Fiscal Deficits and Currency Crises

by

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Abstract

This paper investigates currency and financial crises in an optimizing general
equilibrium model. It is shown that a rise in current and expected future budget
deficits generates a real exchange rate appreciation and a decumulation of external
assets, leading up to a currency crisis when foreign reserves approximate a critical
level. Strong empirical support for our model is obtained by a probit estimation for
Latin American and Asian countries.

JEL classification: F31; F32; F41; E52; E62

Key words: budget deficits, foreign exchange reserves, currency crises

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I. Introduction

The events of the 90’s have cast serious doubts on the validity of standard models of
currency crises. The economic conditions in the Asian and Latin American countries
did not appear to show the kind of macroeconomic and financial distress that typically is at the core of the traditional models of balance of payments crises. Alternative explanations have been provided by a new literature emphasizing moral hazard problems and financial panics. The moral hazard based models stress the role played by the government bailout promise in determining excessive risk taking by financial intermediaries. The channels through which this could have operated in emerging economies are poor banking regulations and the so called “carry trade”, by which banks borrow in international markets at low interest rates and lend at higher rates at home (OECD, 1999, pp. 177-83). This resulted in a lending boom fuelled by large capital inflows, thereby generating overinvestment in risky projects and asset price bubbles. The bubble grew up until an adverse shock burst it, revealing the fragility of the banking system and generating a financial and currency crisis\(^1\).

Proponents of this view argue that if the bailout promise is at the core of moral hazard problems, then the only policy cure is to abolish the lender of last resort facilities.

The problem with the moral hazard view, as pointed out in Radelet and Sachs (1998, pp. 35-42), is that the data in the early 1990s did not show a dramatic deterioration in either loan quality or investment riskiness for the crisis countries. Furthermore, spreads on Asian bonds fell between 1995 and 1997, and ratings of long term government bonds by Moody’s, Standard and Poor’s, and Euromoney remained unchanged until the onset of the crisis, revealing that foreign lenders did not perceive an increase in risk. Finally, no warning of an asset price bubble was present in the reports of the investment houses, showing that expectations of a financial crash and a subsequent bailout were absent\(^2\).

The financial panic based models, on the other hand, stress self-fulfilling prophecies and herding behavior as the determinants of a crisis. According to this view, the crisis may be triggered by either rumors or fundamentals resulting in a massive withdrawal by investors attempting to avoid capital losses. This is rationalized by using multiple rational expectations equilibrium models where the financial panic represents a self-fulfilling bad equilibrium leading to the collapse, along the lines sketched by Diamond and Dybvig (1983) in the context of banking institutions. Crises are thus unavoidable and can occur even when countries show sound or non-deteriorating fundamentals\(^3\).

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1 For an interpretation of financial crises in Southeast Asia along these lines see Calvo, et al. (1994), McKinnon and Pill (1996), Dooley (1997), Corsetti et al. (1998, 1999), Krugman (1998), and Chinn et al. (1999), among others.

2 See also BIS (1997), IMF (1998). For a contrary view on the last point see, however, Sarno and Taylor (1999).

The key factor behind the sudden shifts in expectations is “the excess volatility” in international financial markets. Evidence may be found in the large and, to some extent, unanticipated swings of capital flows that played a critical role in pushing the emerging market economies into crises. The sharp reversal of capital flows from Latin American and Asian countries, respectively in 1994 and 1997, was the start of the currency and banking crises through herding behavior and contagion effects (e.g. Radelet and Sachs, 1998; Kaminsky and Schmukler, 1999). Proponents of this view argue that there is a strong rationale for an international lender of last resort, so that the crisis could be stopped and not be allowed to spread.

The problem with the financial panic view is that the data in the 1990s in many emerging countries do show macroeconomic imbalances, and this must have played some role in the subsequent crises (see, for example, OECD, 1999, pp. 183-91; Corsetti et al., 1998).

However, no satisfactory model based on fundamentals has, to our knowledge, been presented in the literature. Notable exceptions are Burnside, Eichenbaum and Rebelo (1998, 2000) who explain currency crises as the consequence of expected future budget deficits brought about by the implicit bailout promise to failing banking systems. Yet, in their approach self-fulfilling beliefs play the crucial role of determining the timing of the attack, so that the model may also be included within a multiple equilibrium framework.

In this paper we present, on the other hand, a model of currency crises entirely based on fundamentals, where current account deficits brought about by current and prospective fiscal deficits push foreign reserves to a critical level, where the attack starts.

The paper is organized as follows. Section II presents the theoretical model. Section III presents the empirical results. Section IV contains the summary and conclusions of the paper.

II. The Optimizing Model

The semi-small open economy macromodel is composed of households, firms, and the government. There is no bequest motive, population is constant and lifetimes are uncertain as in Yaari (1965) and Blanchard (1985). Agents maximize the discounted value of an expected utility function subject to the appropriate budget constraint. The utility function is logarithmic in aggregate consumption and real money balances and

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4 Other models where both fundamentals and self-fulfilling beliefs play an important role may be found in Cole and Kehoe (1996), Jeanne (1997) and Masson (1999).

5 The approach of entering money in the utility function to allow for money holding behavior within a Yaari-Blanchard framework, is, by now, common to a number of papers including Spaventa (1987), Marini and van der Ploeg (1988), van der Ploeg (1991), Daniel (1993), and Kawai and Maccini (1990, 1995).
labor supply is inelastic. Individuals ensure that the marginal rate of substitution between consumption and real money balances equals the opportunity cost of holding real money balances (i.e. the nominal interest rate). Non-human wealth is the sum of government debt, capital stock, net foreign assets and real money balances.

In this economy, assumed to be operating under a fixed exchange rate regime, agents consume two physical goods, which we denote as \( C^H \) (domestically produced good) and \( C^F \) (foreign or imported good), so that total real consumption, \( C \), can be written as \( C = C^H + \rho C^F = \rho C + (1-q)C \), where \( \rho = \frac{e_p^*}{p} \) is the relative price of foreign goods in terms of domestic goods, or real exchange rate, \( e \) is the nominal fixed exchange rate, \( p^* \) and \( p \) the foreign and domestic price of the consumption goods, respectively, and \( q \) and \( 1-q \) the proportions of domestic and foreign goods over total consumption. Since this is a semi-small open economy, the price of import goods and foreign assets are exogenous but the price of the export good is domestically set.

On the production side, we assume that domestic output is produced by a two factor neoclassical production function with constant return to scale, which can be written as \( Y = Y(K) \) normalizing labor input to unity, where \( Y \) is domestic output and \( K \) the stock of capital. Output equals the sum of private and government consumption, net exports and investment.

For simplicity, we assume that capital and government bonds are owned entirely by domestic residents. External bonds pay an exogenously given world real interest rate, \( r^* \). International capital markets are perfect and uncovered interest parity holds at all times. The monetary authorities adopt policies to sustain the fixed exchange rate regime and finance current account imbalances through changes in foreign reserves. Trend monetary growth is set to zero for analytical convenience.

We can write the aggregate relationships as follows:

\[
\begin{align*}
1. \quad & C = \frac{\delta + \beta}{1 + \eta} \left[ \alpha(K) + K + \rho F + \rho m \right] + \rho d \\
2. \quad & \dot{m} = \frac{1}{\rho} \left[ r^* \rho m - \eta C \right] \\
3. \quad & K = Y(K) - (qC + C^H) - X(p) \\
4. \quad & \dot{F} = \frac{1}{\rho} \left( X(p) - \left[ (1-q)C + \rho C^F \right] + r^* \rho F \right) \\
5. \quad & \dot{\rho} = [Y(K) - r^*] \rho \\
6. \quad & \dot{d} = (r^* - \alpha) - \frac{\dot{\rho}}{\rho} + \left[ \frac{\delta (\delta + \beta)}{\rho (r^* + \delta)(1 + \eta)} \right] Z.
\end{align*}
\]

Similar results should also be obtained, under certain conditions, by use of cash-in advance or liquidity cost models (see Feenstra 1986).
where $\omega$ denotes real labor income, $F$ is the stock of net external assets, $M$ indicates the nominal quantity of money, $m = \frac{M}{p}$ is real money balances, $x$ stands for real exports, $G = G^H + \rho G^F$ represents total government spending, $d$ is an index of fiscal policy that summarize the effects on aggregate demand of the entire sequence of current and anticipated future budget deficits, as described in the Mathematical Appendix, $Z$ represents an exogenous policy variable, $\alpha$ symbolizes a fiscal policy parameter, $\beta$ denotes the subjective discount factor and $\delta$ is the constant instantaneous probability of death. The effective discount factor thus is $(\delta + \beta)$ and $\delta^{-1}$ is the expected lifetime of agents.

The aggregate consumption function (1) is a linear function of total wealth. Equation (2) describes the time evolution of real money balances driven by the Yaari-Blanchard consumption dynamics. Equations (3)-(5) show the dynamic evolution of capital, foreign assets and the rate of depreciation of the real exchange rate, respectively. Equation (6) expresses the dynamics of fiscal policy. This is centered on a lump-sum tax cut, while government spending is set equal to zero on the entire path, so that fiscal policy can have effects only through consumption. Since taxes are modeled as an increasing function of debt, through the $\alpha$ parameter (see equation (B.I.9) in the Mathematical Appendix), the policy considered here is a fiscal deficit at $t = t_0$, generated by a tax cut, followed by future surpluses as debt accumulates, so as to satisfy the intertemporal government budget constraint. This implies $\alpha \geq r^*.$

We examine the dynamic effects of a lump sum tax cut policy on the macrovariables of the model to bring to light the linkages between expected future budget deficits and currency crises in a pegged exchange rate economy. The monetary authorities accommodate any change in money demand in order to keep the relative price of the currency fixed to $\varepsilon$.

A unique stable saddle-point equilibrium path characterizes the model if $\beta \leq r^* < \delta + \beta$ and the transversality conditions are met. Solving the model for short run and steady state equilibrium, we obtain the following set of relationships among the variables of interest, as shown in detail in the Mathematical Appendix.

**Short-Run Equilibrium**

(7) \[ C = C(K, F, r^*, d) \quad C_K > 0, \ C_F > 0, C_{r^*} < 0, C_d > 0 \]

(8) \[ \rho = \rho(K, F, r^*, d) \quad \rho_K > 0, \ \rho_F < 0, \ \rho_{r^*} > 0, \ \rho_d < 0 \]

(9) \[ m = m(K, F, r^*, d) \quad m_K > 0, \ m_F > 0, m_{r^*} < 0, m_d > 0 \]

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6  The effects of government spending in optimizing models may be found, for example, in Frankel and Razin (1987), Obstfeld (1989), Turnovsky and Sen (1991).

7  This condition may also be found in other studies facing similar questions within a framework of forward-looking agents with finite horizons. See, for example, Blanchard (1985), Buiter (1987), Matsuyama (1987), Giovannini (1988), Kawai and Maccini (1995).
Steady-State Equilibrium

(10) \[ C = C(r^*, d) \quad \overline{C}_r^* > 0, \overline{C}_d < 0 \]

(11) \[ \rho = \rho(r^*, d) \quad \overline{\rho}_r^* < 0, \overline{\rho}_d > 0 \]

(12) \[ K = K(r^*, d) \quad \overline{K}_r^* < 0, \overline{K}_d = 0 \]

(13) \[ F = F(r^*, d) \quad \overline{F}_r^* > 0, \overline{F}_d < 0 \]

(14) \[ m = m(r^*, d) \quad \overline{m}_r^* > 0, \overline{m}_d < 0 \], where the upper bars indicate long-run effects.

From equations (7) through (14) we can see that an increase in \( d \) implies a rise in consumption and real money balances and an appreciation of the real exchange rate, in the short-run; in steady state equilibrium consumption, foreign assets and real money balances are however below their original levels. The capital stock is unchanged and there is a real exchange rate appreciation.

The dynamics of this economy can be determined by substituting the short-run solution for \( C, \rho \) and \( m \) into the dynamic equations of the model. The critical equation (4) can be rewritten as

\[
\overline{\rho} = \frac{X}{\rho(W, d, r^*)} \left[ (1-q)C(W, d, r^*) \right] + r^* F,
\]

where \( W = K + \rho F \).

Linearizing around the steady state, for given \( r^* \), we obtain

\[
\dot{F} = \Theta(d_0 - \overline{d})e^{\lambda t} + r^*(F - \overline{F}),
\]

where \( \lambda \) is the negative root associated with the stable arm of the saddle path,

\[
\Theta = \frac{1}{\rho \left[ \left( \nu d_{W} - \rho C_{W} \right) + \left( \nu d_{d} - \rho C_{d} \right) \right]} \oiint, \quad \rho_{W} = \rho_{K} + \rho_{F}, \quad C_{W} = \frac{(1-q)}{\rho} \left( C_{W} - \frac{C}{\rho} \right),
\]

\[
C_{d} = \frac{(1-q)}{\rho} \left( C_{d} - \frac{C}{\rho} \right), \quad \nu = \frac{X - \overline{X}}{\overline{\rho}} > 0 \quad \text{and} \quad \overline{b} = \frac{1 + \eta}{\alpha} \rho < 0 \quad \text{is a parameter linking} \ W \ \text{and} \ d \ \text{along the stable path (see the Mathematical Appendix)}. \]

The current stock of foreign assets is given by

\[
F_{t} = F + \frac{\Theta}{\lambda_{t} - r^*} \left( d_{0} - \overline{d} \right) e^{\lambda t} + \left[ F_{0} - \overline{F} - \frac{\Theta}{\lambda_{t} - r^*} \left( d_{0} - \overline{d} \right) \right] e^{\lambda t}.
\]

The dynamics towards the steady state is described by

\[
F_{t} = \overline{F} + \frac{\Theta}{\lambda_{t} - r^*} \left( d_{0} - \overline{d} \right) e^{\lambda t},
\]

or,

\[
F_{t} = \overline{F} + \frac{\Theta}{\lambda_{t} - r^*} \left( d_{0} - \overline{d} \right) e^{\lambda t}. \]

Equation (15) shows the relationship between the accumulation of foreign assets and the evolution of budget deficits along the path approaching the steady state equilibrium. There are both direct effects on the real exchange rate \( \nu \rho_d \) and
consumption \( (\rho C^F_d) \), and indirect effects via changes in real wealth \( (\nu \rho W - \rho C^F_w) \).

A rise in the budget deficit generates a depletion of external assets (or current account deficits) during the transition to the steady state if \( \Theta > 0 \).

The other dynamic equations of the model are:

\[
\begin{align*}
K_t &= \bar{K} - \frac{\Psi}{\lambda_1 - Y'(K)}(d_{t0} - \bar{d})e^{\lambda_1(t-t_0)}, \\
\rho_t &= \bar{\rho} - \frac{\Phi}{\lambda_1 - [Y'(k) - r*]}(d_{t0} - \bar{d})e^{\lambda_1(t-t_0)}, \\
m_t &= \bar{m} - \frac{\Pi}{\lambda_1 - r*}(d_{t0} - \bar{d})e^{\lambda_1(t-t_0)}, \\
C_t &= \bar{C} + a(W_{t0} - \bar{W})e^{\gamma(t-t_0)}, \\
W_t &= \bar{W} + b(d_{t0} - \bar{d})e^{\lambda_1(t-t_0)},
\end{align*}
\]

where \( \Psi = \left( qC_W + X'\rho_W b + qC_d + X'\rho_d \right) \), \( \Phi = \left( Y' (K) \rho \Psi / (\lambda_1 - r*) \right) \), \( \Pi = \frac{n}{\rho} \left[ C_W - \frac{C}{\rho} \rho_W b + \left( C_d - \frac{C}{\rho} \rho_d \right) \right] \), \( a = \frac{\delta + \beta}{1 + \eta} > 0 \).

Equations (16), (17) and (18) describe the transitional dynamics towards the long-run equilibrium for the capital stock, the real exchange rate and real money balances. Equations (19) and (20) express the dynamical relationship between consumption and wealth along the adjustment path, as shown in the Mathematical Appendix. During the transition to the steady state, the paths for \( K \) and \( \rho \) are positively sloped if \( \Psi, \Phi > 0 \), while that of \( m \) is negatively sloped if \( \Pi < 0 \).

The adjustment process can be better understood by making use of figs. 1(a) – 1(e), where \( d_{t0} \) and \( \bar{d} \) denote the initial and steady state value of the fiscal index, respectively. Assume, for example, a tax cut at \( t = t_0 \). This, on impact, yields an appreciation in the real exchange rate, an increase in consumption and real money balances and a deterioration in the current account. These effects are visualized in figs 1(a), 1(d) and 1(e) as jumps from the initial equilibrium points \( \bar{C}_0, \bar{\rho}_0 \) and \( \bar{m}_0 \) to \( C_{t0}, \rho_{t0} \) and \( m_{t0} \). The capital stock and foreign assets also move to \( K_{t0} \) and \( F_{t0} \), leaving their sum unchanged (see figs. 1(b) and 1(c)). The total amount of \( K \) and \( F \) is predetermined, but not its two components: agents can substitute foreign assets for capital stock instantaneously. Hence, the domestic real interest rate jumps upwards.

As the economy moves towards its new long-run equilibrium point \( (\bar{C}, \bar{F}, \bar{K}, \bar{\rho}, \bar{m}) \) and the government budget goes from deficit to surplus, non-human wealth, consumption and real money balances begin to decline, foreign assets are run

\[\text{8} \text{ Strong empirical support for a positive relationship between the current account} \]
\[\text{deficit and current and expected future budget deficits, as implied by equation} \]
\[\text{(15), is found in Piersanti (2000). See also Baxter (1995) for a more general} \]
\[\text{discussion on this issue.} \]
down, while the real exchange rate and the capital stock rise. Since the current account is in deficit during the period of adjustment, foreign assets end up to a lower level in the new steady state. The capital stock returns back to its original level, $K$, in the long-run, while foreign assets and human wealth decline. Total real wealth thus becomes lower and consumption and real money balances fall below their starting level. The real exchange rate overshoots and the domestic real interest rate is higher than the world rate during the transition to the steady state.

Currency crises occur along the stable path to the new equilibrium when foreign reserves decline below the threshold level, $F^c$, stirring up a speculative attack and the collapse of the peg. Substituting $F^c$ for $\overline{F}$ into equation (15), we can determine the time of the attack, $t^*$, when the government abandons the peg. Solving equation (15) for $t^*$, we find:

$$t^* = t_0 + \frac{1}{\lambda_1} \ln \left( \frac{\overline{F}_0 - F^c}{\omega(d_{t_0} - d)} \right),$$

where $\Omega = \frac{\Theta}{\lambda_1 - \rho^*}$.
The value of $t^*$ depends on both the level of foreign reserves and the magnitude of the deficit being financed. Given $d^*_h$, the larger the foreign reserves holdings $F$, the longer the fixed exchange rate regime will last. On the other hand, given $F$, the larger $d^*_h$ the smaller $t^*$ will be.

The dynamics of foreign reserves until the time of the attack, $t^*$, is depicted in Fig. 2. Following the tax cut, the economy starts (at $t^*_0$) reducing its holding of foreign assets (hence reserves fall) to finance the higher consumption level along the transitional path. If, during the adjusting process, reserves come to a threshold level ($F^*$), a speculative attack takes place, forcing the government to give up the peg. Thereafter the economy shifts to a flexible exchange rate regime and the money supply becomes exogenous.

The main result of our model of currency crises thus appeals to intuition and can be restated as follows. A rise in current and expected future budget deficits
generates both an appreciation of the real exchange rate and current account deficits. Hence, there is a decumulation of foreign reserves along the transitional path to the new steady state. If reserves approach a critical level in the adjusting process, a speculative attack occurs causing the collapse of the fixed exchange rate regime. The next section tests the predictions of the model.

III. The Empirical Analysis

In this section we test the power of our model in predicting financial and currency crises by using a simple probit model linking the onset of a crisis to the relevant macroeconomic variables of the theoretical model. Our empirical investigation is focused on all the Latin American and Asian countries for which we could find reliable data. The data are annual from 1990 through 2000. The countries are: Argentina, Brazil, Mexico, Venezuela, Chile, Colombia, Peru, Uruguay, Bolivia, Honduras, Indonesia, Korea, Malaysia, Philippines, Thailand, Turkey, Singapore, China (P. R. Mainland), India, Pakistan and Sri Lanka.

Our probit framework implies that the left-hand-side variable takes on a value of one if the country fell into a crisis during the year and zero otherwise. For this purpose, we define a crisis as a drastic depreciation of the currency (and/or the collapse of the peg) or a significant balance of payment disruption. Ten cases out of two hundred and thirty-one are set equal to one: Turkey and Venezuela in 1994; Argentina and Mexico in 1995; Indonesia, Korea, Malaysia, Philippines and Thailand in 1997; Brazil in 1999.

On the right-hand-side, as suggested by our model, we use the following variables as determinants of crises:

i) current and expected future government budget deficits or surpluses as a percentage of GDP ($EFGB$);

ii) current account balance as a percentage of GDP ($CA$);

iii) accumulated real exchange rate appreciation ($REXA$);

iv) total reserves as a percentage of imports ($RESR$), which we use as a proxy for the stock of reserves;

v) the domestic real interest rate ($RATE$); and

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10 This variable is an index that starts at 100 in 1990 and then reflects the accumulated real appreciation of the national currency. It may be found in Veiga (1999), who analyzes the causes of failure of stabilization plans in chronic inflation countries.
vi) domestic credit as a percentage of GDP (DCR), modeling the increase in the domestic component of the money stock along the transitional path to the steady state.

The specification used in the estimation of our probit model may thus be written as:

\[
P(CS_i) = \Phi(EFGB_t, CA_t, EXP, RESR_t, RRATE_t, DCR_t), \quad i = 0, 1, \ldots, n,
\]

where \(CS\) is the binary variable (Crisis) and the lagged values for the independent variables encompass the dynamics implied by equation (15).

According to our model, we expect that the probability of a crisis be negatively correlated to \(EFGB, CA, RESR\) and \(RRATE\), and positively linked to \(REXA\) and \(DCR\), so that increases in expected future budget deficits and domestic credit, reductions in the current account, foreign reserves and the domestic real interest rate, or a real exchange rate appreciation raise the probability that a crisis will eventually break up.

Equation (15) predicts that the budget and current account deficits should be causally linked. The hypothesis of no causality between the two deficits is strongly rejected by our tests reported in table 2, where it is shown that budget deficits do cause current account deficits. Preliminary tests investigating the stationarity of the time series employed show that we may accept the hypothesis of stationarity (see table 1).

The specification of equation (22) raises the important issue of modeling and generating data on future expectations of government budget balance. Our model developed in section I implies that we may define \(EFGB\) as:

\[
EFGB = \sum_{i=0}^{n} \phi^i GB_{t+i}^*,
\]

where \(\phi\) is the discounting factor, \(GB_{t+i}^*\) is the expected government budget balance as a percentage of GDP for the year \(t+i\), and \(n\) is the planning horizon of private agents. We have performed GMM estimations of \(EFGB\) under the hypothesis of rational expectations in order to generate data for future expected government budget balances to be used in the estimation of (22). We have considered values of \(\phi\) in the range \([0.9, 0.99]\) and \(n = 5\)\(^{11}\), using the Newey-West (1987a) consistent estimator of the variance-covariance matrix to deal with the presence of MA (4) in the errors. Table 3 reports only the estimates for \(\phi = 0.9\), since both coefficients and statistics were almost identical for alternative values of \(\phi\) within the chosen range.

We have generated data for \(EFGB\) that were used as a proxy for market’s expectations of current and future government budget deficit in the estimation of the probit model formulated in (22), by carrying out the static forecast of the econometric equation described in Table 3.

\(^{11}\) This value for the planning horizon of agents, originally suggested by Feldstein (1986), emerged from robustness checks.
Probit estimates are presented in table 4. We also report the marginal effects of the explanatory variables on the conditional probability of a crisis evaluated at the mean of the data.

We performed all the model estimates with country dummies in order to control for fixed effects.

It can immediately be seen from table 4 that our theoretical model fits the Asian and Latin American crises extremely well. The most striking result is the high statistical significance of the key variables $EFGB$ and $CA$, suggesting that expected future budget deficits and current account deficits do play a critical role in determining crises.

Since the traditional reserves to imports ratio is not regarded as the best measure of the reserves adequacy, we also tried other indicators suggested in the literature, such as the M1 to reserves ratio and the M2 to reserves ratio$^{12}$. We found no statistical significance for these variables. Tests for the presence of interactive effects among the independent variables were equally negative.

In conclusion, the estimates give strong empirical support to the main prediction of our theoretical model, according to which current and expected future budget deficits, current account deficits, foreign reserves, real exchange rate appreciation, domestic credit and domestic real interest rate are the key variables in predicting the onset of currency crises.

We have assessed the power of the above probit model in predicting the likelihood of a crisis. We forecasted the in-sample probability of a crisis for each country and appraised the resulting probability values for the cutoff levels of 0.5 and 0.25. The results of the goodness of fit estimation are reported in table 5.

We also evaluated the in-sample forecasts by three measures of accuracy known as quadratic probability score (QPS), log probability score (LPS) and global squared bias (GSB). Both the QPS and GSB range from 0 to 2, with zero corresponding to perfect accuracy and perfect global calibration, while LPS ranges from 0 to infinity, with zero corresponding to perfect accuracy.

We can see that our model shows both excellent scores and accurate goodness of fit measures from table 5. It correctly calls more than 99% of total observations both at the 0.5 and 0.25 cutoff levels. Nine out of ten country crises are correctly predicted with probability values falling in the range $(0.71, 0.99)$.$^{13}$

Based on this evidence, we may then conclude that the main cause of the financial and currency turmoil of 1990’s in Latin America and Asia has been prospective budget deficits. The empirical results, obtained from estimating and forecasting a probit-based model give strong support to the main implication of our

$^{12}$ These measures of reserves adequacy have been suggested by Krugman (1979) and Calvo and Mendoza (1996a, b).

$^{13}$ A probability below the cutoff levels was found only for Malaysia.
theoretical model, according to which a rise in current and expected future budget deficits generates a real exchange rate appreciation and current account deficits leading up to a depletion of foreign reserves. A currency crisis occurs when foreign reserves approach a critical level. The evidence thus seems to suggest a simple explanation of the crises entirely based on fundamentals, according to the theoretical results of our optimizing model.

IV Summary And Conclusions

In this paper we have used an optimizing general equilibrium model to investigate the currency crises of 1990’s in emerging markets. It is shown that a rise in current and expected future budget deficits generates, during the transition to the steady state, a real exchange rate appreciation and a depletion of foreign reserves, leading up to a currency crisis when reserves decline below a critical level.

The implications of the model are strongly confirmed by probit estimates for a panel of 21 Latin American and Asian countries in the 1990’s.

Table 1 Unit root tests: 1990–2000

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<th>GB</th>
<th>CA</th>
<th>GB</th>
<th>CA</th>
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</thead>
<tbody>
<tr>
<td>ADF</td>
<td>-3.673</td>
<td>-4.524</td>
<td>-4.245</td>
<td>-6.057</td>
</tr>
<tr>
<td>PP</td>
<td>-3.460(1%); -2.874(5%); -2.574(10%)</td>
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**Legend:**

ADF: Augmented Dickey-Fuller unit root test based on OLS regression of the first difference of the dependent variable (budget deficit, or current account) on a constant, the one period lagged level of the depended variable and four lagged difference terms. Similar results are obtained for lagged differences in the range [2,6]

PP: Phillips-Perron unit root test based on OLS regression of the first difference of the dependent variable on a constant and its one period lagged level, using the Newey-West (1987a) adjusted variance-covariance matrix of the parameter estimates. The figures reported for this test have been obtained using a window size (or truncation point) of 4, but similar results are obtained in the range [2,6]

GB and CA denote the ratio of budget deficit to GDP and of current account balance to GDP, respectively.

More detailed definitions and sources of the variables employed are found in the Data Appendix.
Table 2 Causality tests

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<th>GB → CA</th>
<th></th>
<th></th>
<th>CA → GB</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIMS</td>
<td>GRANGER</td>
<td>GRANGER INST.</td>
<td>SIMS</td>
<td>GRANGER</td>
<td>GRANGER INST.</td>
</tr>
<tr>
<td></td>
<td>LM</td>
<td>LMF</td>
<td>LM</td>
<td>LMF</td>
<td>LM</td>
<td>LMF</td>
</tr>
<tr>
<td></td>
<td>4.41(2)</td>
<td>2.16(2,224)</td>
<td>4.40(2)</td>
<td>2.16(2,225)</td>
<td>4.62(3)</td>
<td>1.51(3,224)</td>
</tr>
</tbody>
</table>

**Legend:**

GB → CA denotes the causal relationship running from GB to CA.

SIMS, GRANGER and GRANGER INST. denote Sims, Granger and Granger instantaneous causality test, respectively. Under GB → CA, the Sims’ test has been performed by regressing the budget deficit on its lagged values together with past and future values of the current account balance. The Granger causality test have been carried out by regressing the current account balance on its lagged values together with past values of the budget deficit. Current values of the budget deficit where also included in the regressors for the Granger instantaneous causality test. An analogous procedure have been applied for the case CA → GB.

LM is the Lagrange multiplier statistics used to test the null hypothesis of no causality, asymptotically distributed as $\chi^2(k)$ under the null hypothesis, where $k$ is the maximum lag (lead) term chosen to perform the test.

LMF is the modified LM statistics, asymptotically distributed as $F(k, T-h)$, where $T$ is the sample size and $h$ the number of regressors.

The single and double asterisks denote a statistical level of significance better than 5 and 1%, respectively.

Table 3 GMM estimates of EFGB: 1990-2000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>T-statistic</th>
<th>Sample size</th>
<th>$R^2$</th>
<th>SE</th>
<th>J-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Const.</td>
<td>4.735</td>
<td>0.979</td>
<td>231</td>
<td>0.924</td>
<td>5.364</td>
<td>0.029(1)</td>
</tr>
<tr>
<td>EFGB(-1)</td>
<td>0.379</td>
<td>4.472</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>REXA(-1)</td>
<td>-0.056</td>
<td>1.371</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CA(-1)</td>
<td>-0.378</td>
<td>2.554</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>-0.983</td>
<td>1.226</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T^2</td>
<td>0.078</td>
<td>1.369</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Instrument list: Const., EFGB(-1), CA(-1), REXA(-1), RESR(-1), T, T^2 and 20 country dummies.

**Legend:**

$R^2$: Adjusted R-squared.

SE: Standard error of regression.

J-statistic: Newey and West (1987b) test for the validity of the $(s-h)$ over identifying restrictions, where $s$ is the number of instruments and $h$ the number of regressors. Asymptotically distributed as $\chi^2(s-h)$ under the null hypothesis that the restrictions are not binding.

Equation estimated with twenty country dummies. T and $T^2$ are, respectively, a time trend ad a squared time trend.
Table 4 Probit estimates: 1990-2000

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>z-statistic</th>
<th>slope derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFGB(^\wedge)</td>
<td>-0.909</td>
<td>2.730</td>
<td>1.509</td>
</tr>
<tr>
<td>CA(-1)</td>
<td>-3.591</td>
<td>2.950</td>
<td>5.962</td>
</tr>
<tr>
<td>REXA(-1)</td>
<td>0.204</td>
<td>4.204</td>
<td>0.338</td>
</tr>
<tr>
<td>RESR(-1)</td>
<td>-0.245</td>
<td>2.801</td>
<td>0.407</td>
</tr>
<tr>
<td>DCR</td>
<td>0.997</td>
<td>3.163</td>
<td>1.655</td>
</tr>
<tr>
<td>RRATE</td>
<td>-0.134</td>
<td>2.465</td>
<td>0.222</td>
</tr>
<tr>
<td>LF</td>
<td>-7.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR</td>
<td>52.807(6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(McF. R^2)</td>
<td>0.830</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:

- **EFGB\(^\wedge\)**: Estimated value of EFGB obtained by static forecast of the econometric equation shown in table 3.
- **LF**: Maximized value of the log likelihood function.
- **LR**: Likelihood ratio statistic to test the null hypothesis that all slope coefficients except the constant and the country dummies are zero, asymptotically distributed as \(\chi^2(n)\), where \(n\) is the number of the variables tested.

\(McF. R^2\): McFadden R-squared.

Probit slope derivatives are expressed in percentage values. The model is estimated by maximum likelihood with a constant and twenty country dummies. The z-statistics uses the robust standard errors estimated by quasi-maximum likelihood method.
Table 5 In-sample prediction evaluation

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic probability score (QPS)</td>
<td>0.017328</td>
</tr>
<tr>
<td>Log probability score (LPS)</td>
<td>0.030462</td>
</tr>
<tr>
<td>Global squared bias (GSB)</td>
<td>0.00000002</td>
</tr>
</tbody>
</table>

**Goodness-of-fit (cut-off probability of 0.5)**

| Dependent variable = 0 |  | % of correct observations | 99.55 |
|------------------------|--|--|--------------------------|------|
| (\(\hat{P} \leq 0.5\)) |  | % of incorrect observations | 0.45  |
| (\(\hat{P} > 0.5\)) |  | % of total correct observations | 99.13 |
| % of total incorrect observations |  | 0.87 |

| Dependent variable = 1 |  | % of correct observations | 90.00 |
|------------------------|--|--|--------------------------|------|
| (\(\hat{P} > 0.25\)) |  | % of incorrect observations | 10.00 |
| % of total correct observations |  | 99.13 |
| % of total incorrect observations |  | 0.87 |

**Goodness-of-fit (cut-off probability of 0.25)**

| Dependent variable = 0 |  | % of correct observations | 99.55 |
|------------------------|--|--|--------------------------|------|
| (\(\hat{P} \leq 0.25\)) |  | % of incorrect observations | 0.45  |
| % of total correct observations |  | 99.13 |
| % of total incorrect observations |  | 0.87 |

Legend:

- **QPS**: Quadratic probability scores, defined as \(QPS = (1/T) \sum_{t=1}^{T} 2(\hat{P}_t - P_t)^2\), where \(\hat{P}_t\) is the probability forecast generated by the model shown in table 4 for the year \(t\) and \(P_t\) is our binary variable (Crisis) which is equal to 1 if a crisis occurs in the year \(t\) and zero otherwise.

- **LPS**: Log probability score, defined as \(LPS = (1/T) \sum_{t=1}^{T} (1 - P_t) \ln(1 - \hat{P}_t) + P_t \ln(\hat{P}_t)\).

- **GBS**: Global squared bias, defined as \(GBS = 2(T - \overline{P}^2)\), where \(\overline{P} = (1/T) \sum_{t=1}^{T} \hat{P}_t\) and \(T = (1/T) \sum_{t=1}^{T} P_t\). For a more detailed discussion see Diebold and Lopez (1996).

An observation is classified as “correct” when the predicted probability is less than or equal to the cut-off value and the observed Crisis = 0, or when the predicted probability is greater than the cut-off value and the observed Crisis = 1. Analogously, an observation is classified as “incorrect” when the predicted probability is greater than the cut-off level and the observed Crisis=0, or when the predicted probability is less than or equal to the cut-off level and the observed Crisis=1.
A - DATA APPENDIX -


**Government budget deficit or surplus/GDP:** The ratio of government budget deficit (-) or surplus (+) (IFS line 80) to GDP (IFS line 99b).

**Current Account/GDP:** The ratio of current account (IFS line 78ald) to GDP (IFS line 99b) converted into dollars (using IFS line rf).

**Real exchange rate:** The real exchange rate is the nominal exchange rate (IFS line rf) adjusted for the relative consumer prices (IFS line 64). The measure is defined as the price foreign goods (using United States as the foreign country) to the price of domestic goods. **Reserves/Imports:** The ratio of total reserves (IFS line 1l.d) to imports (IFS line 98c) converted into dollars (using IFS line rf).

**Total reserves:** IFS line 1l.d.

**Domestic credit/GDP:** IFS line 32 divided by IFS line 99b.

**Nominal rate of interest:** Money market rate (IFS line 60b) for Argentina, Brazil, Mexico, Indonesia, Korea, Malaysia, Thailand, Turkey, Singapore; discount rate (IFS line 60) for Venezuela, Philippines, Pakistan, India, Uruguay, Colombia, Peru; deposit rate (IFS line 60l) for Bolivia, Honduras, Sri Lanka, Chile; lending rate (IFS line 60p) for China.

**Real interest rate:** Nominal rate minus annual inflation rate, using consumer prices (IFS line 64).

**M1/Reserves:** IFS line 34 converted into dollars divided by IFS line 1l.d.

**M2/Reserves:** IFS line 35 converted into dollars divided by IFS line 1l.d
B - MATHEMATICAL APPENDIX

B.I) The basic model and the fiscal index

The equations (1)-(6) in the text are derived from the following aggregate model

\[(B.I.1)\quad C = \frac{\delta + \beta}{1 + \eta} \left[ \frac{\omega(K) - T}{r^* + \delta} + K + \rho F + \rho m + \rho D \right] \]

\[(B.I.2)\quad \dot{m} = \frac{1}{\rho} [r^* \rho m - \eta C] \]

\[(B.I.3)\quad \dot{K} = Y(K) - (qC + G^H) - X(\rho) \]

\[(B.I.4)\quad \dot{F} = \frac{1}{\rho} \left\{ X(\rho) - [(1 - q)C + \rho G^F] + r^* \rho F \right\} \]

\[(B.I.5)\quad \dot{p} = [Y(K) - r^*] \rho \]

\[(B.I.6)\quad \dot{D} = \frac{1}{\rho} \left[ r^* \rho D + (G^H + \rho G^F) - T - \frac{M}{\rho} \right] \]

which describes the equations of motion of an open economy satisfying the transversality conditions

\[(B.I.7)\quad \lim_{t \to \infty} F e^{-r^* t} = 0, \quad \lim_{t \to \infty} D e^{-r^* t} = 0. \]

Here equation (B.I.6) is the dynamic budget constraint of government, where \(D\) is the level of debt.

In a sustainable fixed exchange rate regime no seignorage revenues are available to the government. Setting \(M = 0\) in equation (B.I.6), as in Burnside et al. (1998), and integrating it under the constraint implied by (B.I.7) we may write the intertemporal budget constraint of the government as

\[(B.I.8)\quad D = \int_{t=0}^\infty \frac{1}{\rho} (T_v - G_v) e^{-r^* (v-t)} dv , \]

which states that the level of government debt is equal to the present discounted value of future surpluses.

Since current and anticipated fiscal variables affect demand through the effects on wealth, relative price and consumption, it is convenient, following Blanchard (1985), to summarize all these effects by an index of fiscal policy, which may be written as

\[(B.I.9)\quad D = \frac{1}{\rho} \left[ \delta + \beta \right] \left[ \rho D - \int_{t=0}^\infty T_v e^{-(r^* + \delta) (v-t)} dv \right] \]

normalizing, for simplicity, government spending to zero on the entire path.

The specific fiscal policy designed in this paper may be characterized by the following equations

\[(B.I.9)\quad T = \alpha \rho D - Z, \quad \dot{D} = \frac{1}{\rho} (r^* \rho D - T), \quad D_0 = 0 . \]
Taxes are positively linked to the level of debt through the $\alpha$ parameter, while $Z$ is a lump sum tax. Solving (B.I.9) for the time path of $D$ and $T$ and substituting in (B.I.8) we obtain

$$\rho_d = \frac{\delta(\delta + \beta)}{(\alpha - r^*)(1 + \eta)} \left[ \frac{1}{r^* + \delta} - \frac{e^{-(\alpha - r^*)(t - t_0)}}{\alpha + \delta} \right] Z$$

from which we get

$$d_{0} = \frac{\delta(\delta + \beta)}{(r^* + \delta)(\alpha - r^*)(1 + \eta)} Z$$

and

$$d_{\infty} = \frac{\delta(\delta + \beta)}{(r^* + \delta)(\alpha - r^*)(1 + \eta)} Z = \frac{\delta(\delta + \beta)}{(r^* + \delta)(1 + \eta)} D_{\infty} > \rho_0 d_0,$$

where $(\rho_d)_{t_0}$ and $d_{\infty}$ denote, respectively, the initial and steady state value of $d$ and $D_{\infty} = \frac{Z}{(\alpha - r^*)}$ is the steady state value of debt. The initial value of $d$ depends on the entire sequence of current and anticipated future budget deficits. Differentiating equation (B.I.10) with respect to time we obtain

$$\dot{d} = (r^* - \alpha - \frac{\rho}{\rho}) d + \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} Z,$$

which describes the equation of motion of the fiscal index reported in the text.

**B.II) Short-run and long-run equilibrium**

The short-run macroeconomic equilibrium is obtained by combining the aggregate consumption equation (1) together with equation (2) and the product market equilibrium condition

$$Y(K) = qC + G^H + K + X(\rho).$$

These equation can be solved for $C, \rho$ and $m$ obtaining equations (7)-(9) in the text.

The partial derivatives are

$$\frac{\partial C}{\partial K} = \frac{X}{\Lambda} \left[ \frac{\omega(K)}{r^* + \delta} + 1 \right] + \frac{Y(K)}{\Lambda} \left( F + \frac{1 + \eta}{\rho} \right) > 0; \quad \frac{\partial C}{\partial F} = \frac{\rho X'}{\Lambda} > 0; \quad \frac{\partial C}{\partial q} = \frac{\rho X'}{\Lambda} > 0.$$  

$$\frac{\partial C}{\partial \rho} = -\frac{X}{\Lambda} \left[ \frac{\omega(K)\rho m}{(r^* + \delta)^2 + \rho m} \right] - \frac{\partial q}{\Lambda} < 0; \quad \frac{\partial m}{\partial F} = -\frac{q\rho}{\Lambda} < 0; \quad \frac{\partial m}{\partial d} = \frac{\rho X'}{\Lambda} > 0; \quad \frac{\partial m}{\partial \rho} = \frac{q m}{\Lambda} > 0.$$  

$$\frac{\partial \rho}{\partial q} = \frac{\partial \rho}{\partial d} = \frac{q}{\Lambda} < 0; \quad \frac{\partial \rho}{\partial q} = \frac{q}{\Lambda} > 0.$$  

$$\frac{\partial \rho}{\partial m} = -\frac{\eta}{\Lambda} \rho X' + q \frac{m}{\Lambda} > 0.$$  

19
\[
\frac{dm}{dK} = \left[ \frac{\omega(K)}{r^*+\delta} + 1 \right] \left( \frac{\eta}{r^*} X^* + q \frac{m}{\rho} \right) + \dot{Y}(K) \left[ F_m + m + \frac{1+\eta}{\delta+\beta} \left( d - \frac{m}{\rho} \right) \right] > 0;
\]
\[
\frac{dm}{dr^*} = -\frac{m}{r^*} \left[ q(F + m) + \left( \frac{1+\eta}{\delta+\beta} \right) \left( d + X^* \right) \right] - \frac{\omega(K)}{(r^*+\delta)^2} \left( \frac{n}{r^*} X^* + q \frac{m}{\rho} \right) < 0;
\]

where: \( \Lambda = X^* \left[ \frac{1+\eta}{\delta+\beta} - \frac{\eta p}{r^*} \right] + q \left( F + \frac{1+\eta}{\delta+\beta} d \right) > 0 \) if \( \frac{1+\eta}{\delta+\beta} \geq \frac{\eta p}{r^*} \).

The long-run equilibrium is reached when \( K = \dot{K} = \dot{m} = \rho = \dot{d} = 0 \) in the model given by equations (1)–(6). The partial derivatives reported in (10)–(14) are obtained as

\[
\frac{\partial C}{\partial r^*} = \frac{\left( \delta + \beta \right) X^*}{(1+\eta) Y(K) \Lambda} \left[ Y''(K) \left[ Y''(K) \left[ \frac{\omega(K) r^*}{(r^*+\delta)^2} + \rho(m + F) \right] - r^* \left[ \left( \frac{1+\eta}{\delta+\beta} \right) \left( \frac{r^*}{\delta+\beta} \right) + \omega(K) \right] \right] \right] > 0;
\]
\[
\frac{\partial \rho}{\partial d} = \frac{r^* q d}{\Lambda} > 0; \quad \frac{\partial K}{\partial r^*} = \frac{1}{Y(K)} < 0; \quad \frac{\partial K}{\partial d} = 0;
\]
\[
\frac{dF}{d\rho} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right] +
\]
\[
\frac{dF}{d\rho} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right] > 0
\]
\[
\frac{dF}{d\rho} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right]
\]
\[
\frac{\partial \rho}{\partial d} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right] > 0
\]
\[
\frac{\partial \rho}{\partial d} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right]
\]
\[
\frac{\partial \rho}{\partial d} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right]
\]
\[
\frac{\partial \rho}{\partial d} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right]
\]
\[
\frac{\partial \rho}{\partial d} = \frac{1}{Y(K) \Delta} \left[ \frac{\omega(K)}{r^*} \left( q \frac{(1-q) r^*}{\rho} \right) + \frac{\omega(\delta) r^*}{\rho} \left( F + m + 1+\eta \frac{d}{\rho} \right) \right]
\]

**B.III) Stability conditions and transitional dynamics**

We can demonstrate that our model has a saddle point equilibrium path by analyzing the stability conditions when \( d = 0 \) and \( d \neq 0 \). We assume, for simplicity, \( Y(K) = r^* \), so that \( \dot{\rho} = 0 \) and \( \omega \) is constant. Denoting the sum of \( K, F \) and \( m \) by \( W \), the dynamics is given by the two linear differential equations
\[ \dot{C} = (r^* - \beta)C - \delta \left( \frac{\delta + \beta}{1 + \eta} \right) W \]

\[ W = r^* W + \omega - (1 + \eta)C, \]

when \( d = 0 \).

The first equation is obtained by differentiating equation (1) in the text, while the second is obtained combining equations (2), (3) and (4), setting \( G = 0 \). If \( \beta < r^* < \delta + \beta \) the determinant of the coefficient matrix

\[
\begin{pmatrix}
(r^* - \beta) & -\delta \left( \frac{\delta + \beta}{1 + \eta} \right) \\
(1 + \eta) & r^*
\end{pmatrix}
\]

is negative and the steady state equilibrium \( (\bar{C}, \bar{W}) \) is a saddle point with eigenvalues \( \gamma_1 = r^* - (\delta + \beta) < 0 \) and \( \gamma_2 = r^* + \delta > 0 \).

The stable locus associated with the negative root is

\[ C - \bar{C} = a(W_{t0} - \bar{W}) e^{\gamma_1(t-t_0)}, \]

where \( a = \frac{\delta + \beta}{1 + \eta} > 0 \). The stable path is positively sloped and consumption and wealth move in the same direction.

Stability conditions, however, do not change if we drop the above restriction \( \dot{Y}(K) = r^* \). Under the sign restrictions \( \beta < r^* < \delta + \beta \), the linear approximation of the four dimensional system in \( \dot{C}, \dot{K}, \dot{F} \) and \( \dot{\rho} \) near the steady state has two positive and two negative roots, ensuring a saddle point equilibrium path in the neighbourhood of the steady state (the proof is available from the authors upon request).

The dynamics of the system when \( d \neq 0 \) and \( \dot{Y}(K) = r^* \) is given by

\[ C = \frac{\delta + \beta}{1 + \eta} \left( \frac{\alpha}{r^* + \delta} + W \right) + \rho d \]

\[ W = r^* W + \omega - (1 + \eta)C \]

\[ \dot{d} = (r^* - \alpha)d + \left[ \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} \right] Z \]

which reduces to a system of two linear differential equations by substituting the first equation into the second. The coefficient matrix is

\[
\begin{pmatrix}
r^* - \alpha & 0 \\
(1 + \eta)\rho & r^* - (\delta + \beta)
\end{pmatrix}
\]

and the two roots are both negative, with \( \lambda_1 = r^* - \alpha \) and \( \lambda_2 = r^* - (\delta + \beta) \). The linear system, thus, is globally stable when \( \alpha > r^* < (\delta + \beta) \).

The path describing the transition to steady state is given by the equation

\[ W - \bar{W} = b(d_{t_0} - \bar{d}) e^{\lambda_1(t-t_0)}, \]

where \( b = \frac{(1 + \eta)\rho}{\alpha - (\delta + \beta)} < 0 \) if \( \alpha < (\delta + \beta) \). Non-human wealth and expected future budget deficits are negatively correlated along the adjustment path.
The long-run equilibrium, however, will be a saddle point if we take the following system of dimension three
\[
\dot{d} = (r^* - \alpha)d + \left[ \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} \right] Z
\]
\[
\dot{C} = (r^* - \beta)C - \delta \left( \frac{\delta + \beta}{1 + \eta} \right) W - (\delta + \alpha)pd + \left[ \frac{\delta(\delta + \beta)}{\rho(r^* + \delta)(1 + \eta)} \right] Z
\]
\[
\dot{W} = r^* W + \omega - (1 + \eta)C,
\]
where the second equation is obtained by differentiating equation (1) in the text under \( d \neq 0 \). The coefficient matrix
\[
\begin{bmatrix}
(r^* - \alpha) & 0 & 0 \\
-(\delta + \alpha)p & (r^* - \beta) & -\delta \left( \frac{\delta + \beta}{1 + \eta} \right) \\
0 & -(1 + \eta) & r^*
\end{bmatrix}
\]
has two stable roots (\( \lambda_1 = r^* - \alpha, \lambda_2 = r^* - (\delta + \beta) \)) and one unstable root (\( \lambda_3 = r^* + \delta \)) when \( \alpha > r^* < (\delta + \beta) \). The path towards the steady state is a saddle.
REFERENCES


