Technological externalities and first-best policy rules

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Abstract

The present paper analyzes optimal policy rules in models with technological externalities. It is shown that first-best policy restores efficiency by correcting the divergence of market prices for investment from the social value of capital stocks. The optimal policy rule, correcting both for the wedge between private and social returns and for the discrepancy between market and shadow prices, is a non-linear tax/subsidy.

1. Introduction

Market forces do not always ensure the maximization of social welfare in the long-run. Typical causes for this kind of global inefficiency are coordination failures, expectational errors and the presence of technological externalities. Underdevelopment traps may arise from the impossibility of coordinated investment behavior on the part of private agents in a free market regime. In these cases, a benevolent planner is required to overcome inefficiency. When technological externalities are at work, the global efficient path may be selected either by the intervention of a planner endowed with perfect foresight or by the adoption of Pigouvian tax/subsidies correcting for the wedge between private and social returns. This latter kind of policy is analyzed in Romer (1986). The desirability of Pigouvian measures has, however, been questioned by Ciccone and Matsuyama (1999) and Judd (1999). In fact, the Pigouvian policies cannot ensure the selection of the first-best solution, when multiple paths satisfy the first-order conditions for dynamic maximization of social welfare.

The study of sub-optimal accumulation paths arising from technological externalities has mainly focused on local stability issues. This was initiated by Boldrin and Rustichini (1994) and Benhabib and Rustichini (1994). The main result is that the saddle-path is not a general dynamic property of inefficient paths. This result holds even when mild externalities are at work, independently of the existence of increasing returns (Benhabib and Farmer, 1996, Benhabib and Nishimura, 1998, Nishimura and Venditti, 1999). Comparatively little work, with notable exceptions such as Ciccone and Matsuyama (1999), has been carried out to analyze efficiency of models with externalities.

The present paper studies corrective policies that ensure the selection of the globally efficient path when the market solution is not the first-best because of the presence of technological externalities. When multiple paths satisfy the first-order optimality conditions, first-best policy ensures efficiency by taking account of the divergence of market prices for investment from the social value of capital stocks. This policy is characterized by a non-linear tax/subsidy that selects the unique globally efficient path.

The scheme of the paper is as follows : section 2 sets forth the assumptions of the model. Section 3 reviews the properties of inefficient *laissez-faire* dynamic equilibria in the presence of technological externalities. Two examples are presented to illustrate the results. Section 4 analyzes the case where demand complementarities cause global inefficiencies. Section 5 concludes with a brief summary.

2. Assumptions

Let $n \in N$ be the number of different capital goods in the economy. The vector of private levels of capital at time $t \in [0, \infty)$ is denoted by $k(t)$, while the vector $K(t)$ gives the levels of aggregate or social capital levels. $\hat{p}_i \in \hat{P} \subset R^n$ and $\tilde{p}_i \in \tilde{P} \subset R^n$are the social and private utility prices of capital good i , while $\hat{P}$ and $\tilde{P}$ are the sets of social and private utility prices, respectively. Technology and utility are assumed to be stationary and to satisfy the standard assumptions :

(A.1) Both the set of feasible private capital stocks $k \subset R_+^n$ and the set of feasible aggregate capital stocks $K \subset R_+^n$ are convex.

(A.2) $f : R_+^n \times R_+^n \longrightarrow R_+^n$ is of class C^2 and defines the technological constraints faced by private agents. The vector of private marginal products of capital is f_k.

(A.3) $F : R_+^n \times R_+^n \longrightarrow R_+^n$ is of class C^2 and defines the social production function. The vector of social marginal products of capital is $F_k + F_K$, where $F_k = f_k$.

(A.4) The time preference rate δ is positive.

(A.5) The indirect utility function $u : D \subset \overset{\circ}{R_+^n} \times R_+^n \rightarrow R$ is C^2 and concave; D is convex and has nonempty interior.[1]

Assumption (A.1) ensures boundedness of optimal trajectories. Assumptions (A.2) and (A.3) imply that a technological externality creates a difference between private and social productivities.

3. Inefficient dynamic equilibria and policy

Decentralized equilibria can be described by the path $(\tilde{p}, k)$ which is the optimal solution to the pseudo-planner's problem (see Becker, 1985)

$$max \int_0^\infty u\left(k(t), \dot{k}(t)\right) e^{-\delta t} dt \tag{1}$$

subject to

$$k(0) = k_0, \tag{2}$$

[1] For a characterization of the class of preferences and technologies underlying the concavity properties of indirect utility functions, see Venditti (1997).

$$\lim_{t\to\infty} e^{-\delta t}\tilde{p}(t)\, k(t) = 0 \tag{3}$$

The solution to the competitive optimal control problem is given by the following Hamiltonian dynamical system (see Cass and Shell, 1976):

$$\begin{aligned} \dot{k}_i &= H_{\tilde{p}_i} \\ \dot{\tilde{p}}_i &= \delta\tilde{p}_i - H_{k_i},\ i = 1, ..., n \end{aligned} \tag{4}$$

Private agents take the path of social capital stocks as fixed parameters entering the private investment equation

$$\dot{k}(t) = g(k, K, \tilde{p}) \triangleq f(k, K) - c(\tilde{p})$$

and denoted by K, while $c(\tilde{p})$ is consumption.

The standard policy prescription in the presence of externalities is to compensate for the productivity difference according to the principles of Pigouvian taxation. However, as Judd (1999) points out, this approach to optimal policy has no dynamic flavor. A similar argument is presented in Ciccone and Matsuyama (1999) who show that conventional linear taxes are ineffective in selecting the globally efficient development path, when there are divergences between social and private returns.

4. Optimal policy

The optimal dynamic taxation aimed at correcting technological externalities can be characterized under the simplifying assumption that the policymaker can use lump sum taxes to subsidize the accumulation of capital goods.

The efficient policy must ensure that the path $(\tilde{p}, k)$ is Pareto-optimal. The intervention rule can be implemented through a perturbation S (as described below) to the private investment equation ensuring the rental rate for capital paid by private firms equals the social rental rate. Policy intervention can thus be formulated as a modification to the investment equation of private agents. In particular, the Hamiltonian function associated with the solution to the private agents problem in the presence of corrective policy can be written as

$$H(\tilde{p}, k) \triangleq u\left(k, \dot{k}\right) + \tilde{p}\left(g(k, K, \tilde{p}) + S - \tau\right)$$

where τ is a lump-sum tax financing S.

Theorem 1. *The optimal vector of subsidies is*

$$\sigma \triangleq \frac{\hat{p}}{\tilde{p}} (f_k + f_K) - f_k$$

Proof: see Appendix. ■

The optimal subsidy rule can be interpreted as follows. The value of the social marginal productivity of capital good i deflated by the private utility price $\tilde{p}_i$ is measured by the expression

$$\frac{\hat{p}_i}{\tilde{p}_i} (f_{k_i} + F_{K_i})$$

Private firms in sector i only obtain f_{k_i}. Hence, the optimal subsidy is

$$\sigma_i \triangleq \frac{\hat{p}_i}{\tilde{p}_i} (f_{k_i} + F_{K_i}) - f_{k_i} \quad i = 1, ..., n \tag{5}$$

and the market values of the subsidies are

$$\tilde{p}_i \sigma_i \triangleq \hat{p}_i (f_{k_i} + F_{K_i}) - \tilde{p}_i f_{k_i} \quad i = 1, ..., n \tag{6}$$

The competitive dynamic equilibrium is supported by utility prices for investment $\tilde{p}$ that differ from social prices $\hat{p}$. The policymaker must take this difference into account to select a globally efficient path. It then follows that the adoption of a Pigouvian subsidy compensating for the productivity difference is not sufficient to obtain efficiency unless $\tilde{p}_i = \hat{p}_i$.

From optimal control theory, candidate efficient paths must satisfy the necessary condition for optimality $\hat{H}_k = \delta \hat{p} - \dot{\hat{p}}$.

When $n = 1$, we can state

Theorem 2. *The optimal policy brings about a higher intertemporal welfare than the planned solution, when utility is strictly concave and the initial market price for investment is higher than the initial shadow price a planning board (see Cass, 1966) would impose.*

Proof: see Appendix. ■

Two examples are presented below.

5. Example 1: global external effects

When external effects are as in Romer (1986), Pigouvian intervention eliminates the wedge between private and social productivities. Let the private marginal product of capital be

$$f_k = \alpha k^{\alpha-1} K^{\beta}$$

where α and β are the elasticities of output w.r.t. private and social capital.

The social marginal product equals

$$f_k + F_K = (\alpha+\beta)\, k^{\alpha+\beta-1}$$

Equality of private and social marginal product requires a policy instrument ϕ to be set such that

$$\phi + \alpha k^{\alpha-1} K^{\beta} = (\alpha+\beta)\, k^{\alpha+\beta-1}$$

or, in symmetrical equilibrium

$$\phi + \alpha k^{\alpha-1} k^{\beta} = (\alpha+\beta)\, k^{\alpha+\beta-1}$$

The Pigouvian subsidy must be equal to

$$\phi = \beta k^{\alpha+\beta-1}$$

This is equivalent to a marginal subsidy ϕ' to the private productivity of capital

$$(1+\phi')\, \alpha k^{\alpha-1} k^{\beta} = (\alpha+\beta)\, k^{\alpha+\beta-1}$$

i.e.

$$\phi' = \frac{\beta}{\alpha}$$

Equality between the social and the market rental rates for capital implies

$$u_k + \hat{p} f_k + \hat{p} F_K = u_k + \tilde{p} f_k + \tilde{p}\sigma$$

or

$$u_k + \hat{p}\left((\alpha+\beta)\, k^{\alpha+\beta-1}\right) = u_k + \tilde{p}\left(\alpha k^{\alpha-1} k^{\beta}\right) + \tilde{p}\sigma$$

The efficient subsidy on capital in place is

$$\sigma = \frac{\hat{p}}{\tilde{p}}\left((\alpha+\beta)\, k^{\alpha+\beta-1}\right) - \alpha k^{\alpha-1} k^{\beta}$$

which cannot be equal to ϕ unless the market price for investment coincides with its shadow price. Pigouvian taxation thus can only be effective when market and shadow prices are identical.

6. Example 2: congestion effects

Judd (1999) assumed negative external effect in the form of highway congestion reducing private marginal productivities. In particular, let the aggregate production function be

$$\begin{aligned} y &= F(k,l,k,l,g) \\ &= f(k,l,g) = k^{\alpha} l^{1-\alpha} k^{-\beta} l^{-\gamma} g^{\beta+\gamma} \end{aligned}$$

where l and h denote population and public expenditure in highways, respectively. The firm level production function is

$$\begin{aligned} Y &= F(K,l,k,l,h) \\ &= k^{\alpha} l^{1-\alpha} K^{-\beta} L^{-\gamma} h^{\beta+\gamma} \end{aligned}$$

A marginal Pigouvian instrument affecting the competitive equilibrium is a tax on private marginal productivity ξ such that

$$(1-\xi) F_k(K,L,k,l,h) = f_k(k,l,h)$$

i.e.

$$\xi = \frac{\beta}{\alpha}$$

On the other hand, the selection of the globally efficient path requires

$$\sigma = \frac{\hat{p}}{\tilde{p}} f_k(k,l,h) - F_k(K,L,k,l,h)$$

which, in turn, implies a subsidy to capital in place with market value equal to

$$\tilde{p}\sigma = \hat{p} f_k(k,l,h) - \tilde{p} F_k(K,L,k,l,h)$$

The equivalent marginal tax on private productivity is ν such that

$$\begin{aligned} (1-\nu) F_k(K,L,k,l,h) &= f_k(k,l,h) \\ (1-\nu) F_k(K,L,k,l,h) &= \frac{\hat{p}}{\tilde{p}} (\sigma + F_k(K,L,k,l,h)) \end{aligned}$$

which can be expressed as

$$v = 1 - \frac{\hat{p}}{\tilde{p}} \left(1 + \frac{\sigma}{F_K(K,L,k,l,h)}\right)$$

The marginal policy instrument v is the Pigouvian tax ξ only if market and shadow prices coincide.

The two examples illustrate the inadequacy of marginal Pigouvian instruments to select globally efficient paths, as conjectured in Ciccone and Matsuyama (1999) and Judd (1999).

7. Demand complementarities and inefficiencies

The above analysis can be extended to the important case of demand complementarities analyzed by Ciccone and Matsuyama (1999), who show that dynamic equilibria can be affected by inefficiencies which cannot be eliminated by Pigouvian taxation.

In particular, they compare the optimal allocation

$$\begin{aligned} \frac{\dot{c}}{c} &= \gamma(c)\left(\hat{F}'(m) - \delta\right) \\ \dot{m} &= \hat{F}(m) - c \end{aligned} \tag{7}$$

with the dynamic equilibrium system

$$\begin{aligned} \frac{\dot{c}}{c} &= \gamma(c)\left(F'(m) - \delta\right) \\ \dot{m} &= F(m) - c \end{aligned} \tag{8}$$

where m is the composite stock of a variety of differentiated intermediate inputs, $\hat{F}(m)$ is the aggregate social production function, $F(m)$ is the aggregate private production function, $\gamma(c)$ is the intertemporal elasticity of substitution in consumption. The assumption is that global inefficiencies arise when Hicks-Allen complementarity between differentiated inputs implies that the rate of return to investment in new differentiated inputs increases with aggregate investment. Since each private firm only has a negligible effect on aggregate investment, a convenient way to introduce this hypothesis in our framework is to assume $\hat{F}'(m) = F'(m) + f_m$ where f_m is the effect of aggregate investment on the rate of return $\hat{F}'(m)$.

Dynamic aggregate increasing returns are associated with a multiplicity of feasible paths satisfying the necessary conditions for optimality. Hence, when the initial market price for investment is higher than the social price, the non-linear optimal policy is sufficient to select the globally efficient path.

8. Conclusions

This paper analyzed the problem of efficient policy in the presence of technological externalities. It was shown that the standard Pigouvian approach is ineffective in restoring efficiency when market prices differ from shadow prices and multiple paths satisfy the first-order conditions for optimality. Rather, the policymaker should adopt optimal non-linear policy instruments correcting both for the wedge between private and social returns and for the discrepancy between market and shadow prices. Such tax/subsidies are sufficient to select the globally efficient path.

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A. Appendix

Proof of Theorem 1: The Hamiltonian associated to the social planning-command optimum problem, assuming symmetric equilibrium, $K = k$, is

$$\hat{H}(\hat{p}, k) \triangleq u\left(k, \dot{k}\right) + \hat{p}g(k, k, \hat{p})$$

The first order condition

$$\hat{H}_k = \delta\hat{p} - \dot{\hat{p}} \tag{9}$$

implies

$$u_k + \hat{p}g_k = u_k + \hat{p}(f_k + F_K) \tag{10}$$

The condition for optimal policy

$$H_k = \hat{H}_k \tag{11}$$

or, equivalently,

$$\dot{\tilde{p}} - \delta\tilde{p} = \dot{\hat{p}} - \delta\hat{p}$$

gives

$$u_k + \hat{p} f_k + \hat{p} F_K = u_k + \tilde{p} f_k + \tilde{p} \sigma \tag{12}$$

Hence, the optimal subsidy can be expressed as

$$\sigma \triangleq \frac{\hat{p}}{\tilde{p}} \left(f_k + F_K \right) - f_k \tag{13}$$

Q.E.D.

Proof of Theorem 2: In the one sector model, the initial price for investment equals marginal utility from initial consumption

$$\tilde{p}_0 = u^{'} \left(\tilde{c}_0 \right), \ \hat{p}_0 = u^{'} \left(\hat{c}_0 \right)$$

Both the non-linearly subsidized and the planned economy satisfy the first-order conditions from optimal control. The intertemporal welfare along the subsidized path equals, from Skiba (1978),

$$\tilde{W} \triangleq \delta \int u \left(\tilde{c}_t \right) e^{-\delta t} dt = u \left(\tilde{c}_0 \right) + \tilde{p}_0 \left(g \left(k_0, K_0, \tilde{p}_0 \right) + S_0 - \tau_0 \right) \tag{14}$$

while under central planning we would have

$$\hat{W} \triangleq \delta \int u \left(\hat{c}_t \right) e^{-\delta t} dt = u \left(\hat{c}_0 \right) + \hat{p}_0 \left(f \left(k_0, k_0 \right) - c \left(\hat{p} \right) \right)$$

In symmetric equilibrium, i.e. $k = K$, and under balanced budget, i.e. $S = \tau$, eq. (14) can be rewritten as

$$\tilde{W} = \delta \int u \left(\tilde{c}_t \right) e^{-\delta t} dt = u \left(\tilde{c}_0 \right) + \tilde{p}_0 \left(f \left(k_0, k_0 \right) - c \left(\tilde{p} \right) \right)$$

>From Ciccone and Matsuyama (1999), we know that

$$\tilde{W} - \hat{W} > \left(u^{'} \left(\tilde{c}_0 \right) - u^{'} \left(\hat{c}_0 \right) \right) \left(f \left(k_0, k_0 \right) - c \left(\hat{p} \right) \right)$$

or, equivalently

$$\tilde{W} - \hat{W} > \left(\tilde{p}_0 - \hat{p}_0 \right) \left(f \left(k_0, k_0 \right) - c \left(\hat{p} \right) \right)$$

But

$$\left(\tilde{p}_0 - \hat{p}_0 \right) \left(f \left(k_0, k_0 \right) - c \left(\hat{p} \right) \right) > 0 \Longleftrightarrow \tilde{p}_0 > \hat{p}_0$$

Q.E.D.

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