Derivatives
# Derivatives

## Course Introduction and Overview

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1 Course Objectives

This course introduces you to the analysis of derivatives in financial markets. In studying the course you will learn the main features of the most commonly used financial derivatives and will understand how to use them for the management of risk.

An important proviso is, however, in order. This course focuses on the conceptual and analytical aspects of derivatives. After studying this course, you will be able to understand the main characteristics of derivatives, and you should be able to avoid some of the most dangerous pitfalls from their use. However, the course is not a substitute for the professional expertise that can only be acquired by directly working in financial markets. Still, you will find that a sound grounding in the principles of derivatives will enable you to understand much better the practical aspects of derivatives investment and risk management.

2 Course Content

This course presents some of the main types of derivatives. Unit 1 introduces the basic derivatives contracts – i.e. forwards, futures and options. Unit 2 discusses in more detail the properties of stock options, which are among the most commonly traded derivative contracts, and can help you to understand more complex types of derivatives.

Unit 3 presents some mathematical models that have been applied to the analysis and pricing of derivatives products, including the Black-Scholes-Merton model. These models make use of concepts from stochastic calculus for the valuation of derivatives. This is necessary because the value of a derivative contract depends on the realisation of the value of the underlying asset, which is itself a random variable.

Unit 4 discusses how you can measure the sensitivity of a derivative portfolio to its risk factors and explains how you can effectively reduce the risk of your position. Unit 5 explains interest rates and swaps, and discusses the management of bond portfolios. Unit 6 examines the concept of credit risk and introduces credit derivatives. Unit 7 presents some exotic options, and explains how the methods for the analysis of financial options can also be relevant for real investment decisions under uncertainty. Unit 8 introduces some more advanced numerical procedures for the analysis of options, including alternatives to the Black-Scholes-Merton model and the application of Monte Carlo methods to the pricing of American options.
3 Study Materials

This Study Guide is your central learning resource as it structures your learning unit by unit. Each unit should be studied within a week. It is designed in the expectation that studying the unit and the associated core readings will require 15 to 20 hours during the week, but this will vary according to your background knowledge and experience of studying.

Course Units

Unit 1 Derivatives Contracts
1.1 Introduction
1.2 Forward Contracts
1.3 Futures Contracts
1.3 Options
1.4 Types of Traders
1.5 A ‘Health Warning’
1.6 Conclusions
1.7 Solutions to Exercises

Unit 2 Properties of Stock Options
2.1 Options
2.2 Stock Options
2.3 Warrants, Employee Stock Options and Convertibles
2.4 Basics of Pricing Stock Options
2.5 Trading Strategies Involving Options
2.6 Conclusions
2.7 Solutions to Exercises

Unit 3 The Behaviour of the Stock Price and the Black-Scholes-Merton Model
3.1 Introduction
3.2 The Wiener Process
3.3 The Behaviour of Stock Prices
3.4 Itô’s Lemma
3.5 The Lognormal Property of Stock Prices
3.6 The Black-Scholes-Merton Equation and the Black-Scholes-Merton Formula
3.7 Conclusions
3.8 Solutions to Exercises

Unit 4 Greek Letters and Trading Strategies
4.1 Naked and Covered Positions
4.2 Delta \( \Delta \) Hedging
4.3 Theta \( \Theta \)
4.4 Gamma \( \Gamma \)
4.5 Vega \( \nu \)
4.6 Rho \( \rho \)
4.7 Hedging and Portfolio Insurance
4.8 Conclusions
4.9 Solutions to Exercises
Unit 5 Interest Rate Models
5.1 Interest Rates
5.2 Forward Rates
5.3 Management of Bond Portfolios
5.4 Swaps
5.5 Currency Swaps
5.6 Bond Options
5.7 Conclusions
5.8 Solutions to Exercises

Unit 6 Credit Derivatives and Credit Risk
6.1 Credit Ratings and Default Probabilities
6.2 Mitigation of Credit Risk and Default Correlation
6.3 Credit Default Swaps
6.4 Asset-Backed Securities and Collateralized Debt Obligations
6.5 Correlation and the Gaussian Copula
6.6 Conclusions
6.7 Solutions to Exercises

Unit 7 Some Exotic Options
7.1 Exotic Options
7.2 Barrier, Binary, and Lookback Options
7.3 Asian Options
7.4 Some Other Exotic Options
7.5 Weather and Energy Derivatives
7.6 Insurance Derivatives
7.7 Conclusions
7.8 Solutions to Exercises

Unit 8 Further Numerical Procedures
8.1 Binomial Trees
8.2 Alternative Procedures for Constructing Trees
8.3 Monte Carlo Simulations
8.4 Finite Difference Methods
8.5 Alternatives to Black-Scholes-Merton
8.6 Stochastic Volatility Models
8.7 American Options
8.8 Conclusions
8.9 Solutions to Exercises

📖 The Textbook

The reference for this course is a textbook by John Hull:


This is a classic textbook on derivatives. It is written by an authority in the field, and covers both theoretical and practical aspects in the use of derivatives. It discusses all the most commonly used derivative instruments, and
contains a large number of examples and exercises. The latest edition also explains a number of recent events and analyses relevant case studies in financial markets.

The textbook itself is very thorough. In this course you will not study the book by Hull in its entirety, but will instead concentrate on a selected number of key chapters. If you have a professional interest in derivatives, however, you will find it extremely useful to study all the remaining chapters as well.

**Software**

The textbook by Hull comes with a CD that contains the proprietary software DerivaGem. This software enables you to compute the prices of a large number of derivatives and to draw the relevant graphs. You should familiarise yourself with DerivaGem, and you should make extensive use of it as you study the textbook and the course units.

Please now install the DerivaGem software on your PC. You may wish to refer to the appendix on DerivaGem Software on pages 818–22 of the textbook by Hull for a general description of the software, although for the moment all you will need are the general instructions on page 818.

### 4 Studying the Course

In the units, you will be asked to answer questions and solve exercises related to the course materials. The exercises are an essential part of the course, and it is important that you take your time to answer them. A solution to the exercises is provided at the end of each unit.

There are two assignments, after Unit 4 and Unit 8. These count together for 30% of the course grade. A three-hour unseen examination counts for the other 70%. To gain good marks it is essential that you make use of the materials in the textbook and reader and apply ideas and techniques to real-world circumstances. You will be asked to show your analytical skills and, of course, you will be judged only on the quality of your knowledge and argument and not on your opinions.

The examination will follow the format of the specimen examination paper provided at the end of this introduction. You must answer three questions. When you write the examination, please remember:

- to write logically, setting out your argument and evidence, using the materials you have studied, including the case studies
- to show the sources that your remarks are based on
- to plan your answers before you start writing
- to read carefully the questions that require the solution to problems, and to make sure that your explain carefully the procedure that you follow for solving them
to manage your time, so that each question is allocated roughly the same amount of attention.

5 The Course Author


Professor Scaramozzino has taught Risk Management for the on-campus MSc in Finance and Financial Law in London and has contributed to several off-campus CeFiMS courses, including Mathematics and Statistics for Economists, Portfolio Analysis and Derivatives, Quantitative Methods for Financial Management, Managerial Economics and Risk Management: Principles and Applications.

6 Assessment

Your performance on each course is assessed through two written assignments and one examination. The assignments are written after week four and eight of the course session and the examination is written at a local examination centre in October.

The assignment questions contain fairly detailed guidance about what is required. All assignment answers are limited to 2,500 words and are marked using marking guidelines. When you receive your grade it is accompanied by comments on your paper, including advice about how you might improve, and any clarifications about matters you may not have understood. These comments are designed to help you master the subject and to improve your skills as you progress through your programme.

The written examinations are ‘unseen’ (you will only see the paper in the exam centre) and written by hand, over a three hour period. We advise that you practise answering questions on the Specimen Exam in these conditions as part of your examination preparation, as it is not something you would normally do.
You are not allowed to take books or notes into the exam room. This means that you need to revise thoroughly in preparation for each exam. This is especially important if you have completed the course in the early part of the year, or in a previous year.

**Preparing for assignments and exams**

There is good advice on preparing for assignments and exams and writing them in Sections 8.2 and 8.3 of *Studying at a Distance* by Talbot. We recommend that you follow this advice.

The examinations you will sit are designed to evaluate your knowledge and skills in the subjects you have studied: they are not designed to trick you. If you have studied the course thoroughly, you will pass the exam.

**Understanding assessment questions**

Examination and assignment questions are set to test different knowledge and skills. Sometimes a question will contain more than one part, each part testing a different aspect of your skills and knowledge. You need to spot the key words to know what is being asked of you. Here we categorise the types of things that are asked for in assignments and exams, and the words used. All the examples are from CeFiMS examination papers and assignment questions.

### Definitions

Some questions mainly require you to show that you have learned some concepts, by setting out their precise meaning. Such questions are likely to be preliminary and be supplemented by more analytical questions. Generally ‘Pass marks’ are awarded if the answer only contains definitions. They will contain words such as:

- Describe
- Define
- Examine
- Distinguish between
- Compare
- Contrast
- Write notes on
- Outline
- What is meant by
- List

### Reasoning

Other questions are designed to test your reasoning, by explaining cause and effect. Convincing explanations generally carry additional marks to basic definitions. They will include words such as:

- Interpret
- Explain
- What conditions influence
- What are the consequences of
- What are the implications of
Judgment
Others ask you to make a judgment, perhaps of a policy or of a course of action. They will include words like:
- Evaluate
- Critically examine
- Assess
- Do you agree that
- To what extent does

Calculation
Sometimes, you are asked to make a calculation, using a specified technique, where the question begins:
- Use the single index model analysis to
- Using any financial model you know
- Calculate the standard deviation
- Test whether

It is most likely that questions that ask you to make a calculation will also ask for an application of the result, or an interpretation.

Advice
Other questions ask you to provide advice in a particular situation. This applies to law questions and to policy papers where advice is asked in relation to a policy problem. Your advice should be based on relevant law, principles, evidence of what actions are likely to be effective.
- Advise
- Provide advice on
- Explain how you would advise

Critique
In many cases the question will include the word ‘critically’. This means that you are expected to look at the question from at least two points of view, offering a critique of each view and your judgment. You are expected to be critical of what you have read.

The questions may begin
- Critically analyse
- Critically consider
- Critically assess
- Critically discuss the argument that

Examine by argument
Questions that begin with ‘discuss’ are similar – they ask you to examine by argument, to debate and give reasons for and against a variety of options, for example
- Discuss the advantages and disadvantages of
- Discuss this statement
- Discuss the view that
- Discuss the arguments and debates concerning
The grading scheme

Details of the general definitions of what is expected in order to obtain a particular grade are shown below. Remember: examiners will take account of the fact that examination conditions are less conducive to polished work than the conditions in which you write your assignments. These criteria are used in grading all assignments and examinations. Note that as the criteria of each grade rises, it accumulates the elements of the grade below. Assignments awarded better marks will therefore have become comprehensive in both their depth of core skills and advanced skills.

70% and above: Distinction As for the (60-69%) below plus:
- shows clear evidence of wide and relevant reading and an engagement with the conceptual issues
- develops a sophisticated and intelligent argument
- shows a rigorous use and a sophisticated understanding of relevant source materials, balancing appropriately between factual detail and key theoretical issues. Materials are evaluated directly and their assumptions and arguments challenged and/or appraised
- shows original thinking and a willingness to take risks

60-69%: Merit As for the (50-59%) below plus:
- shows strong evidence of critical insight and critical thinking
- shows a detailed understanding of the major factual and/or theoretical issues and directly engages with the relevant literature on the topic
- develops a focussed and clear argument and articulates clearly and convincingly a sustained train of logical thought
- shows clear evidence of planning and appropriate choice of sources and methodology

50-59%: Pass below Merit (50% = pass mark)
- shows a reasonable understanding of the major factual and/or theoretical issues involved
- shows evidence of planning and selection from appropriate sources,
- demonstrates some knowledge of the literature
- the text shows, in places, examples of a clear train of thought or argument
- the text is introduced and concludes appropriately

45-49%: Marginal Failure
- shows some awareness and understanding of the factual or theoretical issues, but with little development
- misunderstandings are evident
- shows some evidence of planning, although irrelevant/unrelated material or arguments are included

0-44%: Clear Failure
- fails to answer the question or to develop an argument that relates to the question set
• does not engage with the relevant literature or demonstrate a knowledge of the key issues
• contains clear conceptual or factual errors or misunderstandings

[approved by Faculty Learning and Teaching Committee November 2006]

Specimen exam papers

Your final examination will be very similar to the Specimen Exam Paper that is printed on the following pages. It will have the same structure and style and the range of question will be comparable.

CeFiMS does not provide past papers or model answers to papers. Our courses are continuously updated and past papers will not be a reliable guide to current and future examinations. The specimen exam paper is designed to be relevant to reflect the exam that will be set on the current edition of the course.

Further information

The OSC will have documentation and information on each year’s examination registration and administration process. If you still have questions, both academics and administrators are available to answer queries.

The Regulations are also available at www.cefims.ac.uk/regulations.shtml, setting out the rules by which exams are governed.
FINANCE

Derivatives

Specimen Examination

This is a specimen examination paper designed to show you the type of examination you will have at the end of this course. The number of questions and the structure of the examination will be the same, but the wording and requirements of each question will be different.

The examination must be completed in THREE hours. Answer any THREE questions.

The examiners give equal weight to each question; therefore, you are advised to distribute your time approximately equally between three questions.

Statistical tables are provided at the end of this examination paper.

Candidates may use their own electronic calculators in this examination provided they cannot store text. The make and type of calculator MUST BE STATED CLEARLY on the front of the answer book.

Please do not remove this Paper from the Examination Room.
It must be attached to your answer book at the end of the examination.

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Answer any THREE questions.
Answer all parts of multi-part questions.

1. **Answer both parts of this question.**
   a. Describe in detail what is meant by a futures contract. Explain the relationship between futures price and spot price, and give reasons to justify the requirement that margin accounts are held.
   b. Explain what position can be regarded as equivalent to a long forward contract to buy an asset at a price $K$ on a given future date and a short position in a call option with a strike price $K$ on the same date.

2. Explain the upper and lower bounds that must be satisfied by option prices. Illustrate the put-call parity formula and discuss its applications.

3. **Answer all parts of this question.**
   Suppose that a stock price $S$ follows a geometric Brownian motion:
   \[ dS = \mu S dt + \sigma S dz \]
   where $dz$ is a standard Wiener increment and where $\mu$ and $\sigma$ are constant parameters.
   a. Provide a motivation for the above stochastic process for $dS$.
   b. Using Itô’s Lemma, find the distribution for $x$ in the following cases:
      i. $x = \alpha S$, where $\alpha$ is a constant;
      ii. $x = S^\gamma$, where $\gamma$ is a constant;
      iii. $x = e^{\beta S}$, where $\beta$ is a constant.

4. Define the delta, theta, and gamma of an option portfolio. Explain under what conditions theta can be regarded as a proxy for gamma in a portfolio.

5. **Answer both parts of this question.**
   a. Carefully explain the relationship between forward and spot rates. Show how forward rates can be measured in terms of changes in the price of zero-coupon bonds.
b. Suppose that zero interest rates with continuous discounting are as given in the following table:

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<td>3.4</td>
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</table>

Calculate the forward interest rates for the second, third, fourth, fifth and sixth years.

6. What is meant by the Gaussian copula model for time of default? Carefully explain this model and discuss the assumptions upon which it is based.

7. Explain what is meant by barrier, binary and lookback options. Describe their properties and discuss under what conditions they can be used for the hedging of risks and for speculation. How do these options compare with the corresponding regular options?

8. Illustrate in detail two alternatives to the Black-Scholes-Merton model. Discuss under what conditions they can be appropriate for the modelling of asset prices.
Normal Curve Areas  
Standard normal probability in right-hand tail

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Unit Content

Unit 1 will introduce some fundamental terminology relating to derivatives, and will describe some basic features of derivatives contracts. You will examine the characteristics of forward contracts, futures contracts and options, and you will also learn about the types of traders who deal in derivatives, and some of the dangers of their misuse.

Learning Outcomes

When you have completed your study of this unit and its readings, you will be able to

- discuss and differentiate between the most common types of financial derivatives: forward contracts, futures and options
- explain the advantages of long and short positions in these contracts
- distinguish between put and call options
- outline three main reasons for the use of derivatives
- discuss the potential dangers in misusing derivatives, and how to avoid them.

Reading for Unit 1

You will read from Chapter 1 ‘Introduction’, Chapter 2 ‘Mechanics of futures markets’ and Chapter 35 ‘Derivatives mishaps and what we can learn from them’ of your textbook, John C. Hull (2012) *Options, Futures, and Other Derivatives,*
1.1 Introduction

Derivatives are a general class of financial contracts, which are written in terms of an underlying financial or real asset. Their payoff over a given period of time will depend on the performance of the underlying asset, which could include stocks, interest rates or exchange rates. The value of a financial derivative, therefore, depends on the performance of the underlying asset. Financial derivatives include futures and options. They are a flexible form of financial instrument, and can be very effective in enabling investors to achieve a complex risk profile. In particular, derivatives can be a very powerful instrument for reducing risk, or hedging. However, derivatives can also be used to increase the risk of investors, if they are used for speculation. Finally, derivatives allow investors to make riskless profits by exploiting mispricing of assets when they are used for arbitrage.

We begin by looking at types of derivative contracts.

1.2 Forward Contracts

The simplest derivative instrument is a forward contract. This is ‘an agreement to buy or sell an asset at a certain future time for a certain price’ (Hull, page 5). You can see that the main feature of forward contracts is that it enables you to fix now the price at which a transaction will take place at a future date. You can therefore make use of a forward contract in order to eliminate the uncertainty associated with the future price of an asset. A forward contract can be contrasted to a spot contract, where delivery of the asset is immediate. Thus, a forward contract can effectively eliminate the uncertainty of the future spot price.

The reduction of uncertainty could be desirable from the point of view of risk management. However, it is important you realise that the elimination of uncertainty does not imply that you would always be better off by entering a forward contract. Indeed, if you successfully hedge your position you will be able to eliminate or reduce the losses associated with adverse movements in the price of the underlying asset, but at the cost of foregoing the profits associated with potentially favourable asset price changes.

In order to see this point, denote the delivery price written in the forward contract by $K$ and the future spot price on the delivery date by $S_T$. If you have a long position in the forward contract – that is, if you commit yourself to purchasing the underlying asset at contract maturity – your payoff upon maturity will be

$$S_T - K$$

(1.1)

The value $K$ is written into the contract, but $S_T$ is the spot price that will prevail in the market at the future delivery date, and cannot be known in advance. If $S_T$ turns out to be greater than $K$, you will be able to purchase at a
price $K$ an asset that has the greater market price $S_T$, and will therefore make a profit: $S_T - K > 0$. However, if the market price $S_T$ happens to be less than $K$, you will have to pay the price $K$ for an asset whose market price is only $S_T$, and will therefore make a loss: $S_T - K < 0$.

If you commit yourself to selling an asset, you will have a short position in the forward contract. In this case, your payoff at maturity will be given by:

$$K - S_T$$

Your profits and losses will now be the mirror image to those in a long position. You will make a gain if the market price $S_T$ is less than the forward price $K$, and will make a loss if $S_T$ is greater than $K$ on maturity.

If you have a long position in an asset, the gains or losses from a long position in a forward contract written on that asset will offset the losses or gains from your position in the underlying asset. You will therefore be able to use forward contracts in order to achieve a hedged position, so that your payoff is no longer subject to the asset price uncertainty.

**Reading**

Please now stop and read the introduction to Chapter 1 to the end of the section on forward contracts of your textbook by Hull, on pages 1–7. Pay special attention to the numerical examples that motivate the use of forward contracts.

**Exercise 1.1**

Please now solve problem 1.5 on page 18 of your textbook. An answer is provided at the end of this unit.

### 1.3 Futures Contracts

The forward contracts that you studied in section 1.2 are usually arranged over-the-counter (OTC) by two financial institutions, or by a financial institution and one of its clients (see the discussion in section 1.2 on pages 3–4 of your textbook by Hull). This implies that the characteristics of the contract can be customised to the specific needs of the two parties. However, over-the-counter contracts present the potential disadvantage of involving some credit risk (that is, the risk that the counterparty may default on its obligations), and could also suffer from limited liquidity.

By contrast, futures contracts have similar features to forward contracts, but are normally traded on an exchange market. This requires that futures contracts be standardised – that is, they must specify in detail the characteristics of the particular underlying asset, and must follow standard conventions regarding the contract size, the delivery arrangements and the delivery dates. This both significantly reduces the credit risk associated with futures contracts and makes them much more liquid.
Futures contracts are traded on markets such as the CME Group (created in 2007 from the merger between the Chicago Board of Trade (CBOT) and the Chicago Mercantile Exchange (CME)), where trade instructions are carried out by brokers. As a *bona fide* insurance against the risk of default, brokers usually require that the investor deposits a fund known as the *initial margin*. The initial margin can be modified over the life of the contract, depending on the changes in the market value of the futures contract (*variation margin*). In turn, brokers must maintain margins with a *clearing house*, which acts as an intermediary in futures transactions. These are the *clearing margins*. The main role of margins is to reduce the credit risk in futures contracts.

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**Reading**

Please now read section 1.4 on page 7 and Chapter 2, pages 22–43 of your textbook by Hull. Section 1.4 provides a brief introduction to futures contracts, whereas Chapter 2 describes the mechanics of futures margins, including a detailed discussion of the workings of margins. Table 2.3 on page 41 usefully summarises the main differences between forward and futures contracts.

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**Exercise 1.2**

Please solve problem 1.6 on page 18 of the textbook by Hull.

### 1.4 Options

Forward and futures contracts constitute a binding commitment for investors. If an investor has a long position in a forward contract, for instance, she is committing herself to completing the contract at maturity and to purchasing the underlying asset. Forward and futures contracts therefore require final delivery, unless they are closed out prior to maturity (which is typically the case for futures contracts). By contrast, *options contracts* confer onto their holders the right to buy or sell an asset at or before maturity, but do not prescribe that the purchase or sale must be executed. In other words, there is no obligation involved to carry out the transaction. If it is not profitable to do so, the holder of an option contract could choose to let the contract expire unexercised. Their only loss would be the initial cost of the option contract.

An option to buy an asset is a *call option*, and an option to sell an asset is a *put option*. The price at which the purchase or the sale can be carried out is called the *strike price*, or the *exercise price*. The price of the option contract is called the *option premium*. Option contracts that can be exercised any time up to and including the expiration date are called *American options*, whereas contracts that can only be exercised on the expiration date are called *European options*.

Investors who have the right to buy or to sell an asset (in call options or in put options, respectively) are said to have a *long* position. Their counterparties, who have issued the options, are said to have a *short* position: the options would be exercised against them.
The main feature of options contracts is that their payoffs can be strongly asymmetric. For instance, if you have a long position in a call option, the most you can lose is the premium that you paid for the option. By contrast, your gains could potentially be very large, since if the price of the asset increases on maturity you could profit from the whole difference between the spot price of the asset and the exercise price. The asymmetry in the payoff profiles of options makes them a very versatile financial instrument.

In this course you will examine in detail a number of different option contracts, and you will see how they can be combined to achieve a large variety of risk profiles for your financial portfolios.

Reading

Please now stop and read section 1.5 of your textbook by Hull, pages 7–9. Pay particular attention to the two diagrams in Figure 1.3 on page 10, which illustrate the payoff profiles associated with a long position in a call option and a short position in a put option on a stock.

Exercise 1.3

Please solve problem 1.7 on page 18 of the book by Hull.

1.5 Types of Traders

As noted already, derivatives are very flexible financial instruments. They can be used for a variety of purposes, and can be very effective in modifying the risk profile of investors.

There are three main reasons for the use of derivatives.

- First, they can be used to reduce risk – that is, for hedging. For instance, if an investor has a long position in an asset she can hedge her risk by taking a short position on a derivative written on the underlying asset.

- Second, derivatives can be used in order to increase risk – that is, for speculation. If you believe that the market is mispricing the probability that the price of an asset will change – for instance, you could believe that it is more likely that the price of an asset will fall, relative to what is expected by the market and is reflected in the asset price. In this case, you could speculate by directly taking a short position in the asset. However, an alternative strategy could involve taking a short position on a derivative written on the asset. Taking a short position on a derivative could be more profitable, because futures and options can provide a form of leverage. The financial consequences are much larger, given the size of the initial investment.

- Finally, derivatives can be used to exploit potential arbitrage opportunities, which could arise, for instance, from small discrepancies in the pricing of assets across different markets, or from inconsistency in the pricing of similar assets. Since arbitrage involves little or no risk,
large amounts of resources have been devoted to developing software programmes that are able to execute automatic transactions whenever arbitrage opportunities are seen to arise.

**Reading**

Please now read sections 1.6 to 1.9 of the book by Hull, pages 9–16. You should study carefully the examples in Tables 1.4 and 1.5, which illustrate the potential effect of leverage when using futures and options for speculative investment.

**Exercise 1.4**

Please solve problem 1.9 on page 18 of the book by Hull.

**Exercise 1.5**

Please solve problem 1.16 on page 19 of the book by Hull.

**Exercise 1.6**

Please answer question 1.21 on page 19 of the book by Hull.

### 1.6 A ‘Health Warning’

While derivatives are very flexible instruments, if they are misused they can lead to potentially catastrophic losses. There has been a number of high-profile instances of reputable financial institutions being forced to fold because of the enormous losses suffered from speculation in the derivatives market. It is therefore essential that proper measures are put in place to monitor and control the risks associated with trading in derivatives. These measures include

- setting proper risk limits
- monitoring traders
- separating trading execution from book keeping and accounting
- being aware of the usefulness but also of the limitations of quantitative models, and
- taking liquidity risk seriously.

These measures will not completely eliminate all risks, but can go a long way towards reducing the likelihood of negative outcomes from the trading in derivatives.

**Reading**

Please now read section 1.10 and the Summary on pages 16–18 and chapter 35, pages 779–90 of your textbook by Hull. Chapter 35 is towards the end of the book. You may not be able to understand yet all the references in the chapter, but it is very useful that you are aware of the potential issues in the use of derivatives, and of some of the main
measures that should be taken in order to avoid the most serious adverse consequences; and you should therefore read this chapter now.

1.7 Conclusions

This unit has introduced the most common types of financial derivatives: forward contracts, futures and options. You have studied their different characteristics and have started to learn how you can use them to modify the payoff of your investment profile. In the next units of this course you will build on the basic concepts presented in this unit and will learn how you can construct very complex risk profiles by making use of derivatives instruments.
1.8 Solutions to Exercises

Here are the answers to the exercises you’ve been assigned for this unit, but please don’t check the answers until you’ve worked them out for yourself.

Exercise 1.1
a) At the end of the contract the investor sells 100,000 British pounds for US dollars at an exchange rate of 1.4000 US dollars per pound, whereas the spot exchange rate on maturity is 1.3900. The investor therefore gains $1,000.

b) At the end of the contract the investor sells 100,000 British pounds for US dollars at an exchange rate of 1.4200 US dollars per pound, when the spot exchange rate on maturity is 1.4000. The investor therefore loses $2,000.

Exercise 1.2
a) The trader commits herself to selling 50,000 pounds of cotton at the end of the contract at a price of 50 cents per pound. If the spot price of cotton at maturity is only 48.20 cents per pound, she will gain $900.

b) The trader must sell 50,000 pounds of cotton at the end of the contract at a price of 50 cents per pound, when the spot price of cotton at maturity is 51.30 cents per pound. She will therefore lose $650.

Exercise 1.3
You have sold a put option, and therefore you have a short position on a put contract. You have received the option premium, and have committed yourself to buying 100 shares at a price of $40, if the party who has purchased the put option decides to exercise her right to sell the shares at the exercise price. At present, it would not be profitable to exercise the option because the market price of each share is $41.

The investor who has purchased the put option will only find it profitable to exercise it if the stock price falls below $40. For instance, if the stock price falls to $30, the holder of the put option could purchase the shares at $30 and sell them to you at $40. She will have made a profit of $1,000, minus the price of purchasing the option contract. You will have to buy the shares at $40, although their market price is only $30. You will therefore have made a loss of $1,000, minus the premium that you have received from the option holder.

Exercise 1.4
One strategy that you can implement is to purchase 200 shares (5,800/29); the alternative strategy would be to purchase 2,000 call options (5,800/2.9). The table below illustrates the payoffs associated with each one of the two strategies when the stock price increases to $40 and when it declines to $25.
When the stock price increases to $40, your profit from the first strategy will be $200 \times ($40 - $29) = $2,200, and $2,000 \times ($40 - $30)] - $5,800 = $14,200 from the second strategy, if you exercise your call option and purchase stocks at the strike price of $30. By contrast, when the stock price falls to $25 your losses will be $200 \times ($29 - $25) = $800 from the first strategy, but will amount to your whole investment of $5,800 because you will have to let your options expire unexercised.

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<th>Strategy</th>
<th>Stock price</th>
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<td>Buy 200 shares</td>
<td>$25, $40</td>
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<tr>
<td>($800)</td>
<td>$2,200</td>
</tr>
<tr>
<td>Buy 2,000 call options</td>
<td>$5,800</td>
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You can verify that the second strategy yields larger gains when the stock price increases, but also larger losses when the stock price falls.

**Exercise 1.5**

The put option will be exercised against the trader if the stock price is less than $30 in December. If we consider the price that the trader has received for issuing the option, she will make a profit provided the stock price in December is greater than $26 (ignoring the time value of money).

**Exercise 1.6**

With futures and options, the gain to one party must always be equal to the losses suffered by the other party. Hence, if we add up the payoffs to both parties, they must always add up to zero. Graphically, the payoff from a short position is the mirror image of the payoff from a long position (that is, it is symmetrical about the horizontal axis).