Budget Deficits and Exchange-Rate Crises*

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Abstract

This paper investigates currency crises in an optimizing general equilibrium model with overlapping generations. It is shown that a rise in government budget deficits financed by future taxes generates a decumulation of external assets, leading up to a speculative attack and forcing the monetary authorities to abandon the peg.

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1 Introduction

The financial and currency turmoil of the 90’s in many European, Latin-American and Asian countries have called into question the viability of fixed exchange-rate regimes and led to the development of new models on the causes of currency collapses. We can now identify at least two main theoretical explanations of crises, one based on the view that a collapse is the inevitable outcome of inconsistent macroeconomic policies and the other based on the view that a collapse results from self-fulfilling expectations. According to the so-called ‘first-generation’ models of currency crises, if a government finances its fiscal deficits by printing money in excess of money demand growth, while following a fixed exchange-rate policy, a gradual loss of reserves will occur, leading to a speculative attack against the currency that forces the abandonment of the fixed-rate regime eventually (see, e.g., Krugman 1979; Flood and Garber 1984; Obstfeld 1986a; Calvo 1987; van Wijnbergen 1991; Calvo and Végh 1999).

The ‘second-generation’ models of currency crises, on the other hand, show that the government’s decision to give up a fixed exchange rate depends on the net benefits of pegging; hence, the fixed rate is likely to be maintained as long as the benefits of devaluing are smaller than the costs. However, changes in market beliefs about the currency sustainability can force the government to go out of the peg. For example, if agents expect a devaluation, a speculative attack will start, forcing the government to abandon the peg, since the costs of keeping the exchange rate fixed outweigh the benefits. On the other hand, if agents expect no change in the currency rate the fixed peg will be preserved. Private expectations are self-fulfilling and multiple equilibria can occur, for given fundamentals (see, e.g., Obstfeld 1996; Velasco 1996; Cole and Kehoe 1996; Jeanne 1997; Jeanne and Masson, 2000).1

1The partition between first and second generation models is consistent with the classification scheme of currency crises proposed by Flood and Marion (1999), and Jeanne (2000). From this point of view the so called “third-generation” models, elaborated to explain the more recent financial turmoil in Asia, Latin America and Russia, are considered extensions of the existing setups that explicitly include the financial side of the economy.
More recently, the debate about the role played by fundamentals and/or self-fulfilling expectations in triggering a speculative attack has been enriched by a new set of models analyzing currency crises in the context of a change in the expectations of future policy (see, e.g., Daniel 2000, 2001; Burnside, Eichenbaum, and Rebelo 2001, 2003). According to this view, explanations of crises do not necessarily require a period of fundamental misalignments. All that is needed is that the path of current and future government policies becomes inconsistent with the fixed peg.

This literature typically uses static models, or models with extrinsic dynamics, that is models where the dynamics of the system come exclusively from current or anticipated future changes in exogenous variables. Such systems are, in fact, always in steady-state equilibrium in the absence of external shocks. This is in contrast to the so-called intrinsic dynamics of the system, where the economy evolves from some initial stationary state due to, e.g., the accumulation of capital stock or foreign assets. Models with intrinsic dynamics are useful to understand currency crises and to predict the exact time of an attack. They enable us to study the dynamics of relevant macroeconomic variables.

The purpose of this paper is to deal with this kind of dynamics using a modified version of the Yaari (1965)-Blanchard (1985) model. Our approach has three main advantages. First, it allows a ‘nondegenerate’ dynamic adjustment in the basic monetary model of exchange rate determination. Second, it shows that the macroeconomic equilibrium is dependent on the timing of fiscal policies. Third, the current account is allowed to play a crucial role in transmitting fiscal disturbances to the rest of the economy.

A central finding of the paper is that a collapse may occur even as a consequence of a temporary tax cut fully financed by future taxes. In particular, following a fiscal expansion, current account imbalances and the expected depletion of foreign assets will lead up to a currency crisis forcing the mon-

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2This distinction may be found, for example, in Turnovsky (1977, 1997) and Obstfeld and Stockman (1985).
etary authorities to adopt a floating exchange-rate regime. This result is in sharp contrast with the existing literature, where monetary or fiscal policies are inconsistent or expected to be inconsistent with the exchange-rate policy. On the other hand, our theoretical results are consistent with the evidence that Asian countries that came under attack in 1997 were those which had experienced larger current account deficits on the eve of the crisis.\(^3\)

The main conclusion that emerges from our analysis is that crises can occur in a flexible price fully optimizing framework even when both monetary and fiscal policies are correctly designed, that is when the intertemporal budget constraint of the government is always respected and monetary policy obeys the rules of the game. The crisis, however, is not driven by self-fulfilling expectations, as the collapse results from a well-defined dynamics in the fundamentals. The main implication of our model is that the sustainability of a fixed exchange-rate system may require not only giving up monetary sovereignty, but also strongly restraining the conduct of fiscal policy.

The paper is organized as follows. Section II presents the theoretical model. Section III describes the dynamics of the model and the time of the speculative attack. Section IV contains the summary and conclusions of the paper.

## 2 The Model

Consider a small open economy described as follows. Agents have perfect foresight and consume a single tradeable good. Domestic supply of the good is exogenous. Household’s financial wealth is divided between domestic money (which is not held abroad) and internationally traded bonds. There are no barriers to trade, so that purchasing power parity (PPP) holds at all times, that is \( P^*S = P \), where \( S \) is the nominal exchange rate (defined as units of domestic currency per unit of foreign currency), \( P \) is the domestic price.

\(^3\)For an exhaustive overview of the economic fundamentals in Asian countries in the years preceding the financial and currency crisis, see Corsetti, Pesenti and Roubini (1999) and World Bank (1999).
level and $P^*$ is the foreign price level. There is perfect capital mobility and domestic and foreign assets are perfect substitutes, thus uncovered interest parity (UIP) is always verified, $i = i^* + \dot{s}$, where $i$ and $i^*$ are the domestic and foreign (constant) nominal interest rate, respectively and $\dot{s} \equiv d (\ln S) / dt$ is the rate of exchange depreciation. In the absence of foreign inflation the external nominal interest rate is equal to the real rate.

The demand side of the economy is described by an extended version of the Yaari-Blanchard perpetual youth model with money in the utility function.\(^4\) There is no bequest motive and financial wealth for newly born agents is assumed to be zero. The birth and death rates are the same, so that population is constant. Let $\delta$ denote the instantaneous constant probability of death and $\beta$ the subjective discount rate. For convenience, the size of each generation at birth is normalized to $\delta$, hence total population is equal to unity.

Each individual of the generation born at time $s$ at each time period $t \geq s$ faces the following maximization problem:

$$\max \int_0^\infty \log \left( \varepsilon_{s,t}^e \frac{m_{s,t}}{P_t} \right) e^{-(\beta+\delta) t} dt,$$

subject to the individual consumer’s flow budget constraint

$$\frac{d (w_{s,t}/P_t)}{dt} = (i^* - \dot{p}^* + \delta) \frac{w_{s,t}}{P_t} + y_{s,t} - \tau_{s,t} - c_{s,t} - (i^* + \dot{s}) \frac{m_{s,t}}{P_t},$$

and to the transversality condition

$$\lim_{t \to \infty} \frac{w_{s,t} e^{-\int_0^t (i^* - \dot{p}^* + \delta) dv}}{P_t} = 0,$$

where $0 < \epsilon < 1$, $\beta < i^* - \dot{p}^*$, $\dot{p}^*$ is the external inflation rate, while $c_s, y_s, m_s, w_s$ and $\tau_s$ denote consumption, endowment, nominal money balances, total

\(^4\)The approach of entering money in the utility function to allow for money holding behavior within a Yaari-Blanchard framework, is common to a number of papers including Spaventa (1987), Marini and van der Ploeg (1988), van der Ploeg (1991), and Kawai and Maccini (1990, 1995). Similar results could also be obtained by use of cash-in-advance models (see Feenstra, 1986).

\(^5\)Following Blanchard (1985) this condition ensures that consumers are relatively patient, in order to ensure that the steady state-level of aggregate financial wealth is positive.
financial wealth and lump-sum taxes, respectively. Notice that the effective discount rate of consumers is given by \( \beta + \delta \), where \( \beta + \delta > i^* - \dot{p}^* \). Each individual is assumed to receive for every period of her life an actuarial fair premium equal to a fraction \( \delta \) of her financial wealth from a life insurance company operating in a perfectly competitive market. At the time of her death the remaining individual’s net wealth goes to the insurance company. For simplicity, both the endowment and the amount of lump-sum taxes are age-independent; hence, individuals of all generations have the same human wealth. In addition, the endowment is assumed to be constant over time.

The representative consumer of generation \( s \) chooses a sequence for consumption and money balances in order to maximize (1) subject to (2) and (3) for an initial level of wealth. Solving the dynamic optimization problem and aggregating the results across cohorts yield the following expressions for the time path of aggregate consumption, the portfolio balance condition, the aggregate flow budget constraint of the households and the the transversality condition, respectively:

\[ C_t = (i^* - \beta)C_t - \delta \beta \frac{W_t}{1 + \eta P_t}, \quad (4) \]

\[ \frac{M_t}{P_t} = \eta \frac{C_t}{i^* + \dot{s}_t}, \quad (5) \]

\[ \frac{d(W_t/P_t)}{dt} = (i^* - \dot{p}^*) \frac{W_t}{P_t} + Y - T_t - C_t - (i^* + \dot{s}_t) \frac{M_t}{P_t}, \quad (6) \]

\[ \lim_{t \to \infty} \frac{W_t}{P_t} e^{-\int_0^t (i^* - \dot{p}_s) \, ds} = 0, \quad (7) \]

where, \( \eta \equiv (1 - \xi) / \xi \). The upper case letter denotes the population aggregate of the generic individual economic variable. For any generic economic variable at individual level, say \( x \), the corresponding population aggregate \( X \) can be obtained as \( X_t = \int_{-\infty}^t \delta x_{s,t} e^{\delta(s-t)} \, ds \).

\[ ^6 \text{This assumption ensures that savings are decreasing in wealth and that a steady-state value of aggregate consumption exists. See Blanchard (1985) for details.} \]
Letting $B$ denote traded bonds denominated in foreign currency, total financial wealth of the private sector can be expressed as:

$$\frac{W_t}{P_t} = \frac{M_t}{P_t} + \frac{S_t B_t}{P_t} = \frac{M_t}{P_t} + \frac{B_t P^*}{P_t}. \quad (8)$$

where we have used the PPP condition.

The public sector is viewed as a composite entity consisting of a government and a central bank. Let $D$ denote the net stock of government debt in terms of foreign currency given by the stock of foreign-currency denominated government bonds ($D_G$) net of official foreign reserves ($R$), that is $D = D_G - R$. Under the assumption that government bonds and foreign reserves yield the same interest rate, the public sector flow budget constraint can be expressed as:

$$\frac{d(S_t D_t / P_t)}{dt} = G_t + (i^* - \dot{p}^*_t) \frac{S_t D_t}{P_t} - \mu_t \frac{M_t}{P_t} - T_t, \quad (9)$$

where $G$ is public spending, $\mu$ is the growth rate of nominal money and $M \equiv M^H + SR$ with $M^H$ being the domestic component of the money supply (domestic credit) and $SR$ is the stock of official reserves valued in home currency. Henceforth public spending is assumed to be time-invariant.

Subtracting (9) from (6) yields the current account balance:

$$\frac{d(S_t F_t / P_t)}{dt} = (i^* - \dot{p}^*_t) \frac{S_t F_t}{P_t} + Y - C_t - G_t, \quad (10)$$

where $F = B - D$ is the net stock of the external assets of the domestic economy. Since $D = D_G - R$ the net stock of external assets can also be expressed as $F = B + R - D_G$.

### 2.1 Fiscal and Monetary Regimes

In order to close the model we need to specify the fiscal and the monetary regimes. The government is assumed to adopt a tax rule of the form:

$$T_t = \alpha D_t - Z_t - \mu_t \frac{M_t}{P_t}, \quad (11)$$
where \( Z \) is a transfer and \( \alpha > i^* - \dot{p}^* \). Taxes are an increasing function of net government debt adjusted for seigniorage.\(^7\) The parameter restriction on \( \alpha \) rules out any explosive paths for the public debt.

The monetary regime is described by a fixed exchange-rate system. The central bank pegs on each date \( t \) the exchange rate at the constant level \( \tilde{S} \), standing ready to accommodate any change in money demand in order to keep the relative price of the currency fixed. In other words, money supply is endogenously determined so as to meet any change in the demand of domestic money according to equation (5) for a given \( \tilde{S} \). Under a permanently fixed exchange rate, \( \dot{s} = 0, \dot{i} = i^* \) and the domestic price level is \( P = \tilde{S}P^* \). The foreign price \( P^* \) is assumed constant and normalized to one. It follows that \( P = \tilde{S} \) and that \( \dot{p}^* = 0 \).

### 2.2 Macroeconomic Equilibrium

By combining (4)-(5) with (8)-(11), under the fixed exchange-rate regime, the macroeconomic equilibrium of the model is described by the following set of equations:

\[
\begin{align*}
\dot{C}_t & = (i^* - \beta)C_t - \delta \frac{\beta + \delta}{1 + \eta} \left( \eta \frac{C_t}{i^*} + F_t + D_t \right), \\
\dot{D}_t & = (i^* - \alpha)D_t + Z + G, \\
\dot{F}_t & = i^*F_t + Y - C_t - G, \\
M_t & = \eta \frac{C_t}{i^*} \tilde{S},
\end{align*}
\]

(12)  
(13)  
(14)  
(15)

given the initial conditions on net public debt, on official foreign reserves, on the stock of foreign assets and on nominal money balances: \( D_0 = 0, R_0, F_0 \) and \( M_0 \).

The dynamic system described by equations (12)-(14) consists of one jump variable \( C \) and two predetermined or sluggish variables \( D \) and \( F \). The system must have one positive and two negative roots in order to generate a

\(^7\) Similar fiscal rules are frequently adopted in the literature. See Benhabib, Schmitt-Grohé and Uribe (2001) among others.
unique stable saddle-point equilibrium path. It is shown that this condition is always satisfied in the Appendix.

3 Fiscal Deficits and Currency Crises

In this Section we examine the dynamic effects of fiscal policy on the macrovariables of the model to derive the links between expected future budget deficits and currency crises in a pegged exchange-rate economy. The policy is centered on an unanticipated lump-sum tax cut, that is a once and for all increase in \( Z \). There is a fiscal deficit at time \( t = 0 \), generated by the tax cut, followed by future surpluses as debt accumulates, so as to always satisfy the intertemporal government budget constraint without recourse to seigniorage revenues. For sake of simplicity it is assumed that up to time zero the economy has been in steady-state.

Similarly to Burnside, Eichenbaum, and Rebelo (2001) the maintenance of the peg relies on a threshold rule. In particular, it is assumed that the monetary authorities abandon the fixed exchange-rate regime when net foreign assets \( F \) reach a critical level \( F^c \). The flexible exchange-rate regime that follows the fixed-rate regime’s collapse is permanent. Furthermore, it is assumed that changes in the demand for domestic currency are entirely met by variations in the domestic credit component of money supply, while current account imbalances are backed by changes in the level of internationally traded bonds. These assumptions together ensure that till the timing of the speculative attack the level of official reserves remains at its initial level \( R_0 \).

Since the analysis is based on the assumption of perfect foresight, the transitional dynamics of the economy depend on the expectations of the long-run steady-state relationships.

The steady state equilibrium is described by the set of equations (12)-(14) when \( \dot{C} = \dot{D} = \dot{F} = 0 \) and the portfolio balance condition (15). Let \( \overline{C}, \overline{D}, \overline{F} \).

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8The effects of government deficit in optimizing models can be found, for example, in Frenkel and Razin (1987), Obstfeld (1989), Turnovsky and Sen (1991).
\( \bar{F} \) and \( \Phi \) denote the long-run levels of consumption, public debt, net foreign assets and real money balances, respectively. The Appendix shows that the long-run effects of a tax cut are described by the following set of derivatives:

\[
\frac{dC}{dZ} < 0, \quad \frac{dD}{dZ} > 0, \quad \frac{dF}{dZ} < 0, \quad \frac{d\Phi}{dZ} < 0. \tag{16}
\]

From the above relationships we can see that a tax cut implies that in the new steady-state equilibrium consumption, real money balances and foreign assets are below their original levels. Under the assumption that \( \bar{F} < F^c \) the declining stock of net foreign assets, that follows the fiscal expansion, will force the central bank to stop defending the current fixed exchange rate at some point in time.\(^9\) Rational agents will anticipate the collapse of the system. Since the seminal papers of Salant and Henderson (1978) and Flood and Garber (1984) it is well-known that, in order to avoid losses arising from a sudden depreciation of the currency, speculators will force a currency crisis before the critical level of foreign assets is reached, driving the level of official foreign reserves to zero. At the time of the attack there will be a jump increase in net government debt and a decline in both net foreign asset and money supply.

3.1 Solution Procedure and the Time of the Speculative Attack

In order to analyze the adjustment of the economy to the initial tax cut, when the public anticipate the collapse of the exchange-rate regime at some point in the future, we need to proceed backward in time. In particular, we first solve the model under the floating exchange-rate regime and find the exact time of the attack, say \( t^* \). We can then use the results to solve the model under the fixed exchange-rate regime, that is, for \( 0 < t < t^* \). Notice that perfect foresight requires that all jump variables be continuous at \( t = t^* \).

\(^9\)Strong empirical support for a positive relationship between the current account deficit and current and expected future budget deficits is found Piersanti (2000). See also Baxter (1995) for a more general discussion on this issue.
Consider first the model under the floating regime, which applies for $t > t^\ast$.

The relevant equations of the system are (13)-(14) together with:

\[ C_t = (i^* - \beta)C_t - \delta \frac{\beta + \delta}{1 + \eta} (\Phi_t + F_t + D_t), \]

\[ \Phi_t = i\Phi_t - \eta C_t, \]

where $\Phi = \frac{M}{S}$. Notice that the system of equations describing the economy under the peg (12)-(14) and under the float (13)-(14), (17) and (18) have the same steady-state solution, that is $C, D, F$ and $\Phi$.

As shown in the Appendix the system of differential equations (13)-(14), (17) and (18) is saddle path stable. The solution can be easily obtained by using the initial conditions on the predetermined variables, $D, F$ and $M$, under a zero level of official reserves. In this way it is possible to compute the time path for the floating exchange rate before and after the speculative attack. According to the Salant-Henderson’s criterion the currency crisis will occur at the point where the shadow exchange rate (i.e. the exchange rate that would prevail in the economy after the fiscal expansion under a flexible regime if the official reserves had fallen to zero) is equal to the prevailing fixed rate, that is $S_t^\ast = \tilde{S}$.

The time path of the shadow floating rate is given by (see the Appendix):

\[ S_t = \frac{M_0^+}{\Phi + \eta v (D_0^+ - D) (e^{\lambda_1 t} - e^{\lambda_2 t}) + \eta (F_0^+ - F) e^{\lambda_2 t}}, \]

where $\lambda_1 = i^* - \alpha, \lambda_2 = i^* - \beta - \delta, M_0^+ = M_0 - R_0, D_0^+ = D_0 + R_0, F_0^+ = F_0 - R_0$ and $v \equiv \frac{-\delta (\beta + \delta)}{(1 + \eta)([\beta - \alpha] \alpha + \delta (\beta + \delta))}$. Using the above result and imposing the condition $S_t^\ast = \tilde{S}$ one can compute the time of the speculative attack.

Proceeding backward in time, it is now possible to describe the time path for the economy under the fixed regime immediately after the fiscal expansion by solving the system of differential equations (12)-(14), given the initial conditions on the predetermined variables and by imposing the condition associated with the assumption of perfect foresight which requires that consumption be
continuous at the time of the attack \( t^* \). The analytical solutions are described in detail in the Appendix.

### 3.2 Numerical Example

The adjustment process can be better described by making use of a simple numerical example.\(^{10}\)

Consider Figures 1-4 and assume, for example, an unanticipated tax cut at \( t = 0 \). Figures 1 and 2 plot the current time paths for consumption and the nominal exchange rate (bold lines) and for their related “shadow” levels before the attack occurring at time \( t^* \). There is, on impact, an increase in consumption while the shadow exchange rate, after the initial appreciation, starts depreciating steadily. Current generations profit from the lump-sum tax cut, since they share the burden of future increases in taxation with yet unborn individuals.

The dynamics of net foreign assets is depicted in Figure 3. Following the tax cut, the economy starts reducing its holding of foreign assets to finance the higher consumption level along the transitional path. At time \( t^* \) there is an abrupt decline in net foreign assets as a consequence of the speculative attack and the depletion of official reserves. The peg is abandoned and the economy shifts to a flexible exchange-rate regime, where the money supply becomes exogenous, foreign assets decline towards the new long-run equilibrium, and the nominal exchange rate depreciates until the current account is brought back in equilibrium. Figure 4 plots the time path for net government debt which increases gradually, at the time of the attack jumps upward because of the sudden exhaustion of official reserves, and then continues increasing converging to a new long-run equilibrium above its starting level.

Under the baseline calibration, Figures 5-8 show how the time of the attack \( t^* \) is affected by the initial level of reserves \( R_0 \), the amount of the tax cut

\(^{10}\)Figures 1-4 illustrate a numerical example based on the following parametrization: \( \delta^* = 0.03, \eta = 0.25, \delta = 0.02, \alpha = 0.04, Y = 1, G = D_0 = 0, \tilde{S} = 1, F_0 = 0.5 \) and \( R_0 = 0.2 \). The rate of time preference, \( \beta = 0.024 \), and the initial stock of nominal money balances, \( M_0 = 8.46 \), are implied. The fiscal expansion consists in an increase in \( Z \) of 0.05 from zero.
Z, the responsiveness of taxes to public debt \( \alpha \) and the liquidity preference parameter \( \eta \), respectively. The time of the attack depends positively on the level of official foreign reserves and on the responsiveness of taxes to public debt \( \alpha \), but negatively on the magnitude of the fiscal expansion and on the liquidity preference parameter.

Notice that in this model a crisis may occur even when the fiscal budget is showing a surplus.\(^{11}\) This is because along the transitional path a sequence of fiscal surpluses will replace the initial sequence of deficits, in order to satisfy the government intertemporal budget constraint.

### 4 Conclusion

In this paper we have used an optimizing general equilibrium model with overlapping generations to investigate the relation between fiscal deficits and currency crises. It is shown that a rise in current and expected future budget deficits generates a depletion of foreign reserves, leading up to a currency crisis.

Crises can thus occur even when policies are ‘correctly’ designed, that is a monetary policy fully committed to maintain the peg and the fiscal authorities respecting the intertemporal government budget constraint. The sustainability of fixed exchange-rate systems may thus require not only giving up monetary sovereignty but also imposing a more severe degree of fiscal discipline than implied by the standard solvency conditions.

\(^{11}\)This result is consistent with the evidence that in most Asian countries, during the years preceding the crisis, fiscal imbalances were either in surplus or in modest deficit. See World Bank(1999).
References


Appendix

Long-Run Steady-State

The steady-state equilibrium is described by the set of equations (12)-(14), together with the portfolio balance condition (15) or analogously by (13), (14), (17), (18) for $\dot{C} = \dot{D} = \dot{F} = \dot{\Phi} = 0$:

\[
(i^* - \beta)C - \delta \frac{\delta + \beta}{1 + \eta} (\Phi + F + D) = 0, \quad (A1)
\]

\[
G + (i^* - \alpha)D + Z = 0, \quad (A2)
\]

\[
i^* \Phi - \eta C = 0, \quad (A3)
\]

\[
i^* F + Y - C - G = 0. \quad (A4)
\]

Totally differentiating the above system of equations yields

\[
\begin{pmatrix}
dC \\
dD \\
d\Phi \\
dF
\end{pmatrix} = \begin{pmatrix}
0 \\
-1 \\
0 \\
0
\end{pmatrix} dZ, \quad (A5)
\]

where

\[
J \equiv \begin{pmatrix}
i^* - \beta & -\delta \frac{\delta + \beta}{1 + \eta} & -\delta \frac{\delta + \beta}{1 + \eta} & -\delta \frac{\delta + \beta}{1 + \eta} \\
0 & i^* - \alpha & 0 & 0 \\
-\eta & 0 & i^* & 0 \\
-1 & 0 & 0 & i^*
\end{pmatrix}
\]

The long-run effects of an increase in $Z$ are:

\[
\frac{dC}{dZ} = -i^* \frac{\delta (\beta + \delta) i^*}{(1 + \eta) \Delta} < 0, \quad (A6)
\]

\[
\frac{dD}{dZ} = \frac{1}{\alpha - i^*} > 0, \quad (A7)
\]

\[
\frac{d\Phi}{dZ} = -\eta \frac{\delta (\beta + \delta) i^*}{(1 + \eta) \Delta} < 0, \quad (A8)
\]
\[
\frac{dF}{dZ} = -\frac{\delta(\beta + \delta)i^*}{(1 + \eta)\Delta} < 0, \quad (A9)
\]
where \(\Delta \equiv (\alpha - i^*)(\beta + \delta - i^*)(i^* + \delta)i^* > 0\).

The Model under the Float

The characteristic equation of the Jacobian of the system of differential equations (13), (14), (17), (18) describing the economy under the float is

\[
(i^* - \alpha - \lambda)(i^* - \lambda)(i^* - \beta - \lambda) - \delta(\beta + \delta) = 0, \quad (A10)
\]
where the eigenvalues are \(\lambda_1 = i^* - \alpha < 0\), \(\lambda_2 = i^* - \beta - \delta < 0\), \(\lambda_3 = i^* > 0\) and \(\lambda_4 = i^* + \delta > 0\). Under the float the economy displays two jump variables \((C \text{ and } \Phi)\) and two predetermined variables \((D \text{ and } F)\). The signs of the eigenvalues ensure that the model is saddle path stable.

The saddle path solution for the system (13), (14), (17), (18) is:

\[
C_t = \overline{C} + v(D_0^+ - \overline{D}) [\alpha e^{\lambda_1 t} - (\beta + \delta) e^{\lambda_2 t}] + (\beta + \delta)(F_0^+ - \overline{F}) e^{\lambda_2 t}, \quad (A11)
\]
\[
D_t = \overline{D} + (D_0^+ - \overline{D}) e^{\lambda_1 t}, \quad (A12)
\]
\[
\Phi_t = \overline{\Phi} + \eta v(D_0^+ - \overline{D})(e^{\lambda_1 t} - e^{\lambda_2 t}) + \eta(F_0^+ - \overline{F}) e^{\lambda_2 t}, \quad (A13)
\]
\[
F_t = \overline{F} + v(D_0^+ - \overline{D})(e^{\lambda_1 t} - e^{\lambda_2 t}) + (F_0^+ - \overline{F}) e^{\lambda_2 t}, \quad (A14)
\]
where \(v \equiv -\frac{\delta(\beta + \delta)}{(1 + \eta)(\beta - \alpha)[\alpha + \delta(\beta + \delta)]}\), \(D_0^+ = D_0 + R_0\) and \(F_0^+ = F_0 - R_0\).

The above solution describes the behavior of the “shadow” economy after the fiscal expansion until the time of the attack \(t^*\) and the behavior of the “real” economy from time \(t^*\) onward.
The Timing of the Attack

Using (A13) and recalling that $\Phi = \frac{M}{S}$ the time path of the “shadow” exchange rate prevailing in the economy after the fiscal expansion is

$$S_t = \frac{M_0^+}{\Phi + \eta v (D_0^+ - D) (e^{\lambda_1 t} - e^{\lambda_2 t}) + \eta (F_0^+ - F) e^{\lambda_2 t}}.$$  \hfill (A16)

where $M_0^+ = M_0 - R_0$.

The speculative attack will occur at time $t^*$ when the shadow exchange rate is equal to the pegged value $\tilde{S}$:

$$S_{t^*} = \frac{M_0^+}{\Phi + \eta v (D_0^+ - D) (e^{\lambda_1 t^*} - e^{\lambda_2 t^*}) + \eta (F_0^+ - F) e^{\lambda_2 t^*}} = \tilde{S}.$$  \hfill (A17)

The Model under the Peg

Consider the Jacobian of system of differential equations describing the economy under the peg (12)-(14):

$$\Lambda = \begin{pmatrix}
  i^* - \beta - \delta \frac{\delta + \beta}{1 + \eta} & -\delta \frac{\delta + \beta}{1 + \eta} & -\delta \frac{\delta + \beta}{1 + \eta} \\
  0 & i^* - \alpha & 0 \\
  -1 & 0 & i^*
\end{pmatrix},$$  \hfill (A18)

Let $\mu_1$, $\mu_2$ and $\mu_3$ denote the eigenvalues of the Jacobian. It follows that:

$$\mu_1\mu_2\mu_3 = \det \Lambda = [\delta (\delta + \beta) - i^* (i^* - \beta)] (\alpha - i^*) > 0,$$  \hfill (A19)

Since the equation of motion of net public debt is recursive one root, say $\mu_1$, is equal to $i^* - \alpha < 0$. From (A19) the remaining eigenvalues have opposite sign and are such that:

$$\mu_2\mu_3 = - [\delta (\delta + \beta) - i^* (i^* - \beta)] < 0.$$  \hfill (A20)

Let $\mu_2 < 0$ and $\mu_3 > 0$.

The general solution for the system of linear differential equations (12-14)
is:

\[ C_t = \overline{C} + H_1 \alpha u e^{(i^*-\alpha)t} + H_2 (i^* - \mu_2) e^{\mu_2 t} + H_3 (i^* - \mu_3) e^{\mu_3 t}, \quad (A21) \]

\[ D_t = \overline{D} + H_1 e^{(i^*-\alpha)t}, \quad (A22) \]

\[ F_t = \overline{F} + H_1 u e^{(i^*-\alpha)t} + H_2 e^{\mu_2 t} + H_3 e^{\mu_3 t}, \quad (A23) \]

where \( u \equiv -\frac{\delta^2 + \delta \bar{\delta} + \delta}{\alpha (\beta - \alpha + \delta + \delta \bar{\delta} + \delta \bar{\delta})} \) and \( H_1, H_2 \) and \( H_3 \) are constants to be determined given the initial conditions on the predetermined variables, \( D \) and \( F \), and the continuity condition on the forward looking variable \( C \) at the time of the attack \( t^* \).

**Determination of \( H_1 \)**

From (A22) computed at time \( t = 0 \):

\[ D_0 = \overline{D} + H_1 = 0 \rightarrow H_1 = D_0 - \overline{D}. \quad (A24) \]

Substituting the above result into (A21)-(A23) yields:

\[ C_t = \overline{C} + \alpha u (D_0 - \overline{D}) e^{(i^*-\alpha)t} + H_2 (i^* - \mu_2) e^{\mu_2 t} + H_3 (i^* - \mu_3) e^{\mu_3 t}, \quad (A25) \]

\[ D_t = \overline{D} + (D_0 - \overline{D}) e^{(i^*-\alpha)t}, \quad (A26) \]

\[ F_t = \overline{F} + u (D_0 - \overline{D}) e^{(i^*-\alpha)t} + H_2 e^{\mu_2 t} + H_3 e^{\mu_3 t}. \quad (A27) \]

**Determination of \( H_2 \) and \( H_3 \)**

At time \( t = 0 \) the RHS of (A27) must be equal to the given initial condition on \( F \):

\[ F_0 = \overline{F} + u (D_0 - \overline{D}) + H_2 + H_3. \quad (A28) \]
At $t = t^*$ the RHS of (A25) must be equal to the RHS of (A11) in order to rule out any discrete jumps of consumption when the fixed regime is abandoned:

$$
\alpha u \left( D_0 - \overline{D} \right) e^{(i^* - \alpha)t^*} + H_2 (i^* - \mu_2) e^{\mu_2 t^*} + H_3 (i^* - \mu_3) e^{\mu_3 t^*} = v \left( D_0^+ - \overline{D} \right) \left[ \alpha e^{\lambda_1 t^*} - (\beta + \delta) e^{\lambda_2 t^*} \right] + (\beta + \delta) \left( F_0 - \overline{F} \right) e^{\lambda_2 t^*}.
$$

The constants $H_2$ and $H_3$ can be easily obtained by solving the system of equations (A28)-(A29).
Figure 1: Exchange Rate Dynamics, $S$

Figure 2: Consumption Dynamics, $C$

Figure 3: Net Foreign Assets Dynamics, $F$

Figure 4: Net Government Debt Dynamics, $D$