, one should recommend an intergenerational redistribution aimed at slowing the accumulation of capital.

Independently of the endless debate on whether economies are dynamically efficient or not¹, the fact that market economies might fail to achieve what Phelps defines *Golden Age* constitutes a general motivation for the large literature focusing on welfare properties of long run growth paths and related issues of government intervention. Much of this literature has been based on the hypothesis of inelastic labour supply. Empirical evidence suggests that aggregate labour supply moves over time both at high frequencies (see Lucas and Rapping (1969), Hansen (1985) and Rogerson (1988)), and low frequencies (see Maddison (1991), (1995) and Evans et al. (2001)). Such evidence, motivates the study dynamic models with endogenous labour supply at least on two different grounds: *i.* what are the

¹Blanchard and Fisher (1989) argue that the debate is still open.

consequences of labour supply decisions for the steady state stock of accumulable inputs; *ii.* how labour supply changes with the steady state level of accumulable inputs. In relation to point *i*, there is a well established literature, that reconsiders, under the hypothesis of endogenous labour supply, various benchmark results in growth theory obtained with inelastic labour supply like for instance the growth-effects of taxation (Stokey and Rebelo 1995)), the accumulation of human capital (Ortiguera 2000, Duranton, 2001), and the interplay between portfolio and retirement decision (Kingston 2000). Some of these studies are however based on Ramsey-type of models and have therefore little to say about the dynamic efficiency properties of the decentralised steady-state equilibria. Others, while based on OLG frameworks, neither look at the golden value of individual labour supply, nor they provide an analysis of how labour supply differs from its golden value depending on the steady state level of capital accumulation. Both these two aspects appear to be relevant from a welfare view-point: *a.* What is the *golden rule* level of labour supply? *b.* Do we tend to work too much or too little when the economy is dynamically efficient or inefficient? *c.* Can labour supply be exogenously modified to achieve the *Golden Age*?

This paper constitutes a prime attempt to address these three questions. We restrict our attention to a standard Diamond model in which production is characterised by decreasing returns in accumulable inputs and agents, who are identical and live for two periods, have a linearly-separable utility function defined over consumption in both periods, and leisure; they offer labour in their first period of time and they finance life-time consumption with the resulting salary. From a technical point of view the paper is very close to Nourry (2001) and Nourry and Venditti (2001) who provide a more general analysis of the dynamics of OLG models with endogenous labour supply. However, differently from these contributions, we focus our analysis on the welfare properties of the steady state achieved by the *laissez faire* economy.

The standard OLG model we consider yields a the steady state equilibrium such that the level of labour supply is a function of the steady state level of capital. The *Golden Age* is consequently defined as the pair of capital labour ratio and labour supply which maximises the steady state utility of the representative agent subject to the typical resource constraint faced by a benevolent dictator. Clearly enough, steady state labour supply being determined by steady state capital implies that labour does not play any role as a policy to achieve the *Golden Age*. But, how much do we work with respect to the golden rule level of labour supply if our decentralised economy is typically characterised by a rule of capital accumulation which is generally different from the golden one? Our finding is that individual labor supply will be increasing (decreasing) in capital labor ratio according to whether the relative change (elasticity) of wages per

unit of labour is bigger (smaller) than the relative change of the value of the fraction of labour income saved. Noting that the elasticity of wages with respect to capital is positive, while the relative change in the value of savings per unit of income with respect to capital is negative due to decreasing marginal returns to capital accumulation, provides the intuition behind the result. As capital increases, leisure becomes a relatively cheaper good whenever the sum of the effects on the level of wages and the value of savings per unit of income is negative. If so, a person who cares about leisure will decide to work less. Accordingly, given a certain level of the elasticity of wages and interest rates with respect to capital, in a situation of dynamic efficiency agents will work too much compared to the golden rule level of labour supply if the propensity to save is sufficiently low. From another perspective, agents work too hard in a dynamically efficient economy, if the labour share of income is, other things equal, sufficiently high. This result has some potential implications for the 35 hours-debate in Europe, which up until now has been questioned from an academic perspective mainly within standard labour economics literature such as in the case of Marimon and Zilibotti (1999). Is the request for a reduction in the length of the working week set by law justifiable on the basis of dynamic efficiency considerations? Assuming European countries are dynamically efficient, the answer depends on the magnitude of their labour share, and on whether in these economies the ratio of capital per unit of effective labour is moving towards its golden level. A rough calibration of the model, suggests that workers in dynamic efficient economies work too much compared to the golden rule level of labour supply if the labour share is greater than a critical value which lies between 69.7 and 77.5 depending on parameter values. The empirical evidence about OECD countries proposed by Bentolilla and Saint-Paul (1999) suggests that in the 90s countries like the US or Japan have a labour share around 67%, UK, Sweden and Finland have a labour share greater than 70%, while Italy, France and Germany have 63%, which is smaller than the figures for the 80s and the 70s. According to these figures, and given the rough calibration exercise we did, countries like the UK, Sweden, and Finland would be working too hard compared to the golden rule level of labour supply. In our view, such a comparison suggests that it might probably worth to obtain more accurate estimates of the labour share of income for European countries today, as well as on whether these countries are moving toward the golden age, in order to establish whether we should argue against or in favour of a reduction of the working week based on the simple dynamic efficiency considerations we developed in this paper.

The paper is organised as follows. Section 2 presents the model. Section 3 describes the dynamics and the steady state properties. Section 4 carries out the welfare analysis. A final section concludes.

2. The Model

We consider a perfectly competitive one-good closed economy populated by a continuum of size 1 of infinitely-lived atomistic firms and overlapping generations of individuals. At each time t a new generation of N_t individuals is born. Each of these individuals lives for two periods and gives birth to $1 + n$ individuals in her/his second period of life. Hence, the generation-size evolves according to $N_{t+1} = (1 + n)N_t$. All individuals are born with no inheritance. In the first period of their life they supply labour to firms, consume part of the resulting income and save the rest to finance second period consumption. They all have identical preferences described by the following CES function

$$U_t = \frac{1}{1 - \sigma} \left(c_{1t}^{1-\sigma} + \frac{1}{1 + \rho} c_{2t+1}^{1-\sigma} + b(1 - l_t)^{1-\sigma} \right), \quad (2.1)$$

where U_t is the individual utility of a member of generation t , c_{1t} and c_{2t+1} represent consumption in the first and the second period of life respectively, $l_t \in [0, 1]$ is labour supply, ρ is the subjective discount rate, b and σ are positive parameters.

Firms use a constant return to scale technology described by

$$Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}, \quad (2.2)$$

where Y_t is production, K_t is physical capital, L_t is labour and $\alpha < 1$ is the capital's product-share. Labour and capital are paid their marginal productivity:

$$w_t = (1 - \alpha)K_t^\alpha L_t^{-\alpha} \quad (2.3)$$

$$r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha} - \delta, \quad (2.4)$$

where $\delta > 0$ is the depreciation rate of capital.

2.1. The individual problem

Each young individual maximises (2.1) subject to the standard constraint

$$c_{1t} + \frac{c_{2t+1}}{R_{t+1}} = w_t l_t, \quad (2.5)$$

where $R_{t+1} = 1 + r_{t+1}$ is the gross interest rate. Condition (2.5) states that the present value of consumption should be equal to the labour income $w_t l_t$. The Lagrangean associated with the maximisation problem is

$$L = \frac{1}{1 - \sigma} \left(c_{1t}^{1-\sigma} + \frac{1}{1 + \rho} c_{2t+1}^{1-\sigma} + b(1 - l_t)^{1-\sigma} \right) + \lambda(w_t l_t - c_{1t} - \frac{c_{2t+1}}{R_{t+1}}), \quad (2.6)$$

where λ is the lagrangean multiplier. The solution implies the following saving (S_t^*) and labour supply (l_t^*) optimal choices

$$S_t^* = \frac{1}{(1 + \rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}} + 1} w_t l_t^* = s(R_{t+1}) w_t l_t^* \quad (2.7)$$

$$l_t^* = \frac{\left((1 + \rho)^{\frac{1}{\sigma}} + R_{t+1}^{\frac{1-\sigma}{\sigma}} \right)}{\left((1 + \rho)^{\frac{1}{\sigma}} + R_{t+1}^{\frac{1-\sigma}{\sigma}} \right) + w_t^{\frac{\sigma-1}{\sigma}} (1 + \rho)^{\frac{1}{\sigma}} b^{\frac{1}{\sigma}}} = l(R_{t+1}, w_t), \quad (2.8)$$

where

$$s(R_{t+1}) = \frac{1}{(1 + \rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}} + 1} \quad (2.9)$$

is the propensity to save. As usual, savings are increasing in R_{t+1} for $\sigma < 1$ and decreasing otherwise.² Similarly, for the labour supply, $\sigma < 1$ implies $\partial l_t^* / \partial w_t > 0$ and $\partial l_t^* / \partial R_{t+1} > 0$, while the opposite is true for $\sigma > 1$.³ Finally, S_t^* is increasing in w_t if $\sigma < 1$ holds while the impact of a change in w_t is ambiguous if $\sigma > 1$ holds.⁴ Summing up we have,

²Differentiation of S_t^* with respect to R_{t+1} yields

$$\frac{\partial S_t^*}{\partial R_{t+1}} = \frac{ds(R_{t+1})}{dR_{t+1}} w_t l_t^* + \frac{\partial l_t^*}{\partial R_{t+1}} s(R_{t+1}) w_t.$$

Since $s(R_{t+1}) w_t$ and l_t^* are always nonnegative, we have that $\frac{\partial S_t^*}{\partial R_{t+1}}$ is positive (negative) if both $\frac{ds(R_{t+1})}{dR_{t+1}}$ and $\frac{\partial l_t^*}{\partial R_{t+1}}$ are positive (negative). Moreover, since, these derivatives have always a common sign, $\frac{\partial S_t^*}{\partial R_{t+1}} > 0$ follows if $\sigma < 1$.

³Obviously, if $\sigma = 1$, the optimal work effort and the optimal propensity to save are constant: $l_t^* = (2 + \rho) / (2 + \rho + b(1 + \rho))$, $s = 1 / (2 + \rho)$.

⁴The derivative of S_t^* with respect to w_t is

$$\frac{\partial S_t^*}{\partial w_t} = l_t^* + w_t \frac{\partial l_t^*}{\partial w_t}.$$

Hence, $\partial S_t^* / \partial w_t$ is surely positive if $\sigma < 1$ so that $\partial l_t^* / \partial w_t > 0$ holds. However as long as $\sigma > 1$, $\partial l_t^* / \partial w_t < 0$ follows so that the sign of $\partial S_t^* / \partial w_t$ becomes ambiguous. In this case the sign will depend on the magnitude of $|\partial l_t^* / \partial w_t|$. In particular $\partial S_t^* / \partial w_t < (>) 0$, if $e_w^l < (>) - 1$, where

$$e_w^l = \frac{\partial l_t^*}{\partial w_t} \frac{w_t}{l_t^*}$$

is the elasticity of the work effort with respect to wages. Notice that $\partial S_t^* / \partial w_t < 0$ only if $\sigma \gg 1$.

$$\begin{aligned} S_t^* &= \widehat{S}(R_{t+1}^+, \bar{w}_t^+) \\ l_t^* &= l\left(R_{t+1}^+, \bar{w}_t^+\right) \text{ if } \sigma < 1 \end{aligned}$$

and

$$\begin{aligned} S_t^* &= \widehat{S}(R_{t+1}^-, \bar{w}_t^?) \\ l_t^* &= l\left(R_{t+1}^-, \bar{w}_t^-\right) \text{ if } \sigma > 1. \end{aligned}$$

3. Dynamics and Steady state

In equilibrium $L_t = N_t l_t$ holds⁵. Hence, output per effective worker, $Y_t/N_t l_t$, is equal to $y_t = f(k_t) = k_t^\alpha$, where $k_t = K_t/N_t l_t$ is capital per effective worker. The equilibrium values of the interest rate and the wage rate are given by

$$r_t = \alpha k_t^{\alpha-1} - \delta \quad (3.1)$$

$$w_t = (1 - \alpha) k_t^\alpha. \quad (3.2)$$

Capital evolves over time according to

$$K_{t+1} = S_t N_t, \quad (3.3)$$

which, given expression (2.7), implies

$$(1+n)k_{t+1}l_{t+1} = \frac{1}{(1+\rho)^{\frac{1}{\sigma}} R_{t+1}^{\frac{\sigma-1}{\sigma}} + 1} w_t l_t. \quad (3.4)$$

Substituting for the equilibrium values of w_t and r_{t+1} we obtain the following accumulation equation

$$(1+n)k_{t+1}l_{t+1} = \frac{(1-\alpha)k_t^\alpha}{(1+\rho)^{\frac{1}{\sigma}} (1+\alpha k_{t+1}^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1} l_t. \quad (3.5)$$

The equilibrium level of labour supply evolves according to equation (2.8). Hence, the dynamics of the economy is described by the following system

⁵We drop the "*" for simplicity.

$$k_{t+1} = \frac{l_t (1 - \alpha) k_t^\alpha}{(1 + n) l_{t+1} \left[(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k_{t+1}^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1 \right]} \quad (3.6)$$

$$l_{t+1} = \frac{(1 + \rho)^{\frac{1}{\sigma}} + (1 + \alpha k_{t+2}^{\alpha-1} - \delta)^{\frac{1-\sigma}{\sigma}}}{(1 + \rho)^{\frac{1}{\sigma}} + (1 + \alpha k_{t+2}^{\alpha-1} - \delta)^{\frac{1-\sigma}{\sigma}} + [(1 - \alpha) k_{t+1}^\alpha]^{\frac{\sigma-1}{\sigma}} [(1 + \rho) b]^{\frac{1}{\sigma}}} \quad (3.7)$$

3.1. Steady state: existence and stability

A *dynamic equilibrium* is a sequence $\{k_t, l_t\}_{t=0}^\infty$ that satisfies equations (3.6) and (3.7) with k_0 exogenously given. Given the accumulation equation (3.5), any steady state level of capital k satisfies

$$\phi(k) = (1 + n)k - \frac{(1 - \alpha) k^\alpha}{(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1} = 0. \quad (3.8)$$

Notice that in the standard Diamond model, $k = 0$ is a steady state equilibrium if $f(0) = 0$. But, as Nourry (2001) points out, with endogenous labor supply, agents do not work if there is no production so that $f(k)$ is no longer defined for $k = 0$. Thus no trivial steady state exists ($k = 0$) and the dynamical system can be such that there is no steady-state equilibrium. Therefore, in order to investigate the existence of steady state values we focus on

$$\psi(k) = \frac{\phi(k)}{k} = (1 + n) - \frac{(1 - \alpha) k^{\alpha-1}}{(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1}. \quad (3.9)$$

Proposition 1. *Our economy experiences a unique steady state equilibrium k^* : $\psi(k^*) = 0$. A sufficient condition for this unique steady state to be saddle-point stable is that $\sigma < 1$.*

Proof. $\psi(k)$ is a continuous function of k for $k \in (0, \infty)$. It can be easily verified that

$$\lim_{k \rightarrow \infty} \psi(k) = \lim_{k \rightarrow \infty} (1 + n) - \frac{(1 - \alpha) k^{\alpha-1}}{(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1} = (1 + n) \quad (3.10)$$

$$\lim_{k \rightarrow 0} \psi(k) = \lim_{k \rightarrow 0} (1 + n) - \frac{(1 - \alpha) k^{\alpha-1}}{(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1} = -\infty \quad (3.11)$$

where the second limit, in the case of $\sigma > 1$, follows from the application of the Hôpital's rule. Therefore, there exist at least one value of k , call it k^* , such that

$\psi(k^*) = 0$ holds. and our economy admits at least a steady state equilibrium k^* . As for uniqueness, we only need $\psi(k)$ to be a monotonic function of k . It can be easily verified that

$$\psi'(k) = \frac{(1 - \alpha)^2 k^{\alpha-2} \left[\left(\frac{1 + \rho}{1 + \alpha k^{\alpha-1} - \delta} \right)^{\frac{1}{\sigma}} \left(1 - \delta + \frac{\alpha}{\sigma} k^{\alpha-1} \right) + 1 \right]}{\left[(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1 \right]^2} > 0 \quad (3.12)$$

for all $k \in (0, \infty)$ which directly implies that k^* is unique

As for stability, Nourry (2001) and Nourry and Venditti (2001) show that if: 1) the utility and production functions satisfy the Inada conditions, 2) present and future consumption are strongly normal goods, 3) the elasticity of labor with respect to the rate of interest is positive, then the unique non trivial steady state $k^* > 0$ is saddle point stable. In our economy, 1) is always satisfied and 2) and 3) are always satisfied for $\sigma < 1$. ■

4. Welfare analysis

Since seminal work by Diamond (1965) we are aware of the fact that OLG economies may well suffer of dynamic inefficiency, a situation which, applying the standard golden-rule definition to our model, happens if

$$k > k^{GR} \equiv \left(\frac{\alpha}{n + \delta} \right)^{\frac{1}{1-\alpha}}, \quad (4.1)$$

where k^{GR} is the golden rule level of capital labour ratio, i.e. that particular level of k that maximises each generation per-capita consumption, a sufficient condition for individual welfare maximisation when utility depends only on consumption. However, in our model, individual utility depends both on consumption and labour supply. Therefore, labour supply should be taken into account when computing the golden rule for this economy. In particular, whenever all generations carry the same weight in the social welfare function, in order to maximise the steady state utility of each generation, the central planner will choose a pair $\{k^{GR}, l^{GR}\}$. In other words, the golden rule for our economy is defined as, $g = (k^{GR}, l^{GR})$. Formally, g solves the following problem

$$\begin{aligned} \max_{c_1, c_2, l, k} U_t &= \frac{1}{1 - \sigma} \left(c_1^{1-\sigma} + \frac{1}{1 + \rho} c_2^{1-\sigma} + b(1 - l)^{1-\sigma} \right) \\ \text{s.t. } c_1 + \frac{c_2}{1 + n} &= l(k^\alpha - k(n + \delta)) \end{aligned} \quad (4.2)$$

where the steady-state resource constraint has been derived by imposing the steady-state conditions on the following aggregate constraint

$$(1 - \delta)K_t + K_t^\alpha L_t^{1-\alpha} = K_{t+1} + c_{1t}N_t + c_{2t}N_{t-1}. \quad (4.3)$$

The lagrangean expression for the above problem is

$$L = \frac{1}{1 - \sigma} \left(c_1^{1-\sigma} + \frac{1}{1 + \rho} c_2^{1-\sigma} + b(1 - l)^{1-\sigma} \right) + \lambda [l(k^\alpha - k(n + \delta)) - c_1 - \frac{1}{1 + n} c_2] \quad (4.4)$$

where λ is a lagrangean multiplier. Accordingly, the centralised solution implies

$$k^{GR} = \left(\frac{\alpha}{n + \delta} \right)^{\frac{1}{1-\alpha}} \quad (4.5)$$

$$l^{GR} = \frac{(1 + \rho)^{\frac{1}{\sigma}} + (1 + n)^{\frac{1-\sigma}{\sigma}}}{(1 + \rho)^{\frac{1}{\sigma}} + (1 + n)^{\frac{1-\sigma}{\sigma}} + \left[(1 - \alpha) \left(\frac{\alpha}{n + \delta} \right)^{\frac{\alpha}{1-\alpha}} \right]^{\frac{\sigma-1}{\sigma}} [b(1 + \rho)]^{\frac{1}{\sigma}}} \quad (4.6)$$

4.1. How does labour supply behave getting closer to the golden rule?

The steady state values for k and l in the decentralised economy are

$$k = \frac{(1 - \alpha) k^\alpha}{(1 + n) (1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1} \quad (4.7)$$

$$l = \frac{(1 + \rho)^{\frac{1}{\sigma}} + (1 + \alpha k^{\alpha-1} - \delta)^{\frac{1-\sigma}{\sigma}}}{(1 + \rho)^{\frac{1}{\sigma}} + (1 + \alpha k^{\alpha-1} - \delta)^{\frac{1-\sigma}{\sigma}} + [(1 - \alpha) k^\alpha]^{\frac{\sigma-1}{\sigma}} [b(1 + \rho)]^{\frac{1}{\sigma}}} \quad (4.8)$$

A comparison with the expressions (4.5) and (4.6) suggests that, as in the standard Diamond model, it is not guaranteed that the decentralised economy will spontaneously reach the *golden age* (Phelps, 1961). Also, in our model, for any $k \neq k^{GR}$, the agents will generally choose a level of labour supply l which is different from l^{GR} . However, l is a function of k such that if $k = k^{GR}$, then $l(k^{GR}) = l^{GR}$ follows. Therefore, setting k to its golden rule level is a *sufficient* condition for the economy to reach *golden age*: $g = (k^{GR}, l^{GR})$.

Imagine a stationary economy which is not in its *golden age*, i.e. $(k, l) \neq g$. We then face the question of under which conditions is the golden rule level of labour is higher or lower than the steady state level reached by the decentralised economy.

The aim of the first question is to investigate whether individuals work more or less than in the golden age, given the steady state level of capital k . In particular how does the steady state level of labour supply l compare with the golden rule level l^{GR} assuming that k is respectively greater (dynamic inefficiency) or lower than k^{GR} ? In other words, we are interested in distinguishing between the cases in which welfare maximisation is associated with a reduction in the level of labour supply as opposed to situations in which the reverse is true.

Let us concentrate on the first issue. In steady state, $l(k) = l(R(k), w(k))$, (see equation (4.8)). Therefore, the labour supply l changes with k according to

$$l_k(k) = l_w(k) \frac{dw}{dk} + l_R(k) \frac{dR}{dk}, \quad (4.9)$$

where $l_w(k)$ and $l_R(k)$ are the partial derivatives of l with respect to w and R respectively, evaluated at steady state, implying that

$$l_k(k) \geq 0 \Leftrightarrow \frac{dw}{dk} \Big/ \frac{dR}{dk} \geq -\frac{l_R(k)}{l_w(k)}. \quad (4.10)$$

Given equation (4.8), we have

$$l_w(k) = \frac{\left(\frac{1-\sigma}{\sigma}\right) w(k)^{-\frac{1}{\sigma}} [b(1+\rho)]^{\frac{1}{\sigma}} \left[(1+\rho)^{\frac{1}{\sigma}} + R(k)^{\frac{1-\sigma}{\sigma}}\right]}{\left[1+\rho\right]^{\frac{1}{\sigma}} + R(k)^{\frac{1-\sigma}{\sigma}} + w(k)^{\frac{\sigma-1}{\sigma}} [b(1+\rho)]^{\frac{1}{\sigma}}]^2}$$

$$l_R(k) = \frac{\left(\frac{1-\sigma}{\sigma}\right) w(k)^{\frac{\sigma-1}{\sigma}} [b(1+\rho)]^{\frac{1}{\sigma}} R(k)^{\frac{1-2\sigma}{\sigma}}}{\left[(1+\rho)^{\frac{1}{\sigma}} + R(k)^{\frac{1-\sigma}{\sigma}} + w(k)^{\frac{\sigma-1}{\sigma}} [b(1+\rho)]^{\frac{1}{\sigma}}\right]^2}.$$

By substituting for l_w and l_R we can re-write condition (4.10) as

$$\varepsilon_{w,k} \geq \varepsilon_{R,k} s(R(k)) \quad (4.11)$$

where

$$\varepsilon_{w,k} = \left| \frac{dw}{dk} \cdot \frac{k}{w(k)} \right|$$

$$\varepsilon_{R,k} = \left| \frac{dR}{dk} \cdot \frac{k}{R(k)} \right|$$

$$s(R(k)) = \frac{R(k)^{\frac{1-\sigma}{\sigma}}}{(1+\rho)^{\frac{1}{\sigma}} + R(k)^{\frac{1-\sigma}{\sigma}}}$$

Note that, in condition (4.11), the LHS measures the relative change of the labour income, while the RHS measures the relative change of the value of fraction of labour income saved. Hence, the condition can be interpreted as follows. If, following a marginal positive change in k , $\varepsilon_{w,k} - \varepsilon_{R,k}s(R(k)) > 0$, this implies that the relative change in labour income (due to the increase in $w(k)$), net of the negative relative change in the value of savings per unit of income (due to the reduction of $R(k)$), is positive. Accordingly, individuals work more since leisure has become more expensive. Interestingly, as long as $\varepsilon_{w,k} - \varepsilon_{R,k}s(R(k)) < 0$, for the very same reason, individuals will work less, as the relative change in labour income net of the interest rate effect on savings is negative, which implies that leisure is now cheaper. Hence, if k moves toward k^{GR} due to exogenous changes in the relevant parameters, individuals find leisure more valuable, and hence they are willing to work less. The crucial task becomes then to check for which values of the relevant parameters, condition (4.10) is satisfied with the “<” sign and with the “>” sign. We can state the following

Proposition 1. (i) For $\alpha > \frac{1}{2}$ we have $l_k > 0$ for all $k \in [0, \infty)$. (ii) for $\alpha < \frac{1}{2}$ and $\sigma < 1$ then there is always a unique k' such that $l_k < 0$ for $k \in [0, k')$ and $l_k > 0$ for $k \in [k', \infty)$.

Proof. Substituting for the equilibrium values of w and R , condition (4.10), considered only with the “<” sign, can be re-written as

$$s(k) \equiv \frac{1}{(1 + \rho)^{\frac{1}{\sigma}} (1 + \alpha k^{\alpha-1} - \delta)^{\frac{\sigma-1}{\sigma}} + 1} > \frac{1 + \alpha k^{\alpha-1} - \delta}{(1 - \alpha)k^{\alpha-1}} = v(k). \quad (4.12)$$

Note that, by definition, the saving rate $s(k) \in (0, 1)$ and since we have

$$\begin{aligned} \lim_{k \rightarrow 0} v(k) &= \frac{\alpha}{(1 - \alpha)} \\ \lim_{k \rightarrow \infty} v(k) &= \infty \end{aligned}$$

then a necessary condition for the above inequality to hold is $\alpha < \frac{1}{2}$ (part *i*). Moreover we have

$$\lim_{k \rightarrow \infty} s(k) = \frac{1}{(1 + \rho)^{\frac{1}{\sigma}} (1 - \delta)^{\frac{\sigma-1}{\sigma}} + 1} \quad (4.13)$$

$$\lim_{k \rightarrow 0} s(k) = \begin{cases} 1 & \text{for } \sigma < 1 \\ 0 & \text{for } \sigma > 1 \end{cases} \quad (4.14)$$

And

$$\begin{aligned} s'(k) &< 0 \quad \forall k \in [0, \infty) \text{ for } \sigma < 1 \\ s'(k) &> 0 \quad \forall k \in [0, \infty) \text{ for } \sigma > 1. \end{aligned} \tag{4.15}$$

Since it can be easily verified that $v'(k)$ is always positive, for $\sigma < 1$ such behaviour of the functions $s(k)$ and $v(k)$ directly leads to the results described in part *ii* of the proposition. ■

Given the above proposition, we can analyse how labour supply behaves as the economy gets closer to the golden rule depending on whether the decentralised steady state equilibrium implies dynamic efficiency or not. In doing so, we focus on the most relevant case in which labour supply is increasing in wages, i.e. $\sigma < 1$.

It should be noted that, as a general principle, a dynamically efficient economy, i.e. an economy with $k_t = k < k^{GR}$, can be brought closer to the golden rule (and more specifically at the golden rule) by discouraging consumption of the current generation (via taxation for instance) to such an extent that $k_{t+1} = k^{GR}$. Such policy would have a beneficial effect on the welfare of all future generations at the expenses of the current generation. It is clear that such policy does not constitute a Pareto improvement. However, Pareto criterion is very stringent and it can be shown⁶ that some degree of intervention to bring k closer to k^{GR} is generally desirable unless no-weight is assigned to future generations. As for the case of dynamic inefficiency, $k_t = k > k^{GR}$, it is a well established result that all generations, including the present, can be made better off, and actually reach the maximum welfare, by means of an intergenerational transfer-scheme such that $k = k^{GR}$.

4.1.1. a) The economy is dynamically efficient, $k < k^{GR}$.

In this case, with labour supply increasing in wages ($\sigma < 1$), we have the following sub-cases:

a1. if $k < k^{GR} < k'$ then $l > l^{GR}$

a2. if $k' < k < k^{GR}$ then $l < l^{GR}$

a3. if $k < k' < k^{GR}$ then there is no clear relationship between l and l^{GR} .

Therefore *a1* is a situation in which getting closer to the golden rule implies a spontaneous reduction of labour supply (we desire a shorter working-week). In case *a2*, by contrast, a longer working-week makes us the most happy. In case *a3* no clear cut conclusions can be drawn. Whether we prefer to work less or more as we move toward the golden age depends on very specific and less robust combinations of the-values of the relevant parameters.

⁶See Pagano and Jappelli (1999).

4.1.2. b) The economy is dynamically inefficient, $k^{GR} < k$.

b1. if $k^{GR} < k < k'$ then $l < l^{GR}$

b2. if $k' < k^{GR} < k$ then $l > l^{GR}$

b3. if $k^{GR} < k' < k$ then there is no clear relationship between l and l^{GR} .

Therefore, under dynamic inefficiency, getting closer to the golden rule motivates a shorter-working week in case *b2*. The opposite is true in case *b1*, while no striking conclusions can be reached in case *b3*.

4.2. Leisure vs. labour supply as the economy approaches the golden age

We focus now on the cases in which, surely, we "want more time", that is, cases *a1* and *b2*.⁷ We need to compare k^{GR} and k' in order to be able to say whether individuals would like to work more or less as the economy moves toward the golden age, where we recall that $l > l^{GR}$ is obtained under dynamic efficiency if $k^{GR} < k'$ while, under dynamic inefficiency, if $k^{GR} > k'$. Noting that $s(k^{GR}) > (<)1 + \alpha k^{GR\alpha-1} - \delta / (1 - \alpha) k^{GR\alpha-1}$ if and only if $k^{GR} < (>) k'$ provides an indirect and yet effective way of checking whether $k' > (<) k^{GR}$. By substituting for the value of k^{GR} given equation (4.5) we obtain

$$s(k^{GR}) \equiv \frac{1}{(1 + \rho)^{\frac{1}{\sigma}} (1 + n)^{\frac{\sigma-1}{\sigma}} + 1} \geq \frac{\alpha (1 + n)}{(1 - \alpha) (n + \delta)}, \quad (4.16)$$

According to this inequality, under dynamic efficiency, we need more time for leisure if, other things being equal, the golden rule propensity to save is sufficiently high. By contrast, under dynamic inefficiency, in order for a reduction of the working time to be welfare improving, the golden rule propensity to save has to be sufficiently low. Calibrating this condition according to the parameter-values suggested by Barro and Sala-i-Martin (1999), $n = 0.35$, $\delta = 0.78$, $\rho = 0.82$, and for $\sigma = 0.9$, we find $s(k^{GR}) = 0.347$, implying that condition (4.16) is satisfied if and only if $\alpha < 0.225$ (see Table 1) which corresponds to a labour share greater than 0.775. With full capital depreciation and imposing an annual growth rate of 2%, i.e. $n = 0.82$ and an annual subjective discount rate of 1%, i.e. $\rho = .35$, yields $s(k^{GR}) = 0.434$, so that condition 4.16) holds for $\alpha < 0.303$, which corresponds to a labour share than greater than 0.697, (see Table 1). These numerical examples suggest that, under dynamic efficiency, agents might work too much compared to the golden rule level of labour supply if and only if the labour share of income

⁷Notice that, if we trust in Abel et al. (1989), case *a* appears as the most relevant one.

is around 70%, while the opposite would be true if the economy is dynamically inefficient. As shown by Bentolilla and Saint-Paul (1999), the labour share in OECD countries has been varying substantially both through time and across countries. In the UK for instance, it has been fluctuating around a stable level of roughly 71%. This is very much in line with the "growth stylised fact" about the constancy of labour share through time. Evidence from other countries is, however, quite different from the UK case. For example, Sweden average labour share goes from 69.7 in the seventies up to 72.6 in the 90s; in Italy it ranges from 67.1 in the 70s to 62.6 in the 90s; while France (similarly to what happened in Germany) had 67.6 in the 70s, 71.7 in the 80s and 62.4 in the 90s.

In our view, these data suggest that for these countries, assuming they are dynamically efficient, there is some room for condition (4.16) to be satisfied. Therefore, we believe that, more generally, an more accurate empirical investigation is needed in order to evaluate whether the condition actually holds for developed countries, as well as to assess whether these countries are moving toward the golden rule. This evidence is crucial to argue in favour or against a reduction of the working-week length on the dynamic efficiency grounds we analysed in this paper.

Table 1:
Golden rule propensity to save and critical values of α :

		$\delta = 0.78$		$\delta = 0.9$		$\delta = 1$	
n	ρ	$s(k^{GR})$	α	$s(k^{GR})$	α	$s(k^{GR})$	α
0.35	0.82	0.347	0.225	0.347	0.243	0.347	0.258
0.57	0.82	0.351	0.232	0.351	0.248	0.351	0.26
0.82	0.82	0.355	0.238	0.355	0.252	0.355	0.262
0.35	0.57	0.385	0.244	0.385	0.263	0.385	0.278
0.57	0.57	0.389	0.251	0.389	0.267	0.389	0.280
0.82	0.57	0.393	0.257	0.393	0.271	0.393	0.282
0.35	0.35	0.426	0.263	0.426	0.283	0.426	0.299
0.57	0.35	0.43	0.270	0.43	0.287	0.43	0.301
0.82	0.35	0.434	0.276	0.434	0.291	0.434	0.303

5. Conclusion

As shown by Samuelson (1958) and Diamond (1965), market economies might fail to reach the long run optimal level of the stock of accumulable inputs despite of being populated by utility maximisers agents. Introducing utility considerations which lead to abandoning the standard assumption of inelastic labour endogenous

labour supply raises a number of questions like for instance, what is the golden rule level of labour supply and how much do agents decide to work in long run situations in which the stock of accumulable inputs differs from its golden rule level. This paper provides a prime attempt to answer these questions by focusing on a standard OLG model where physical capital and labour are the two only inputs, the economy is perfectly competitive, and agents decide how much to work. Our findings are that: i. the golden rule level of work effort is a function of the *golden rule* level of capital; whenever the steady state level of capital differs from its *golden rule* level, agents work either too much or too little compared to the *golden rule*. In particular, starting from a situation of dynamic efficiency and assuming preferences such that labour supply increases in the level of wages, agents work more than they would in the *Golden Age* if the labour share of output is, other things equal, sufficiently high or, from another perspective, if the golden rule level of the propensity to save is, *ceteris paribus*, sufficiently high. Symmetrically, whenever the economy is dynamic inefficient, they might work too much provided the propensity to save evaluated at the golden age, is below some critical threshold, or if, other things equal, the labour share of output is sufficiently low. Both cases seem of some interest as they arise for plausible alternative combinations of the parameters. These conclusions seem to have some potential implications for the 35 working- hours debate in Europe. In this respect we believe that more accurate evidence is needed on the labour income share of European countries as well as on whether these countries are moving toward the golden age or not, in order to evaluate whether we should argue in favour or against a reduction in the working-week length on the basis of the simple dynamic efficiency analysis we developed in this paper. Finally, more work needs to be done, relaxing some of the assumptions of the standard Diamond model, to understand under what conditions, if any, labour supply could become a policy instrument to induce dynamic efficiency.

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