

# Excess Reserves and Macroeconomic Instability

by  
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PRELIMINARY  
**August 2007**

## Abstract

This paper is concerned with international reserves. It makes two main points. Firstly, excess reserves cannot be regarded as a substitute for sound fundamentals because the former may destabilize the economic system in the longer term. Secondly, reserve accumulation financed by public debt can lead to a potential debt crisis in developing countries. Optimal control theory is employed to illustrate the stabilization problem in an economy in which excess reserves are financed by fiscal deficit.

*JEL Classifications: D81, E52, F31, F32, F33, F41, F42, F47*

*Keywords: International Reserves, Fiscal Deficit, Optimal Control Theory, Dynamic Linear Programming*

## 1 Introduction

The negative relationship between international reserves and external crises discussed in Sachs, Tornell and Velasco (1996) (STV) indicates that when a country faces weak fundamentals, the probability of external crises due to self-fulfilling prophecies is higher if reserves are low. Two points are missed in this context. One is that a country with weak fundamentals, in theory, cannot accumulate reserves unless it borrows externally because the creditworthiness with weak fundamentals means the spread will be large and therefore it will be costly to finance it. The other is that a country with sound fundamentals could avoid external crises.

The implication of STV (1996) can be seen from the large increase in the stocks of international reserves held by a number of Asian countries. For example, from the end of 1999 to the end of 2003, there was an approximate US \$1.2 trillion increase in global reserves. These include \$582 billion by developing

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countries in Asia, and another \$375 billion purchased by Japan. The sum of Asian countries' reserves was almost five times larger than those of EU countries and the average ratio of reserves over GDP in Asian countries was greater than 24 percent in 2003. In particular, the five countries most affected by the crisis in 1997-98 – namely, Indonesia, Korea, Malaysia, Philippines and Thailand (the ACA5) – held reserves totalling nearly \$1.24 billion in 2005. By contrast, sovereign debt as a share of the GDP during 2005 was 52.6 percent in Indonesia, 48.5 percent in Malaysia, 30 percent in Korea, 77.4 percent in the Philippines, and 35.9 percent in Thailand<sup>3</sup>.

This situation is due primarily to the newly introduced floating exchange regime and full liberalized capital flows. Consequently, major changes in reserves policies have resulted in reserve accumulation as experienced by those Asian countries. Feldstein (1998, 2002) suggests that emerging markets should accumulate foreign exchange reserves as insurance against the disruptive effects of abrupt capital outflows. Similarly Greenspan (1999a, b) has suggested that countries should hold an amount of reserves equal to all the short-term debt scheduled to fall due over the next 12 months. In the experience of currency crises, there is evidence that reserves do indeed act as a buffer stock against future financial crises, consistent with the precautionary motive. Aizenman and Marion (2002) and Sachs et al. (1996) find that reserve levels have a significantly negative influence on the likelihood of currency crises. Similarly, Berg and Partillo (1999) find that low reserve holdings in relation to short term external debt are a significant source of vulnerability. Bird and Rajan (2003) confirm that those countries which have experienced crises appear to hold significantly higher reserves in the long term than those that have not. Also central banks smooth exchange rate movements as well as “excessive” volatility of trade and foreign direct investment (FDI). Rajan (2002) and Aizenman et al. (2004) regressed foreign exchange reserves on currency volatility for emerging market economies and finds a strong negative effect of reserves adequacy on exchange rate volatility.

Secondly, the other reason for high reserve-holding is to help reduce exchange rate volatility via a signalling effect on creditworthiness. This could be associated with lower external borrowing costs. This effect works both directly through improved confidence and indirectly through improved credit ratings on sovereign foreign currency debt, as the government's default risk is perceived to diminish. Reserve level is one of the determinants of creditworthiness so that higher reserve holdings seem likely to enhance a country's access to private capital markets. Ul Haque et al. (1996), Dooley and Verma (2001) and Hviding et al. (2004) also suggest that the larger the reserves, the more confidence investors will have when market-sentiment turns and the banking system comes under pressure. On the other hand, Willett and Nitithanprapas (2000) question the robustness of the Sachs-Tornell-Velasco (1996) conclusion that high reserves with sound fundamentals could have a powerful effect in protection against crises. But with weak fundamentals, first generation crisis models im-

<sup>3</sup>Figures taken from <http://www.fenews.com/fen50/fe-in-asia/fe-in-asia.html> (2005).

ply that reserve levels should only influence the timing of crises. In principle, the smoothing behavior by central banks should, over time, have no net impact on reserves. Hence, the continuous accumulation of reserves suggests that foreign exchange intervention is largely asymmetric. Furthermore, DeLong and Eichengreen (2002) argue that financial liberalization increases the credit driven by reserve accumulation to hold down the exchange rate, which in turn spills over into excessive investment in the non-traded sector when the export led growth model reaches the point of diminishing returns.

Thirdly, the costs of the Asian crisis in terms of sustained output losses turned out to be high and this may have increased the perceived marginal benefit of holding reserves relative to their cost (Cerra et al. 2005). Asian governments have been accumulating foreign reserves, mostly United States securities, in order to maintain stable exchange rates or to attempt to limit currency appreciation. As the domestic yield rates are higher than those from US treasury securities, the cost of reserve holdings and the position keeping of short-dollar seem too large if the domestic currency needs to be appreciated. Given the high costs of reserve holdings, Asian countries should clearly be concerned about their reserves holdings being above “adequate” levels, or at least the costs of continuing to amass even more reserves.

Finally, the reserve accumulation however would reflect the stance of the preference for risk taking by the central bank’s fiscal, monetary and reserve policy and the individual country’s economic prospects.

Most existing literature have focused on the adequate level and the cost of reserve hoarding based on the above considerations. The main characteristics in this paper are the consideration of the long term impact resulting from public debt financed reserve hoarding into the system. In section 2, a simple dynamic macroeconomic model is used to look at the impact of excess reserves. Optimal control rules for linear discrete-time macro economic system are obtained for a simple IS-LM model in which optimization in a restricted and unstable macroeconomic system which public debt and exchange rate depends on both fiscal and monetary fundamentals. Section 3 discusses the role of weak fundamentals and reserve accumulation for the long-run stability of the system. Some policy implications on international reserves are suggested in the conclusions.

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<sup>3</sup>In order to apply optimal control theory to reduce the accumulation cost of reserves, we assume that: (i) the need to hold reserves can be substantially reduced; (ii) the recent scale of reserve accumulation is unlikely to be sustainable indefinitely as it would also eventually put upward pressure on interest rates which would induce further capital inflows, leading to an increasingly large and costly sterilisation requirement; (iii) foreign currency intervention capacity to reflect real market is partly a reserve management rather than allow currency appreciation due to bond financing and capital inflows because of improved creditworthiness owing to reserve accumulation; (iv) there are alternative goals to official intervention beyond restoring market functionality, which stem from precautionary motives including the possible prevention of crises and contagion in the region; (v) there is some flexibility around the timing and approach to financing public debt; and finally (vi) the importance of timely dissemination of accurate information on countries’ international reserves to assessing countries’ external vulnerability.

## 2 Optimal Control Theory in Macro Configuration

This section makes use of Dynamic Linear Programming (DLP) of optimal theory to address the instability problem in a simplified economic system with excess reserves. To illustrate optimal paths in the system, Dynamic Programming (DP) involves the adjustment behaviors over time based on a pattern emerged in the system with explicitly prescriptive policies for a small open economy<sup>4</sup>. Linear Programming (LP) can be an efficient method of economic calculation for linear functions. The DLP in this framework, for simplicity, seeks to produce general properties without individual treatment as in Pindyck (1973), Turnovsky (1976, 1979a,b)], Chow (1981) and Murata (1977, 1982). In next section 2.1, the DLP of optimal control theory is briefly outlined. In section 2.2, the rules of optimal control are applied in a simple macroeconomic configuration with a fixed exchange rate regime. In section 2.3, we apply the theory in the same linear system under a floating exchange rate regime.

### 2.1 Preliminaries on Dynamic Linear Programming (DLP)

A stochastic linear quadratic (LQ) problem of optimal control theory with Riccati differential equation (RDE) in a finite horizon can be employed to solve optimal values and paths for a target state with multiple control and state variables with respect to an objective function. In the LQ with the RDE, both the control and the state vectors enter into the diffusion terms in the LQ function<sup>5</sup>. Some preliminaries including some lemmas are imposed throughout this paper in formulating the DLP. The main concepts in the control theory for the instability problem are 'controllability' and 'stability' to attain a fixed target  $x(\beta) = x$  at some terminal time ( $\beta > 0$ ). Some useful lemmas can be referred in Rami and Zhou (2000)<sup>6</sup>.

**Lemma 1** (*Moore-Penrose inverse*) *For any square matrix  $M$  with determinant ( $A \neq 0$ ), there exists a unique matrix  $M^{-1}$  such that  $MM^{-1}M = M$ ,  $M^{-1}MM^{-1} = M^{-1}$ ,  $(MM^{-1})^T = MM^{-1}$ , and  $(M^{-1}M)^T = M^{-1}M$ .*

<sup>4</sup>The DP principally involves embedding the optimal control problem in which the system starts in a state at time 0 but with different initial states and initial times with various economic magnitudes per unit of time.

<sup>5</sup>Further discussions about the conditions and features of the RDE, see Anderson and Moore (1990) and Balvers and Mitchell (2003).

<sup>6</sup>The definitions used in the theory include:  $M^T$  is the transpose of a matrix  $M$ .  $Det(M)$  denotes the determinant of a square matrix  $M$ , and  $Tr(M)$  its trace where  $M$  is positive and nonnegative definite,  $M > 0$  and  $M = M^T$ .  $R^n$  is the  $n$ -dimensional Euclidean space,  $R^{m \times n}$  the set of all  $m \times n$  matrices,  $S^n$  the set of all  $n \times n$  symmetric matrices that also assumed to be nonnegative definite matrices. See Rami and Zhou (2000) for some useful lemmas and the proofs. For example, for a symmetric matrix  $S$ ,  $S^{-1} = (S^{-1})^T$ ,  $S \geq 0$  if and only if  $S^{-1} \geq 0$ , and  $SS^{-1} = S^{-1}S$ . Schur's lemma: let matrices  $M = M^T$ ,  $R = R^T$ , and  $N > 0$  be given with appropriate dimensions. The following conditions are equivalent:  $M - NR^{-1}N^T \geq 0$ ;  $\begin{bmatrix} M & N \\ N^T & R \end{bmatrix} \geq 0$ ; and  $\begin{bmatrix} R & N^T \\ N & M \end{bmatrix} \geq 0$ . The matrix  $R$  is then required to be nonsingular (see Murata, 1982 for the details).

**Lemma 2** (*Tinbergen Policy Rule*) *A static economic system  $x = Fv$ , where  $x$  is a  $n \times n$  vector of target variables,  $v$  is a  $n \times m$  vector of policy instruments, and  $F$  is a  $n \times m$  real constant matrix. The system has a solution  $v$  for an arbitrary target  $x$ , if and only if  $\text{rk}(F) = n$ , where  $\text{rk}(\cdot)$  denotes the rank of a matrix.*

**Lemma 3** (*Lyapunov stability*) *For any real symmetric positive definite  $K$ , all eigenvalues of  $A$  are less than one in modulus, if and only if the matrix equation  $A^T B A - B = -K$  has as its solution  $B$ , a symmetric positive definite matrix. A real square matrix  $A_t$  approaches a zero matrix as time ( $t$ ) goes to  $\beta$ . The system is said to be controllable, if and only if the  $[n \times (n\beta)]$  matrix  $M_n \equiv [B, AB, \dots, A^{n-1}B]$  has rank  $[n]$ .  $M_n$  is the state controllability matrix of system  $x(t)$  with initial condition  $x(0)$ . Then, the system is also said to be dynamically controllable for its stability if only if the  $[n \times m\beta]$  matrix  $M_\beta \equiv [B, AB, \dots, A^{\beta-1}B]$  has  $\text{rk}(M_\beta) = [n]$  and the matrix  $A$  approaches a zero matrix as time goes to  $\beta$ .*

Let us consider the following stochastic linear-quadratic (LQ) control problem in (1) subject to (2). The loss function of an arbitrary initial state in (1) is the expected value of cost function  $v(t)$  subject to  $x(t)$  which is a state-space form of linear system (with an  $n$ -dimensional state vector) for a finite time-horizon  $\{1, \dots, \beta\}$ . The  $L(x_0)$  satisfies the backward recursion equation for the expectation ( $E$ ) over  $\xi(t)$  which is conditional on  $x(t-1)$  and  $v(t)$ . The  $\xi$  in (1) indicates a random  $n$ -dimensional vector, and  $v(t)$  is an  $m$ -dimensional control vector, both depend on the future optimization programs. The constant weighting matrices  $\Gamma$  and  $\Theta$  are assumed to be positive semidefinite matrices, and  $\Phi$  in the cost function is positive definite. Hence, these weighting matrices will be the nonnegative diagonal matrices. The function  $g_t(x(t-1), v(t), \xi(t))$  in (1) is equivalent to  $(Ax(t-1) + Bv(t) + \xi(t)^\beta \Gamma x_0 + v^T(t) \Phi v(t))$ . A state vector  $x(t)$  in (2) can be extended to a stochastic discrete form  $Ax(t-1) + Bv(t) + c(t) + D\xi(t)$ , where  $c(t)$  is non-random exogenous vector. A stochastic discrete form of  $x(t)$  can be written by neglecting  $c(t)$ , while a deterministic type can be expressed as  $Ax(t-1) + Bv(t) + c(t)$  by neglecting  $D\xi(t)$ .

$$(1) \quad L(x_0) \equiv \min_{v(t)} E_{\xi(t)} \{ x^T(\beta) \Gamma x(\beta) + \sum_{t=1}^{\beta} x^T(t-1) \Theta x(t-1) + v(t) \Phi v(t) \}.$$

$$(2) \quad x(t) = Ax(t-1) + Bv(t) + \xi(t).$$

$A$  and  $B$  in (2) are constant positive matrices. The objective function can also be expressed in terms of  $v(t)$  as  $\min_{v(1)} E_{\xi(1)} \{ g_1(x(0), v(1), \xi(1)) + \min_{v(2)} [E_{\xi(2)} \{ g_2(\cdot) \}] + \dots + \min_{v(\beta)} [E_{\xi(\beta)} \{ g_\beta(x(\beta-1), v(\beta), \xi(\beta)) \}] \} + x_0^T \Theta x_0$ . The general explicit form of (2) is obtained by an iterative form by substituting  $t = 1, 2, \dots, \beta$ , as (3), where

$$(3) \quad x(t) = A_t x(0) + \sum_{\beta=0}^{t-1} A^T B v(t-\beta) + \sum_{\beta=0}^{t-1} \xi(\beta).$$

A generalised form of the expected cost function based on (1) in a finite time interval  $\beta$ , forms  $L(\beta_\xi, \beta_g)$ :

$$(4) \quad L(\beta_\xi, \beta_g) = E_{\xi(\beta)} g_\beta(x(\beta-1), v(\beta), \xi(\beta)).$$

The differential of (4) with respect to  $v(\beta)$  is set equal to zero:

$$(5) \quad 0 = \partial L(\beta_\xi, \beta_g) / \partial v(\beta) \\ = 2(B^T \Gamma B v(\beta) + B^T \Gamma A x(\beta-1) + \Phi v(\beta)).$$

From (5) an optimal control can be explicitly written in a linear feedback form:

$$(6) \quad v(\beta) = -K(\beta)x(\beta-1),$$

where  $K(\beta)$  is defined as:

$$(7) \quad K(\beta) \equiv [\Phi + B^T \Gamma B]^{-1} B^T \Gamma A.$$

Substituting (6) into (5) and assuming stochastic term,  $\xi(\cdot) \equiv 0$ , we have the  $L(\beta_\xi, \beta_g)$  as<sup>7</sup>:

$$(8) \quad L(\beta_\xi, \beta_g) \equiv x^T(\beta-1)[[A - BK(\beta)]^T \Gamma [A - BK(\beta)] + \\ K^T(\beta) \Phi K(\beta)] x(\beta-1) + \text{tr}(\Gamma R).$$

A multi-period objective function in a dynamic economic systems for a finite time-horizon  $\beta$  with instrument costs ( $\Phi$ ) can be written:

$$(9) \quad \min_{E(L_\beta)} \left\{ x^T(\beta) \Gamma x(\beta) + \sum_{t=1}^{\beta} x^T(t-1) \Theta x(t-1) + v^T(t) \Phi v(t) \right\}.$$

By Lemmas 2 and 3, given a generalized quadratic form (1) with respect to the associated cost to the system (6) with (7), starting from a given arbitrary state  $x(0)$  and  $x_0 \in R^n$ , and for each  $(x_0, v(\cdot)) \in R^n \times V^{x_0}$ , the system (2) is said to be state controllable if  $x$  can reach a preassigned target vector  $\hat{x}$  by controlling policy instruments  $v(\Upsilon)$ , ( $1 \leq \Upsilon \leq \beta$ ) at some time  $\beta > 0$ , i.e.,  $x(\beta) = \hat{x}$ , where  $t = 1, 2, \dots, \beta$ .

**Lemma 4** Let  $A$  and  $B$  be  $[n \times n]$  and  $[n \times m]$  real constant matrices respectively. If  $A$  and  $B$  are controllable, such that  $A$  nonsingular matrix  $M_n \equiv [B, AB, A^2B, \dots, A^{n-1}B]$  has full rank, by Lemma 2, then  $(A - BK, B)$  is controllable for each  $[m \times n]$ , real constant matrix which is referred to  $K$  as shown in (7). Any nonzero state vector ( $x \neq 0$ ) is sufficient to guarantee the controllability of  $A$  and  $B$ , if  $M^T M = B$ ,  $MAx = vMx$ , where  $v$  represents an eigenvalue of  $A$  by Lemmas 1.

<sup>7</sup> Similarly,  $J(\beta, \beta-1)$  is defined as:

$$E_{\xi(\beta-1)} g_{\beta-1} \{ (x(\beta-2), v(\beta-1), \xi(\beta-1)) + \min v(\beta) J(\beta, \beta) \}.$$

Setting equal to zero the differential of equation of  $J(\beta, \beta-1)$  with respect to  $v(\beta)$  will yield:  $0 = \partial J(\beta, \beta-1) / \partial v(\beta-1) = 2(B^T \Gamma S(\beta-1) B v(\beta-1) B^T S(\beta-1) A x(\beta-2) + \Phi v(\beta-1))$ , where  $S(\beta-1)$ . It is equivalent to:  $[A - BK(\beta)]^T \Gamma [A - BK(\beta)] + K^T(\beta) \Phi K(\beta) + \Theta$ . Equation (8) with stochastic term  $E\xi(\beta)$  can be expressed:  $L(\beta, \beta) \equiv E\xi(\beta) \{ ([A - BK(\beta)] x(\beta-1) + \xi(\beta))^T \Gamma [A - BK(\beta) x(\beta-1) + \xi(\beta)] \} + x^T(\beta-1) K^T(\beta) \Phi K(\beta) x(\beta-1)$ .

The optimal control vector  $v(t)$  with non-random exogenous a vector  $c(t)$  in (10) minimizes the objective function (9) subject to (2) for a finite time-horizon  $t = 1, 2, \dots, \beta$ , is  $(-K(\cdot))$  optimal feedback controller and exogenous constant vector  $-\bar{k}(t)$ . For every initial state  $x_0$ , a feedback control  $v(t) = -Kx(t)$ , where  $K$  is a constant matrix, stabilizes and produces the solution of (9) if the corresponding state  $x(t)$  with  $t \in (1, \dots, \beta)$  satisfies  $E[x(t)^T x(t)] = 0$ .

$$(10) \quad v(t) = -K(t)x(t-1) - \bar{k}(t)$$

where  $K(t)$  is defined as in (5):

$$(11) \quad K(t) \equiv [B^T S(t)B + \Phi]^{-1} B^T S(t)A,$$

and  $S(t)$  denotes:

$$(12) \quad S(t) = A^T S(t-1)[A - BK(t-1)] + \Theta.$$

Then, the second term  $\bar{k}(t)$  in (10) can be expressed:

$$(13) \quad \bar{k}(t) = [B^T S(t)B + \Phi]^{-1} B^T \{S(t)c(t) + J(t+1)S(t+1)c(t+1) \\ + J(t+1)J(t+2)S(t+2)c(t+2) + \dots + \Phi\tau = 1\} \beta \\ - J(\beta - \tau + 1)S(\beta)c(\beta)\}$$

where  $J(t)$  denotes  $J(t) \equiv [A - BK]^T$ .

**Remark 1** Similarly, rewriting (2) as a computational form:  $X_t$  equals  $(A - HK_t)X_{t-1}$  and  $X_\beta$ ,  $(A - HK_\beta)$ , ...,  $(A - HK_1)X_0$ . Setting also cost function  $(C_\beta)$  at time  $\Upsilon_t$  equal to  $X_\beta^T \Gamma X_\beta + \sum_{t=1}^{\beta} X_{t-1}^T \Phi X_{t-1} + V_\beta^T \Phi_\beta$ , we have an optimal controller  $-K_\beta$  in  $V_\beta$  as a function of  $(A - HK_1), \dots, (A - HK_{\beta-1})X_0$  ( $\equiv X_{t-1}$ ), which represents an optimal feedback control at  $t = 1, \dots, \beta$ , where  $K_t$  is a constant gain matrix.

Applying Lemmas (2) and (4) for the objective function (9) for a finite horizon ( $t=1, 2, \dots, \beta$ ) subject to the initial state ( $x_0$ ), the target state ( $x_\beta$ ) is attainable by manipulating the policy instruments  $v(t)$ . Also, the dynamic system is asymptotically stable by Lemma 5, if a quadratic form has negative definite  $[A^T B A - B]$ .

## 2.2 Application: Bond financed economic system: fixed exchange rate regime

For the application of the theory in a simple macroeconomic model, it is assumed that the optimal stabilization policy is primarily concerned with the level of national income ( $y$ ) along with the policy instruments reserves ( $r$ ), exchange rate ( $e$ ) or interest rate ( $i$ ), and fiscal spending ( $g$ ). These instruments need to be adjusted in order to offset the effects of past policies via the control feedback system in (6). The IS equation is simplified in a standard form in (14), which says that national income ( $y_t$ ) is equal to consumption plus investment and

fiscal spending. Money supply equals the demand for money in equation (15). Changes of money stocks  $\Delta m_t = m_t - m_{t-1}$  and the amount of new bond issues should be equals the taxation on output and government expenditure. The budget constraint is the sum of the additional money supply and the issuance of new bonds as in (15). Equation (16) shows the sources of international reserves ( $\ddot{r}_t$ ) which would reflect an external balance, such as the cumulated surplus in current account.

$$\begin{aligned}
 (14) \quad y_t &\equiv \alpha(1 - \tau)(y_t + d_{t-1}) - vi_t + g_t + \xi(e_t + 1) - \beta y_t + \varepsilon_t^{(1)} \\
 (15) \quad \Delta m_t + i^{-1} \Delta r_t &\equiv g_t - \tau y_t + d_{t-1}(1 - \tau) \\
 (16) \quad \ddot{r}_t &\equiv \psi y_t - \mu i_t - m_t + \varepsilon_t^{(2)}
 \end{aligned}$$

where  $\alpha$  is marginal propensity to consume,  $\tau$  the taxation, and  $\xi$  the exchange rate elasticity of export ( $x$ ) in period  $t$ . The parameters  $v$ ,  $\beta$ , and  $\mu$  are defined as  $v \equiv -dI/di$ ,  $\beta \equiv \partial l / \partial y$ , and  $\mu \equiv -\partial l / \partial i$ , where  $v, \beta$ , and  $\mu > 0$ .  $\varepsilon_t^{(1, \dots, n)}$  is assumed to be normally distributed independent and identical stochastic disturbance with mean 0 and standard deviation ( $\sigma$ ). The symbols  $m_t$ ,  $\tau y_t$ ,  $g_t$ ,  $e_t$ , and  $i_t$  denote money stock, initial government revenue, government spending, nominal exchange rates, and real domestic interest rate at time  $t$  respectively. Rearranging (14):

$$(17) \quad \psi' y_t + vi_t \equiv \alpha(1 - \tau)d_{t-1} + g_t + \xi(1 + e_t) + \varepsilon_t^{(1)},$$

where  $\psi' \equiv 1 - \alpha(1 - \tau) + \beta > 0$ .

Adding (15) to (17) multiplied by  $\pi$  will yield high powered money ( $h_t$ ):

$$(18) \quad h_t \equiv \beta y_t - \mu i_t.$$

Substituting (18) into (17),

$$(19) \quad \tau' y_t \equiv g_t + (1 - \tau)d_{t-1} - i^{-1} \Delta d_t + (1 + e_t)\pi\xi - \Delta h_t + \pi\kappa i_t + \varepsilon_t^{(2)}$$

where  $\tau' = \tau + \pi\varsigma$ ;  $\varepsilon_t^{(2)} \equiv \varepsilon_{1t} + \pi\varepsilon_t^{(1)}$ ,  $1 - \pi$  is the sterilization rate of foreign reserves ( $0 < \pi \leq 1$ ),  $\varsigma$  marginal propensity to import and  $\kappa$  capital flows, (19) can be rewritten as:

$$(20) \quad (\tau' + \beta)y_t - \mu' i_t \equiv \beta y_{t-1} - \mu i_{t-1} + g_t + (1 - \tau)d_{t-1} - i^{-1} \Delta d_t + \xi\pi(e_t + 1) - \varepsilon_t^{(3)}$$

where  $v \equiv \mu' = \mu - \pi\kappa$  and  $\varepsilon_t^{(3)} = \varepsilon_t^{(2)} - \varepsilon_{t-1}^{(2)}$ .

We derive the following matrix form for (19) and (20) and  $d_t \equiv \Delta d_t + d_{t-1}$ :

$$(21) \quad MX_t \equiv NX_{t-1} + QV_t + \epsilon_t$$

where

$$X_t = \begin{bmatrix} y_t \\ i_t \\ d_t \end{bmatrix}; \quad V_t = \begin{bmatrix} g_t \\ e_t \\ \Delta r_t \end{bmatrix}; \quad \epsilon_t = \begin{bmatrix} \xi + \varepsilon_t^{(1)} \\ \xi\pi - \varepsilon_t^{(4)} \\ 0 \end{bmatrix}$$

and

$$M = \begin{bmatrix} \psi' & v & 0 \\ \tau' + \beta & -\mu' & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad N = \begin{bmatrix} 0 & 0 & \alpha(1 - \tau) \\ \beta & -\mu & 1 - \tau \\ 0 & 0 & 1 \end{bmatrix};$$

$$Q = \begin{bmatrix} 1 & \xi & 0 \\ 1 & \pi\xi & -1/\bar{i} \\ 0 & 0 & 1 \end{bmatrix}.$$

By Lemmas 1 and 3,  $\det(M) \equiv -(v\tau + v\beta + \psi\mu) < 0$  implies that  $M$  is non-singular and so its inverse  $M^{-1}$  exists, then the following state vector  $X_t$  is defined:

$$(22) \quad X_t \equiv AX_{t-1} + HV_t + K\epsilon_t$$

where  $A \equiv M^{-1}N$ ,  $H \equiv M^{-1}Q$  and  $K \equiv M^{-1}$ . Setting  $\delta \equiv v\tau + v\beta + \sigma\mu$ , we compute the composite parameter  $A$  and  $H$  which is a constant matrix respectively :

$$(23) \quad A \equiv \begin{bmatrix} v\beta/\delta & -v\mu/\delta & (\mu'\alpha + v)(1 - \tau)/\delta \\ -\psi'\beta/\delta & \psi'\mu/\delta & (1 - \tau)(\alpha(\tau' + \beta') + \psi')/\delta \\ 0 & 0 & 1 \end{bmatrix}$$

$$(24) \quad H \equiv \begin{bmatrix} (v + \mu')/\delta & \xi(v\pi + \mu')/\delta & v(\pi\xi - 1)(1 - \tau)/\delta \\ (\tau' + \beta' - \psi')/\delta & \xi(\tau' + \beta' - \pi\psi')\mu/\delta & \psi'/\delta \\ 0 & 0 & 1 \end{bmatrix}.$$

The cost function in (9) is to be minimized with respect to control instruments  $(g, e, r)$  in (25) subject to system  $X_t$  in (22) with state variables  $(y, i, d_t)$ , and a given initial condition  $x(0) = X_0$  over the finite time period  $(t = 1, \dots, \Upsilon)$ . Assuming that the initial government budget is balanced,  $\Delta m_0 = \Delta d_0 = 0$ , an optimal control rule  $v_t$ ,  $t = 1, 2, \dots, \Upsilon$ , the optimal control feedback system is:

$$(25) \quad V(t) \equiv -A(t)X(t-1), \text{ for } t = 1, 2, \dots, \Upsilon,$$

where  $A(t) \equiv [\Phi + H^T\Psi(t)H]^{-1}H^T\Psi\Theta(t)A$ , and  $\Theta\Psi(\Upsilon) = \Upsilon$ .

$\Gamma$  and  $\Theta$  are assumed to be positive semidefinite symmetric matrices and  $\Phi$  is a positive definite matrix. The composite noisy term  $(K_t)$  has zero mean and a finite variance. Preassigned values of weighting matrices are primarily focused on  $y_t$  for  $t = 1, 2, \dots, 20$ , while other numerical values given remain unchanged. The final-period values ( $\Upsilon = 20$ ) for the state variables are assumed to be asymptotically approaching zeros as a whole. The paths of optimally controlled state and control variables are illustrated below based on a simulated initialization by setting different initial economic conditions and target scenario.

Using parameter values given in Appendix 1.1<sup>8</sup>, the values of constant composite parameters  $A$  and  $H$  for (23) and (24) have been produced (in Appendix 1.2). By the controller  $(-K)$  at each point in time, all variables are optimized to minimize the expected loss from the instability of the deviation from its equilibrium state of the system. The target values at  $\beta=20$  are assumed to be asymptotically approaching zeros resulting from the optimization of looping. The optimal paths with maximum of 50000 iterations at each point time for the optimally controlled state (economic growth, interest rate, and fiscal debt) and control vector (government spending (set as a constant), exchange rate, and reserves (which is financed by bonds)) are illustrated in Figures 2.1-2.4. The smallest values for state vector in each time  $t=1,2,3,\dots,20$  for the suboptimal structures in (1) feedback synchronously reflect their overall optimality set targets at  $t=20$  and the corresponding initialisation. The iterated outcomes show the long run dynamics of the optimal paths of the state variables (the columns on the left hand side, 'a') and policy instruments (the right hand side columns, 'b') under a flexible exchange rate regime. A finite time interval ( $t=1,\dots,20$ ) is expressed in a discrete form (so as to be regarded a '20 year policy planning'). The penalty imposition on reserves associated with its deviation from a stationary reserves  $r_{\{t\}}$  sets to  $\Phi=1$  or 50.

**Case 1.** Figures 2.1a and b: (i)  $x(0)=X_0=1$ , sound or not-weak-fundamentals, and (ii) no excess reserves  $r_0=0$  at  $t=0$  also  $r_{\{\beta\}}=0$  at  $\beta=20$ . A marginally volatile (i.e., the absolute volatility is about 1.8) is shown in control vector throughout the policy period. This is much less than that under a flexible regime (around 50). The system of state variables will be stabilised from 4th year by controlling the policy instruments ( $r$ ,  $g$ , and  $i$ ).

**Case 2.** Figures 2.2a and b: (i)  $x(0)=X_0=1$ , sound or not-weak-fundamentals, and at the final-period, (ii) excess reserves  $r_0=50$  at  $t=0$  with initial external debt  $d_0=100$ , and (iii) no penalty of reserve accumulation is imposed. As it shows a long term stable economy until its demand-push inflationary pressure will become explosive in the long term.

**Case 3.** Figures 2.3a and b: (i)  $x(0)=X_0=-10$ , weak-fundamentals (ii) excess reserves  $r_0=50$  at  $t=0$  with initial external debt  $d_0=100$ . The magnitude of the instability is however less severe than in case 2. This will lead to a potential worst case scenario in terms of system instability in both control and state vectors. As no penalty is imposed, the volatility in all input variables in the system becomes severe during entire policy period. In particular, there are gradually growing explosive pressure of exchange rate and deterioration of output after 10 years.

<sup>8</sup>For initialization for suboptimal structure of the loss function which has three terms as shown in (1), the basis values of parameters on interest rate, tax rate, export and import elasticity, government spending, and public debt are proxied from the values of a normalized average of some Asian countries for the post crisis period. As Malaysia kept its fixed exchange regime while Thailand and Korea have introduced a flexible exchange regime since the Asian crisis in 1997, they form the basis of the case studies in this paper. For example, the variables in a small open economy are based on the averaged or rough proxy of their recent data (the IMF IFS monthly data, 1999M1-2005M12).

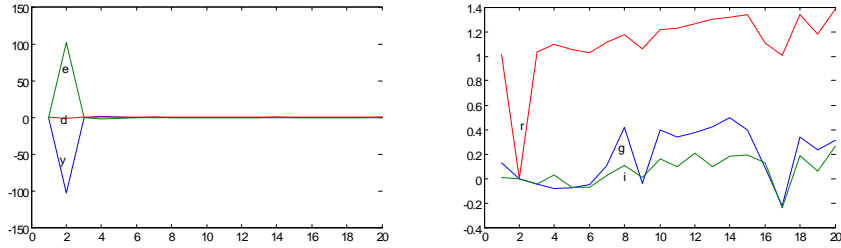


Fig. 2.1a and b: iterative output under fixed exchange rate regime.

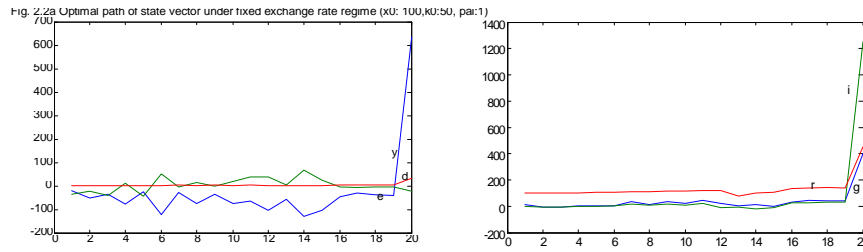


Fig. 2.2a and b: iterative output under fixed exchange rate regime.

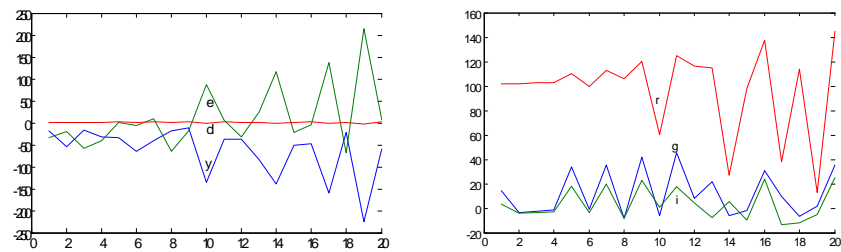


Fig. 2.3a and b: iterative output under fixed exchange rate regime.

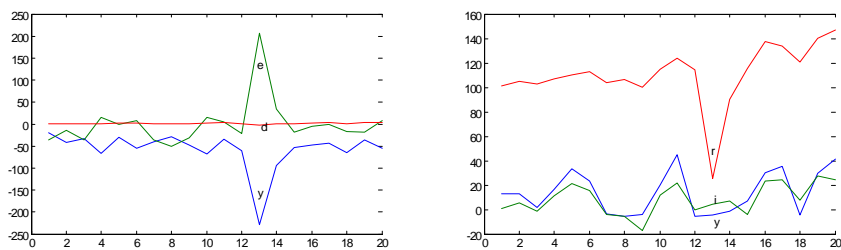


Fig. 2.4a and b: iterative output under fixed exchange rate regime.

**Case 4.** *Figures 2.4a and b: (i)  $x(0)=X_0 = -10$ , weak-fundamentals (ii) excess reserves  $r_0=50$  at  $t=0$  with external debts. The difference this case from case 3 is that the penalty on excess reserves are imposed. Around 13th year there might be an economic crisis if reserves flow out.*

The stylized patterns with different scenarios with given initial settings under a fixed exchange rate regime suggest that (i) regardless of the economic conditions, the instability of the state variables is more severe than that of the policy instruments ( $V_{\{t\}}$ ). (ii) With initial settings of excess reserves (or fiscal deficit) or weak fundamentals, the magnitude of volatility will increase. With initially excess reserves financed by public debt, future instability or some potential liquidity crises may be foreseeable by the terminal time, beta, if fundamentals are weak at  $t=0$ . Furthermore the most severe destabilization to the economy is when there are initially excess reserves and at the same time the reserve authority attempts to increase the level of reserves. The worst case when multiple external crises will occur is when under weak fundamentals and initial and continuous excess reserve holding.

### 2.3 Bond financed economic system: floating exchange rate regime

The same economic system (14-16) in the section 2.2 is adopted. Under a flexible exchange rate regime, letting the exchange rate be a state variable and a lagged effect of exchange rate on trade balance exists, equation (17) is rewritten as:

$$(26) \quad \psi' y_t + v i_{t-1} = \alpha(1 - \tau) d_{t-1} + g_t + \tilde{\xi}_0 e_t + \tilde{\xi}_1 e_{t-1} + \varepsilon_{1t},$$

where  $\psi' \equiv 1 - \alpha(1 - \tau) + \varsigma > 0$ . Notations  $\tilde{e}$ ,  $\tilde{x}$ ,  $z$ ,  $\epsilon$ , and  $\tilde{\xi}$  denote nominal exchange rate, equilibrium exchange rate at equilibrium, export, import, trade balance with respect to the change in exchange rate, and marginal propensity to export over import times equilibrium level of exchange rate respectively.  $e_t = \frac{\tilde{e} - \bar{e}}{\bar{e}}$ ,  $(= e_t - e_{t-1}) = 0$ ,  $\epsilon_0 = \frac{\partial \tilde{x}_t}{\partial e_t} - \frac{\partial z_t}{\partial e_t} > 0$ ,  $\epsilon_1 = \frac{\partial \tilde{x}_t}{\partial e_{t-1}} - \frac{\partial z_t}{\partial e_{t-1}} > 0$ ,  $\tilde{\xi}_0 = \epsilon_0 \bar{e}$ , and  $\tilde{\xi}_1 = \epsilon_1 \bar{e}$ .

The deviation form of system (26) is obtained by substituting (15) for  $\Delta m$  into (27) which yields:

$$(27) \quad (\tau' + \beta) y_t - (\mu + \kappa) i_t - \tilde{\xi}_0 e_t = \beta y_{t-1} - \mu i_{t-1} + \tilde{\xi}_1 e_{t-1} + g_t + (1 - \tau) d_{t-1} - \bar{v}^{-1} \Delta d_t - \varepsilon_t^*$$

We derive the following matrix forms (30) and (31) in terms of the deviations of variables from their stationary values with government expenditure ( $g_t$ ), new issue of bonds ( $\Delta d_t = \Delta b_t$ ) and interest rate ( $i_t$ ) as control variables, while state variables are  $y_t$  and  $e_t$  and  $d_t$  are expressed in terms of deviations from the initial stationary values. Pre-multiplying by the inverse of the coefficient

matrix on the left-hand side, we get a state-space form in state variable vector  $\tilde{X}_t$ <sup>9</sup>, the tilda representing the economic system with a floating exchange rate regime.  $C_t$  is an exogenous variable vector. We minimize the expected loss function (9) over a finite horizon with an initial condition  $x(0)$ . Applying the Lemmas (6) and (7) and the extension in Remarks (8), the state form of the system under flexible exchange rate regime is expressed as:

$$(28) \quad \tilde{M}\tilde{X}_t = \tilde{N}\tilde{X}_{t-1} + \tilde{Q}V_t + C i_{t-1} + \epsilon_t.$$

State and control vectors under flexible exchange rate regime with disturbance term (assumed to be  $\epsilon_t \equiv 0$ ):

$$(29) \quad \tilde{X}_t = \begin{bmatrix} y_t \\ e_t \\ d_t \end{bmatrix}; \quad \tilde{V}_t = \begin{bmatrix} g_t \\ \Delta i_t \\ \Delta d_t \end{bmatrix}; \quad \epsilon_t = \begin{bmatrix} \epsilon_t \\ -\epsilon_t^* \\ 0 \end{bmatrix}.$$

and the matrices  $\tilde{M}$  and  $\tilde{N}$  of the composite parameters are:

$$(30) \quad \tilde{M} = \begin{bmatrix} \psi' & -\tilde{\xi}_0 & 0 \\ \tau' + \beta & -\tilde{\xi}_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \tilde{N} = \begin{bmatrix} 0 & \tilde{\xi}_1 & \alpha(1 - \tau) \\ \beta & \tilde{\xi}_1 & 1 - \tau \\ 0 & 0 & 1 \end{bmatrix},$$

$$\tilde{Q} = \begin{bmatrix} 1 & -v & 0 \\ 1 & \mu + \kappa & -1/\bar{i} \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and } C = \begin{bmatrix} -v \\ \kappa \\ 0 \end{bmatrix}.$$

$\det(M)$  ( $= -(v\tau + v\beta + \psi\mu) < 0$ ) from Lemmas 1-3 implies that  $M$  is non-singular and so its inverse  $M^{-1}$  exists. and  $\tilde{X}_t$  can be written as:

$$\tilde{X}_t = \tilde{A}\tilde{X}_{t-1} + \tilde{H}\tilde{V}_t + c_t + \tilde{K}\epsilon_t.$$

Setting  $\delta = v\tau + v\beta + \psi\mu$ , we find:

$$(31) \quad \tilde{A} = \begin{bmatrix} \delta_0^* & 0 & \delta_1^* \\ -\psi'\delta_0^* & -\epsilon_1/\epsilon_0 & \psi'\delta_0^* - \alpha(1 - \tau)/\bar{\xi}_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(32) \quad \tilde{H} = \begin{bmatrix} 0 & \delta_2^* & \omega_0^* \\ -1/\bar{\xi}_0 & \psi'\delta_2^* + v/\bar{\xi}_0 & \psi'\omega_0^* \\ 0 & 0 & 1 \end{bmatrix}$$

$$(33) \quad \tilde{K} = \begin{bmatrix} -\epsilon_t - \epsilon_1 \\ -\psi'\epsilon_t - (\tau' + \beta)\epsilon_1 \\ 0 \end{bmatrix} \cdot \frac{1}{\rho_0^*}, \quad \text{where } \rho_0 \equiv \frac{\tau + \beta + \psi\mu}{v}.$$

$$(34) \quad \tilde{C}_t = \begin{bmatrix} \delta_2 + \delta_3 \\ -\psi'(\delta_2 + \delta_3) + v/\bar{\xi}_0 \\ 0 \end{bmatrix} \cdot i_{t-1}.$$

Preassigned values of weighting matrices are primarily focused on  $y_t$  for  $t = 1, 2, \dots, 20$ , while the other numerical values remain unchanged. The final-period values ( $\beta = 20$ ) for the state variables are assumed to be asymptotically

<sup>9</sup>For example, the countries (Indonesian, Malaysia, Korea and Thailand) severely affected by the Asian crisis have introduced a floating exchange regime except Malaysia. These countries might therefore be considered as a sort of benchmark for both fixed and flexible regimes.

approaching zeros as a whole. The paths of optimally controlled state and control variables are illustrated below based on a simulated initialization by setting different initial economic condition and the target scenario. Using parameter values given (Appendix 1.1), the constant values  $A$ ,  $H$  and  $K$  for (31), (32) and (33) have been produced (in Appendix 1.2). The optimal paths with maximum of 50000 iterations at each point time for the optimally controlled state and control vectors are illustrated in Figures 2.5-2.8. The target values at  $\beta=20$  are assumed to be asymptotically approaching zeros resulting the optimization of looping. A finite time interval ( $t=1, \dots, 20$ ) is expressed in a discrete form (so as to be regarded a '20 year policy planning'). The penalty imposition on borrowing to finance international reserves associating its deviation from a stationary reserves  $r_{\{t\}}$  sets to  $\Phi=1$  or 50. Setting an economic system with a flexible exchange rate regime, figures 2.5a, 2.6a, 2.7a, and 2.8a (optimal path of stable vector from its computed optimal values) and 2.5b, 2.6b, 2.7b, and 2.8a (optimal path of control vector from its computed optimal values), illustrate the outcomes from different initialisations in economic situations and the existence of excess reserves at time 0.

**Case 5.** 2.5a and b: (i)  $x(0)=X_0=1$ , sound- or not-weak-fundamentals (ii) excess reserves,  $r_0=50$  at  $t=0$ , and (iii) no penalty is imposed on accumulating of excess reserves. This prescribed economy has a narrow range of volatility (absolute range of 40 and 50 for  $x$  and  $v$  vector respectively). The optimal paths at the beginning, domestic output drops due excess reserves, however its stabilisation process takes less than 4 years to smoothing out the entire system if exchange rate depreciates enough to boost the real economy.

**Case 6.** 2.6a and b: (i)  $x(0)=X_0=1$ , (ii) excess reserves,  $r_0=50$  at  $t=0$ , (iii) sound- or not-weak-fundamentals, and (iv) there are costs to accumulate excess reserves,  $\text{pai}=50$ . Assuming the costs to accumulate reserves are not trivial, the range of volatility has increased up to 400 and 180 for state and control vector respectively. To stabilise long term interest rates and total external debts, domestic outcome ( $y$ ) will continuously fluctuate during the policy period.  $y_{\{t\}}$  will initially be improved when interest rate and exchange rate drop at the same time. If reserves are at higher level without experiencing currency appreciation throughout the policy period, the target stabilisation of the economy will be achieved after 6th year.

**Case 7.** 2.7a and b: (i)  $x(0)=X_0=-10$ , weak-fundamentals and (ii) no excess reserves  $r_0=0$  at  $t=0$ . When the fundamentals are weak, an initial depreciation of currency would improve the domestic output although there shows a possibility of economic depreciation around 8th year. The main instrument for the stabilisation of the entire economy will be achieved by adjusting exchange rate.

**Case 8.** 2.8a and b: (i)  $x(0)=X_0=-10$ , weak-fundamentals (ii) excess reserves,  $r_0=50$  at  $t=0$ , and (iii) an imposition of penalty cost to hoard reserves. In order to keep higher level of reserves during the finite interval, domestic currency needs to be appreciated. This appreciation of exchange rate would lead to a slump in domestic output until the currency depreciates enough to improve the output. From 9th year onwards, we observe a stable system of an economy.

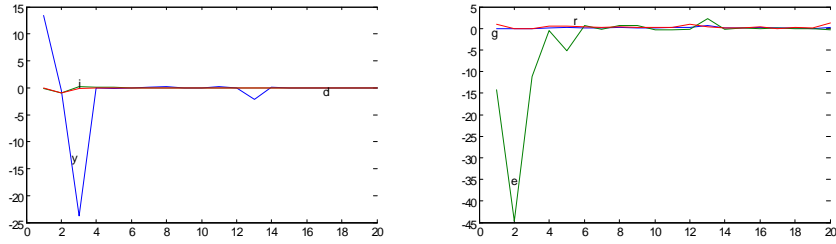


Fig. 2.5a and b: the Iterative outputs under flexible exchange rate system.

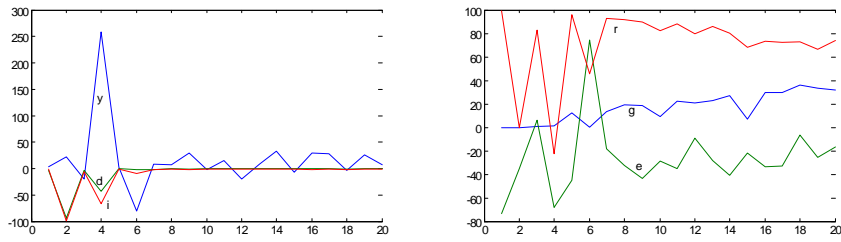


Fig. 2.6a and b: the Iterative outputs under flexible exchange rate system.

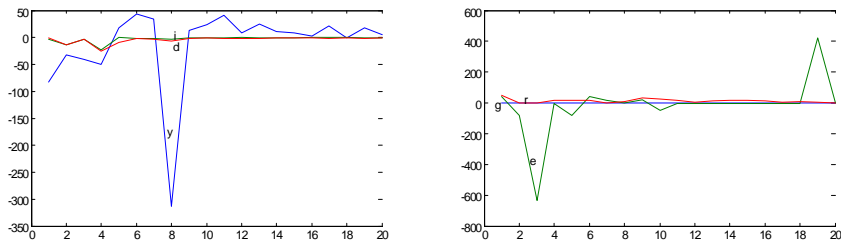


Fig. 2.7a and b: the Iterative outputs under flexible exchange rate system.

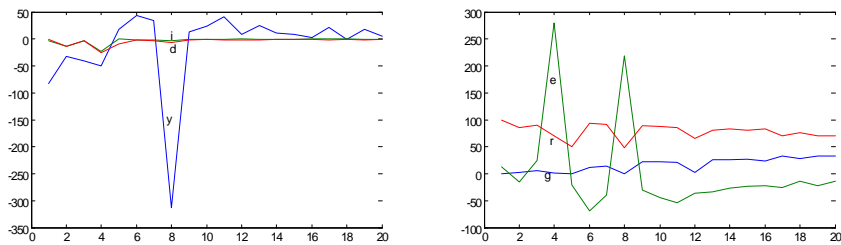


Fig. 2.8a and b: the Iterative outputs under flexible exchange rate system.

The stylized patterns from the DLP suggest that a flexible exchange regime would be more desirable with excess reserves compared to the fixed case. The instability in policy instruments is more or less similar to those of state variables under a flexible regime. However, the magnitude of absolute volatility in both state and policy variables are much lower under a flexible regime. As shown in the first and second rows, it can be considered that holding excess reserves along with sound fundamentals, it takes about 4 – 6 years to stabilise the economy. Where there will be the cost of accumulation of reserves (i.e., financed with a government bond), the stabilisation process will be delayed up to 2 years. In addition, the penalty imposed on reserve accumulation will lead in turn to an increase in the volatility in both stage and control vectors. With weak fundamentals holding excess reserves, the third and fourth rows indicate that there might be a severe economic depression around 8th year. It seems that it takes a decade to stabilize where initial fundamentals are weak with excess reserves. The implication for reserve policy is that, under weak fundamentals, the costs of reserve accumulation will cause instability in the both control and state vectors. Furthermore, if a country holds excess reserves, it would be in a much better position under flexible exchange rate regime against potential external crises and instability problem in the target state vector. The findings with both fixed and flexible regimes indicate that weak fundamentals might be a primary concern to the reserve authority. The simulation does not guarantee the causality where a crisis is caused by the initial excess reserves, it however implies that with initially excess reserves with weak fundamentals, in the long term (say within 20 years), the system will be unstable and might experience multiple liquidity crises.

The application of an optimal stochastic LQ control feedback in finite time horizon economic system might be over-simplifying the solution to the instability problem by forecasting optimal values and the respective optimal paths in a deterministic form. Moreover, the solutions to such a problem would depend on the initialisation of parameter values together with the weighing matrix. However, the DLP of optimal control theory provides straightforward answers with respect to the direction and magnitude of the optimal values and paths to minimize the problem of instability, and therefore it allows us to illustrate a time-invariant comparative static analysis of policy reactions.

### **3 Reserve Accumulation: Offset Weak Fundamentals vs. Destabilize Fundamentals**

In the previous section, we simulated the optimal paths in a particular case where the excess reserves ( $r$ ) are financed by fiscal debts, by showing the likelihood of instability and potential crises in a small open economy. This section focuses the effect of the 'excess reserves' in the same system with (17), (18), (19), (32) and (33) in extreme cases. The following assumptions are made for the simple and restricted equations: (i) optimal reserve policy does not only

depend on fiscal policy and macroeconomic fundamentals, but also on debt, exchange rate, and monetary policies. (ii) Distinguishing the reserves ( $\ddot{r}$ ) and ( $r$ ),  $\ddot{r}$  (say a 'sound reserves') denotes  $ay - di + (\vartheta - i^*) - x (= 0)$ , where sovereign spread ( $\vartheta$ ) can be represented in this context as the real rate of return on capital ( $i - i^*$ ). The net  $\ddot{r}$  therefore reflects the surplus of current account, which will be a stationary process in the long term. On the other hand,  $r$  represents the excess reserves (say a 'weak reserves') financed by fiscal debts, so that the changes of reserves ( $\Delta r(t)$ ) imply an increased external liability ( $de$ ). (iii) The uncovered interest parity (UIP),  $1 + i/p = (1 + i^*)(p/e)^{10}$  holds, where  $i^*$  is the foreign interest rate yields on the reserve stocks, and (iv) prices are fixed in the short run, therefore changes in the nominal exchange rate and interest rate imply the changes in the real terms of both variables. (v) It is also assumed that an explicit intertemporal loss function reflects the trade-off in monetary policy between the stability in output, exchange rate, and interest rate. (vi) The unobserved error terms are excluded<sup>11</sup>. However, it may contain an important combination of omitted factors, such as internal and external shocks, production lag, private investor's expectations, specific risk effects, debt repayment and roll over, and balance sheet effect. A currency crisis is defined in (35) as a function of the changes in the level and volatility (standard deviation) of exchange rate ( $e$ ) and reserves ( $\ddot{r}$ ) with respect to the previous year. The first term of the right-hand-side in (35) shows the misalignment of exchange rate ( $\ddot{e}$ ) from its equilibrium level ( $\bar{e}$ ) which should reflect the fundamentals ( $x$ ). The second term captures the precautionary effect of reserves. Therefore it suggests that countries should accumulate reserves even though the marginal costs ( $c$ ) are potentially high while the marginal return to additional reserve accumulation is low when the weak fundamentals ( $\underline{x}$ ) and the consequent ( $\ddot{e}$ ) exist. Whenever  $\ddot{r} < r$ , we assume that there will be a misalignment in the exchange rates ( $\ddot{e}$ ), as the excess reserves ( $r$  or  $\ddot{r}$ ) will overvalue the exchange rate ( $\hat{e}$ ), and as a result, output will be lower than its potential level ( $y < \hat{y}$ )<sup>12</sup>.

$$(35) \quad C \equiv \left\{ \left[ \left( \frac{1}{\sigma_e} \right) \left( \frac{\Delta e}{e_{t-1}} \right) \right] - \left[ \left( \frac{1}{\sigma_r} \right) \left( \frac{\Delta \ddot{r}}{\ddot{r}_{t-1}} \right) \right] \right\}$$

Rewriting (35), the probability of currency crisis with  $\ddot{r}_{\underline{x}}$  is defined as  $C_{\ddot{r}}$ , where ( $\underline{x}$ ) and ( $x^{-1}$ ) indicates weak fundamentals and the output loss respectively, ( $\underline{x} < 0$ ,  $x > 0$ ,  $\ddot{r}_{\underline{x}} < 0$ , and  $r_{\underline{x}} > 0$ ), and:

$$(36) \quad C_{\ddot{r}} \equiv \{ [\ddot{e}(\underline{x}\ddot{r})] - [\ddot{r}_{\underline{x}}^{-1}(\ddot{r}_{\underline{x}})] \}$$

A simplified form of the first order and the second order conditions of (36) with respect to reserves ( $\ddot{r}$ ) and ( $\underline{x}$ ) is written:

<sup>10</sup> or  $i = i^* + \bar{e}/e$ .

<sup>11</sup> The usual assumption of independent and identical distribution is applied on all disturbance terms.

<sup>12</sup> A Krugman-type balance-of-payments crises indicate that misaligned fundamentals can cause crises when the accumulation of public debt would create fundamentals problems to the economy. The public debt would lead to an increase in interest rates that would initially overvalue the exchange rates before abandoning the exchange rate regime.

$$\begin{aligned}
 (36a) \quad \frac{\partial C_{\bar{r}}}{\partial \bar{r}} &= \frac{x(\ddot{e}_{\underline{x}} + 2\gamma^2)}{\bar{r}} = 0; & (36b) \quad \frac{\partial C_{\bar{r}}}{\partial \underline{x}} &= x(\ddot{e}_{\underline{x}} - 2\bar{r}\gamma^2) = 0, \\
 (36c) \quad \frac{\partial^2 C_{\bar{r}}}{\partial \bar{r}^2} &= \bar{r}^2 > 0, & (37d) \quad \frac{\partial^2 C_{\bar{r}}}{\partial \underline{x}^2} &= 2 + \bar{r} > 0.
 \end{aligned}$$

Suppose a precautionary demand of reserve accumulation ( $\bar{r}_{\underline{x}}$ ) results from the output loss ( $x^{-1}$ ) caused by a currency crisis. The misalignment of exchange rate ( $\ddot{e}$ ) implies an appreciation ( $\ddot{e} < \bar{e} < e > 0$ ) resulting from capital inflows<sup>13</sup>. The magnitude of output loss ( $x^{-1}$ ) determines the level of reserves ( $\bar{r}_{\underline{x}}$ ). A country's fundamental macroeconomic and debt situation remain the primary determinants of sovereign borrowing costs ( $\vartheta = i - i^*$ ).

**Proposition 1.** *The reserve authority would hoard excess reserves when at a lower rate of return ( $i^*$ ) than the cost of borrowing ( $\vartheta = i_c - i_c^*$ ) before a currency crisis, if fundamentals are weak ( $\underline{x}$ ). During a crisis, the loss of output and reserves will be greater than during non-crisis period, thus the inequality will hold  $\frac{1 - C_{\bar{r}}}{(\bar{r})} > \frac{C_{\bar{r}}}{(\bar{r})^2}$  conditional on  $-\underline{x}(\bar{r}) > -(\Delta\underline{x} + \Delta\bar{r})$ , and  $(\underline{x}) > \left(-\frac{C_{\bar{r}}}{\underline{x}}\right)$ , as  $(\bar{r} - \underline{x}) + \bar{r}\underline{x} < 0$  which indicates  $\underline{x} < -\left(\frac{\bar{r}}{\bar{r}C_{\bar{r}}}\right)$ .*

**Proof.** Letting  $\gamma = 1$ , where  $\gamma$  denotes a compensation ratio ( $\bar{r}/\underline{x}$ ), substituting  $\gamma$  in  $\frac{\partial C_{\bar{r}}}{\partial \bar{r}}$  (37a) will give  $((\bar{r}(\ddot{e}_{\underline{x}} + 0.5\bar{r})))x^{-1} (= 0)$ . Similarly, the compensation ratio ( $\gamma$ ) in  $\frac{\partial C_{\bar{r}}}{\partial \underline{x}}$  (37b) will give  $((r(\ddot{e}_{\underline{x}} + 2\bar{r})))x^{-1} (= 0)$ . As  $\underline{x} < 0$ ,  $\ddot{e} > 0$ , and  $\bar{r}_{\underline{x}} > 0$ , the probability of currency crisis ( $C_{\bar{r}}$ ) will be reduced by holding  $\bar{r}$  under weak fundamentals ( $\underline{x}$ ) based on both (36ab) and (36cd). Also  $\frac{\partial^2 C_{\bar{r}}}{\partial \bar{r}^2} \leq \frac{\partial^2 C_{\bar{r}}}{\partial \underline{x}^2}$ , where  $\gamma = 1$ . ■

It is often the case that external debt is private sector liability before a crisis but becomes public debt after a crisis. Assuming that reserves are financed by public debt after a currency crisis, rewriting (35) as  $C_r$ , we have the equation of the probability of currency crisis with  $r\underline{x}^{-1}$ :

$$(37) \quad C_r \equiv \{(\ddot{e}(d^e \underline{x}) - (r\underline{x}^{-1} + (d^e \underline{x})))\}$$

$$\begin{aligned}
 (37a) \quad \frac{\partial C_r}{\partial r} &= -\frac{1}{\underline{x}} = 0; & (37b) \quad \frac{\partial C_r}{\partial \underline{x}} &= d^e(\ddot{e} + r\underline{x}^{-2}) = 0, \\
 (37c) \quad \frac{\partial^2 C_r}{\partial r^2} &= 0, & (37d) \quad \frac{\partial^2 C_r}{\partial \underline{x}^2} &= \frac{-2r}{\underline{x}^3} > 0.
 \end{aligned}$$

**Proposition 2.** *The reserve authority would hoard excess reserves at a lower rate of return ( $i^*$ ) than the cost of borrowing ( $\vartheta = i - i^*$ ) after a currency crisis regardless of the state of fundamentals. Precautionary reserves ( $r$ ) will reduce the probability of currency crisis if  $\frac{1 - C_r}{r} > \frac{C_r}{(r)^2}$  conditional on  $-\underline{x}(d^e) >$*

<sup>13</sup>  $\ddot{e}$  may be due to key shock and additional shocks.

$-(\Delta x + \Delta d^e)$ , and only if  $(\underline{x}) > \left(-\frac{C_{\dot{r}}}{\underline{x}}\right)$  and  $\underline{x}d^e(\vartheta) < -\left(\frac{r(d^e)}{r(d^e)C_r}\right)$  hold.

However with  $r(d^e)$  under prospective weak fundamentals  $(\underline{x})$ , the reserves may not necessarily compensate  $\underline{x}$ , but may increase the probability of currency crisis.

**Proof.** Letting  $\gamma = 1$ , where  $\gamma$  denotes a compensation ratio  $(r/\underline{x})$ , and setting  $\underline{x} < 0$ ,  $\dot{e} > 0$ ,  $rd^e > 0$ , and  $r\underline{x} > 0$ ,  $\gamma$  in (37a) will give  $r(\underline{x}(2r + \dot{e}\underline{x})) (= 0)$ . Similarly, the compensation ratio  $(\gamma)$  in (37b) will give  $\underline{x}(\dot{e}\underline{x} - 2r) (= 0)$ . These indicate that  $r$  will compensate  $\underline{x}$ , only if the exchange rate reflects weak fundamentals. The probability of currency crisis  $(C_r)$  will be reduced if and only if weak fundamentals in (37a), (37b) and (37d) improve. However, excess reserves will not sufficiently reduce the probability, if fundamentals become weak. ■

A debt crisis is defined as the aggregate function of the total external liability  $(d^e)$ , fiscal balance  $(\tau y - g)$ , fundamentals  $(\underline{x})$ , reserves  $(\dot{r})$  and the misalignment in exchange rate  $(\dot{e})$ .

$$(38) \quad D \equiv \{d^e \underline{x} + d^e(\dot{r}y)^{-1} - \tau y + \tau(\dot{e})^2 + g\}$$

$$(38a) \quad \frac{\partial D}{\partial \dot{r}} = -\frac{d^e}{\dot{r}^2 y} = 0; \quad (38b) \quad \frac{\partial D}{\partial \underline{x}} = d^e = 0;$$

$$(38c) \quad \frac{\partial^2 D}{\partial \dot{r}^2} \Big|_{x=-1} = \frac{2d^e}{\dot{r}^3 y} > 0; \quad (38d) \quad \frac{\partial^2 D}{\partial \underline{x}^2} = 0$$

The external debt  $(d^e)$  is the sum of total debt and reserves  $(\dot{r})$  minus tax income  $(d - \tau y + \dot{r})$ . The change in total debt  $(d)$  with respect to sound fundamentals  $\left(\frac{\partial d}{\partial \underline{x}}\right)$  can be expressed as  $\frac{\dot{r}e^{-\dot{r}/x}}{\underline{x}^2} \left(\frac{y}{\underline{x}}\right) - e^{-\dot{r}/x} \left(\frac{y}{\underline{x}^2}\right) > 0$ . A definition of the probability sovereign default  $(\theta)$  is expressed in (40), (see Dreher, Herz and Karb, 2006). A sovereign default rate on the debt service is positively related to the misalignment of exchange rate  $(\dot{e} \equiv \bar{e} - e/\bar{e})$ . Substituting  $y$  in (39) into the budget constraint  $(\tau y)$  into (40), this yields an equation of sovereign default risk  $(\theta)$ . The tax rate  $(\tau)$  and its change depends on the product market,  $\underline{x}\left(\frac{1+y}{i-y}\right)$  and  $(1+i)(i-y)^{-1}$ . As the constraint of the government budget  $(\tau y)$  is  $g + (1+\theta)d^e$ , where the government expenditure  $(g)$  reflects the sum of tax income and fiscal debt,  $y\tau\dot{e}^{-i} + d - \dot{e}^{-i}$ . And therefore the net budget constraint  $(\tau y)$  is equivalent to  $\dot{r} - m - d^e$ , and  $\Delta w$ , the national net wealth, is equivalent to  $\Delta\dot{r} + \Delta\tau - \Delta d^e$ .

$$(39) \quad y \equiv \hat{y} - \alpha_1(\dot{e})^2$$

$$(40) \quad \theta \equiv 1 - \left(\frac{\tau y - \tau \alpha_1(\dot{e})^2 - g}{d^e}\right)$$

The default equation  $(\theta)$  becomes  $\left(-\frac{\hat{y} - \alpha_1 \underline{x} - 1}{d^e} + 1\right)$ , replacing  $r$  by  $\dot{r}$ . Then, differentiate  $(\theta)$  with respective  $\underline{x}$ ,  $\hat{y}$ , and  $d^e$ , the first order conditions, we obtain:

$$(40a) \quad \frac{\partial \theta}{\partial \underline{x}} = 1 - \frac{\hat{y} - \alpha_1 \underline{x}}{d^e} = 0$$

$$(40b) \quad \frac{\partial \theta}{\partial y} = -\frac{1}{d^e} = 0$$

$$(40c) \quad \frac{\partial \theta}{\partial d^e} = \frac{\hat{y} - \alpha_1 \underline{x} - 1}{d^{e2}} = 0.$$

(40a), (40b), (40c) and (40d) suggest that higher costs of a change in the exchange rate and the resulting currency appreciation will lead to a lower output and lower tax revenues: hence, the government has to choose a higher rate of default.

The loss function (41) expresses the minimisation problem in terms of the expected probability of any external crisis consisting of a currency crisis ( $C$ ), a debt crisis ( $D$ ), and a sovereign default ( $\theta$ ). The ability to pay external debt,  $\ddot{r}y$ , is key denominator while  $d^e \underline{x}$  being key nominator in (41). Substituting  $\theta$ ,  $C$  and  $D$  from (35), (38) and (40) into (41), the objective function to minimise any external crisis can be expressed in (42), where  $\gamma \equiv \ddot{r}/\underline{x}$ .<sup>14</sup>

$$(41) \quad L \equiv \min E \{ (\alpha_1 \theta^2 d^e + \alpha_2 (\ddot{e})^2) - (1 - \tau)y + C + D \}$$

$$(42) \quad L \equiv$$

$$\min E \left\{ \alpha_2 \theta^2 d^e + (1 - \tau)y - \ddot{e} \underline{x} \ddot{r} + \underline{x} \ddot{r} (\gamma) - d^e \left( 1 - \frac{1 - 1/\ddot{e}}{\underline{x}} \right) - \frac{d^e}{\ddot{r}y} + \tau(y - \ddot{e}^2) - 1 \right\}.$$

Assuming the tax rate  $\tau$  given  $g$  will not increase for a political reason, the parameters  $\tau$  and  $g$  are treated as constant. Differentiating (42) with respect to  $\ddot{r}$  and  $\underline{x}$ , we get the first order conditions:

$$(42a) \quad \frac{\partial L}{\partial \ddot{r}} = \underline{x}(\ddot{r}^2(2\gamma\ddot{r} - e_{\underline{x}}) + d^e) = 0; \quad (42b) \quad \frac{\partial L}{\partial \underline{x}} = \ddot{r}^2 \left( \frac{2}{d^e} \right) = 0.$$

$$(42c) \quad \frac{\partial^2 L}{\partial \ddot{r}^2} = \frac{2d^e}{\ddot{r}^3} (> 0); \quad (42d) \quad \frac{\partial^2 L}{\partial \underline{x}^2} = \frac{2(\ddot{e}\ddot{r} + d^e\ddot{e} - d^e)}{\ddot{e}\underline{x}^3} (> 0);$$

**Corollary 1.** *Minimising the objective function (42) against an external crisis depends on reserves from sound fundamentals  $\ddot{r}x$  as well as the misalignment of exchange rate with respect to the weak fundamentals  $\ddot{e}x$ . reserves ( $\ddot{r}$ ) will compensate weak fundamentals if and only if the misalignments improve.*

**Proof.** Letting  $\gamma = 0.5$  and 1, where  $\gamma$  denotes a compensation ratio ( $\ddot{r}/\underline{x}$ ), substituting  $\gamma$  into (42a) which normalized setting  $y$  equals 1, will give  $x((\ddot{r}^2(0.5 - \ddot{e}\underline{x}) + d^e)$ , and  $\underline{x}(\ddot{r}(-\ddot{e}\underline{x}\ddot{r} + 2\ddot{r}^2 + d^e)$ . Similarly, the compensation ratio in (42b) will give  $\underline{x}((\ddot{r}^2(2\ddot{r} - \underline{x}) + d^e)$  and  $\underline{x}((\ddot{r}^2(-2\underline{x} + \ddot{r})) + d^e)$ . These imply that the misalignment of exchange rate and the respective weak fundamentals need to be improved, if reserves compensate weak fundamentals and reduce an external crisis<sup>15</sup>. To sufficiently minimise any of the crises in (40), weak fundamentals

<sup>14</sup>Sovereign debt crisis can be defined if the country is in arrears on interest or principal and cannot borrow private capital due to a wide spread ( $\vartheta$ ).

<sup>15</sup> $\gamma = 0.5$ , the smoothing or sterilisation needs to be no less than the adjustment rate of 2.5 times, while  $\gamma = 1$ ,  $\ddot{e}$  needs to be the equilibrium rate at  $\bar{e} = 1$ ,

need to be improved by the reserves accumulated from the surplus of sound fundamentals,  $r(x)$ , and the misalignment of appreciation of exchange rate,  $\ddot{e}$ , needs to be adjusted to improve the weak fundamentals being non-negative,  $\underline{x}^3 > 0$ , as implied in (42c) and (42d).

Let us suppose excess reserves after a crisis are financed by fiscal deficits (i.e.,  $\ddot{r}$  becomes  $r(d^e) \equiv d^e$ ), also assume that the external public debt ( $d^e$ ) is entirely relied on by selling domestic bond ( $b_{t+1}, \dots, b_m$ ). Letting  $\hat{d}$  denotes the collateral as a proportion of the debt level. The elasticity of exchange rate and interest rate, conditional on the real sector ( $x$ ), are assumed to be smaller than for the capital markets. The country's capacity to pay  $\ddot{r}(x)$  and the present value of capacity to pay is defined as  $\hat{d}x$  and  $((1 + \vartheta(e)^2)d^e$ , respectively, where  $i$  is replaced by  $\vartheta(e)^2$  to reflect the balance sheet effect of the debt. The nominal net national wealth or total budget ( $w$ ) can be expressed  $\ddot{r} + \tau - m - d^e$ , where  $\tau = x(\frac{1+y}{i-y})$ , and  $d\tau = (1+i)(i-y)^{-1}$ . Setting the marginal utility of government expenditure equal to the marginal benefit from total wealth ( $y\tau + di$ ) would reduce the economic growth due to the continuous issuance of new bonds ( $d$ )<sup>16</sup>.

**Case 9.** *In line with the debt crisis with the sovereign default as defined in (38) and (40), letting the value of debt,  $v \equiv V(\underline{x}_t, \hat{d}_t)/\hat{d}_t$ , is stochastically determined by  $\int_t^T \min\{\underline{x}_s, \hat{d}_s\} \ddot{e}^{-i(s-t)} ds + \ddot{e}^{-i(T-t)} dt$ , where  $T$  is the time of illiquidity. The current liquidity ( $w(\underline{x}_t)$ ) can be written as  $\int_{ts}^{\infty} \underline{x}^{-i(s-t)} \ddot{e} ds = \underline{x}_t/(\vartheta)$ . If creditors collectively refuse to roll-overs or debt guarantees, ( $\underline{x} < \hat{d}i$ ) or ( $\underline{x} < i$ ), then  $\underline{x} = \underline{x}[\underline{x} - (-\vartheta)]$  and  $\underline{x} = \left( \frac{\dot{\vartheta}\{\vartheta\}\underline{x}}{\underline{x}} \right) - (\underline{x} - (\vartheta))\ddot{e}$ . The current liquidity ( $w(\underline{x})$ ) under weak fundamentals will decrease as debt grows faster than the capacity to pay as  $(\hat{d} > w) = \left( \frac{\underline{x}}{\vartheta} < \hat{d}i \right)$ <sup>17</sup>.*

**Remark 2.** *When  $r_t$  (not  $\ddot{r}_t$ ) is very large, the country may be able to issue new bonds expecting to have little difficulty in repaying the external debt,  $d^e$ , so the debt value goes asymptotically towards par ( $d(d^e)$ ). The emerging countries with excess reserves ( $r$ ) which financed by the government bonds show both debt and a shadow ability of debt service ( $d^e \Rightarrow r \Rightarrow \underline{x}\vartheta^{-1}$ ), by narrowing spread ( $\vartheta$ ) due to excess reserves ( $r$ ), both debt and reserves will grow faster.*

**Conjecture 1.** *Assume that an economy with an entire dependence on bond financing ( $rd^e|_{x\ddot{r}/\vartheta} = d$ ) will have  $\rho = 0$ , where  $\rho$  represents deficit financed by money supply. This, ( $rd^e|_{x\ddot{r}/\vartheta} = d$ ), will lead to an explosive pressure on economy with the increments of debts,  $\Delta d_1 < \Delta d_2 < \Delta d_3 < \dots$ . Excess reserves financed by public debt could destabilize or explode the economy in the long run. Holding a high burden of fiscal debt, governments are more likely to resort to inflationary financing of their fiscal imbalances. If UIP holds, this type of instability might lead to the Krugman-type crisis. For this reason, excess*

<sup>16</sup>The debt burden, ( $di$ ) represents the autonomous component of investment and is the stock of capital which is assumed to follow the UIP.

<sup>17</sup>Unconditional guarantees by debtor countries can have 'moral hazard'. Also a liquidity problem may lead to phases of debt crisis.

reserves will not offset the weak fundamentals but instead might exacerbate the already unstable fundamentals.

**Proof.** From (21) and (22), the immediate implication is that the consecutive  $y_t$  obtained with  $\rho = 0$  will lead to a smaller  $y_1, y_2, \dots, y_T$ , than  $y_t | \rho = 1$  if  $\Delta y_1 < \Delta y_2 < \Delta y_3 < \dots$  and  $\Delta d_1 < \Delta d_2 < \Delta d_3 < \dots$ . Let's  $\lambda \equiv \frac{d}{d + \tau\rho}$ ,  $0 \leq \lambda \leq 1$  and  $\mu \equiv d \equiv \frac{-\partial d}{\partial i} > 0$ ,  $\sigma \equiv 1 - \alpha(1 - \tau) > 0$ ,  $\Delta g_1$  and  $d_t$  being positive. A general form of the changes in output,  $\Delta y_t$  is equivalent to  $(1 - \lambda)\lambda^{t-1} \frac{d - \tau}{d} \Delta g_1 + (1 - \tau)\lambda \frac{((1 - \alpha)\mu)\rho}{d(d + \tau\rho)} \cdot y(\lambda^{t-3} \Delta d_1 + \lambda^{t-4} \Delta d_2 + \dots + \lambda \Delta d_{t-3} + \lambda \Delta d_{t-2}) + ((1 - \tau)\lambda \frac{\alpha\mu\rho}{d \Delta d_{t-1}} \Delta d_{t-1}$ , for all  $t = 1, 2, 3, \dots$ . In the case of  $\rho = 0$ , a direct dependence of  $\Delta y_t$  on  $\Delta b_{t-1}$  indicates  $\Delta y_t |_{\rho=0} = (1 - \tau)\lambda \left( \frac{\alpha\mu}{d} \right) \Delta d_{t-1}$ , for  $t = 2, 3, 4, \dots$ , and  $\Delta y_1 |_{\rho=0} = \left( \frac{\mu}{d} \right) g_1$ .  $\Delta d_3 |_{\rho=0} = i_3(g_3 - \tau y_3 + (1 - \tau)d_2) > \Delta d_2 |_{\rho=0} = i_2(g_2 - \tau y_2 + (1 - \tau)d_1) > \Delta d_1 |_{\rho=0} \frac{d - \tau\mu}{d \Delta g_1 i_t}$ . ■

**Results.** (In Appendix 1.3). The simulated results, based on predetermined initial weights of instrument vector together with the combined inequality signs of parameters, show that the first two equations do not behave stably, if the three conditions are shown not to be held. Due to the composite parameters, the inequality might be biased although it will not change the answer. Thus, the endogenous variables tend to deviate further away from the equilibrium value over time.

Appendix 2 illustrates Conjecture with the interaction of (35), (38) and (40) in a dynamic path of an IS-LM economy. The figure shows that the external debt with the respective weak fundamentals may collapse together in the future while holding excess reserves which is the range from point  $A$  through toward  $E$ . Small changes in the supply of reserves will result in large changes in the external debt (over-borrowing) due to the reduced financing cost. Over time, the demand for reserves converges toward the target of smoothing balance ( $\bar{i} \equiv r^*$ ) at the end of the maintenance period  $r_0$  to  $r_1$ , if there is no precautionary requirement of buffering, i.e.,  $r \equiv r_c$ . The inverse relationships of reserves ( $r$ ), exchange rate ( $r$ ), and interest rates ( $i$ ) with the exchange market pressure ( $EMP$ ) are shown within the  $EMP$  function,  $f(r, x, d^e, \bar{I}, \vartheta)$ , where  $x, d^e, \bar{I}$  and  $\vartheta$  indicates fundamentals, total external debt, intervention (sterilisation) to FX market, and spread. The possible combination of partial derivatives of the function with respect to the respective  $r, x, d_e, \bar{I}$ , and  $\vartheta$  will provide the net pressure in the EMP. The short term EMP can be measured by the FX market supply<sup>18</sup> but the monetary authority might ignore the appreciation of the domestic currency, if the external debt weighs more than that of export oriented output growth. Bond financing for excess reserves would raise the demand of the domestic currency, and in the short term, speculative attacks

<sup>18</sup>The CB net sales domestic currency per dollar.

can be immunised unless the fundamentals are sufficiently weak to widen the spread of external debt. The excess reserve overshoots its steady-state value  $\bar{y}$  at  $E$  through  $A$  and  $B$ . In the course of the adjustment process at point  $C$ , both interest rate and exchange rate will rise which would worsen the fundamentals. The saddle path (from  $f|_r < 0$ ,  $f|_x < 0$ ,  $f|_{de} < 0$ ,  $0 \leq f|_{\bar{y}} \leq 1$ , and  $f|_{\vartheta} \geq 0$ ) indicate that the IS-LM both keep moving leftward but with capital reversals start to flow out at point  $D$ . The slope of  $MA$  (minimal adequate reserves) moves upward while the slope of LM moves downwards until the points  $D$  and  $F$  to fall a slump or a debt crisis at point  $D$  to  $G$ <sup>19</sup>.

## 4 Concluding Remarks

This paper theoretically explores the relationship between excess reserves and a potential debt crisis in a small open economy, when the reserves are financed by fiscal deficit. The main finding is that such reserve accumulation can lead to economic instability. This partly arises due to the fact that accumulation can induce currency appreciation and thus export deterioration and over-borrowing. A second finding is that for countries where reserves are financed by accumulated current account surplus and that have sound fundamentals, a minimum level of reserves is optimal. On the other hand, for many emerging countries a higher level of reserves may be desirable but there remains a trade off between the level of reserves and the fundamentals. The analysis has two implications for policy. Firstly, in managing reserves account should be taken of the implications for economic fundamentals. A ‘hands-off’ policy associated with reserve hoarding by borrowing might not be desirable. Secondly, excessive use of public debt to finance reserve accumulation may lead to long term instability. Our study clearly over-simplifies reserve policy. One issue that has not been addressed is the amount of discretion that should be afforded to the reserve authority to deviate from a minimum adequate level of reserves. Secondly, for simplicity, we have not considered the implication of excess reserves financed by bonds for the implementation of monetary policy under a floating exchange rate regime. Finally, the possible role of a Regional Reserves Pooling System as suggested by Rajan and Siregar (2003, 2004) has not been discussed. These remain important areas for further research.

<sup>19</sup>The external sector is represented by exchange rate ( $e$ ),  $x$  (import ( $z$ ) and export ( $\hat{x}$ )), whereas the demand of reserves will contain import and total debts. Export and spread are two critical variables used to measure the debt service that is reflected by fundamentals ( $x$ ) and therefore low exports ( $\hat{x}$ ), the ability to serve debt due), high external debts, and wider spread ( $\vartheta$ ) are the key warning signs of both currency crisis and debt crisis.

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**APPENDIX 1: Applied optimal control theory:**

1.1 Composite parameter setting in sections 2.2 and 2.3.

Predetermined sets of parameter values and some composite parameters input for computation:

$v$	0.4	$\beta$	0.012	$\mu$	0.4	$v$	0.4	$\rho'_0$	0.73
$\xi(\gamma)$	50	$\phi_1$	0.5	$\phi_2$	1	$\phi_2$	1	$\rho'_1$	0.79
$k$	20	$i$	0.05	$\phi_1$	0.5	$\delta_1$	-1.56	$\rho'_2$	0.81
$\psi$	0.32	$a$	0.68	$\delta_0$	3.15	$\delta'_1$	-1.37	$\omega'_0$	-27.39
$\tau'$	0.24	$\mu'$	0.4	$\delta'_0$	2.74	$\omega'_1$	-29.42	$\epsilon_0$	00.3
$\psi'$	0.32	$\pi;$	0.8	$\bar{\xi}_1$	0.6	$\rho_0$	0.64	$\epsilon_1$	00.8

1.2 Values of constant composite parameters

The values of  $A$  and  $H$  for (23) and  $A$ ,  $H$  and  $K$  for (24) have been produced based on the above Appendix 1.1.

$$(23a) \quad A \equiv \begin{bmatrix} 0.0066 & -0.2219 & 0.8213 \\ -0.0164 & 0.5493 & -0.3327 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(23b) \quad H \equiv \begin{bmatrix} 1.1098 & 0.9988 & -11.0987 \\ -0.24694 & 0.0277 & 27.4694 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$(24a) \quad \tilde{A} = \begin{bmatrix} -0.0162 & 0 & -0.1626 \\ -0.1609 & -0.6000 & -0.8409 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(24b) \quad \tilde{H} = \begin{bmatrix} 0 & -14.6341 & 27.1002 \\ -1.0000 & -14.0878 & 26.8292 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(41) \quad \tilde{K} = \begin{bmatrix} -14.0921 \\ -13.5512 \\ 0 \end{bmatrix}$$

$$(24c) \quad C(t) = 0.05 \begin{bmatrix} -66.66 \\ -24.0 \\ 0 \end{bmatrix}.$$

1.3 Solution of Conjecture

By substituting  $-vi_t$  by investment ( $I_t$ ) which is a function of interest rate ( $i$ ) based on equations (1) and (2), the following (S1) to (S2) behave unstably. Thus, the endogenous variables ( $y_t$ ,  $b_t$  ( $r_t$ ) and  $m_t$ ) tend to deviate further away from the equilibrium value over time. The wealth effect of the bond ( $b_t$ ) on the demand for money ( $m_t$ ) and the consequent incremental public debt ( $d_t$ ) are substituted by reserves ( $r_t$ ). The differentials of  $m$  and  $r$  are also replaced with differences  $\Delta m$  and  $\Delta r$  in the budget (S3) where  $g_t$  and  $i$  are taken to be given.

$$(S1) \quad y_t = \alpha(1 - \tau)(y_t + r_t) + I_t(i) + g_t,$$

$$(S2) \quad m_t = l(y_t, r_t, i),$$

$$(S3) \quad \frac{dm_t}{dt} + i^{-1} \frac{dr_t}{dt} = g_t + r_t - \tau(y_t + r_t).$$

We assume that  $l_r \triangleq \partial l / \partial r > 0$ ,  $l_i \triangleq \partial l / \partial i < 0$ ,  $l_y \triangleq \partial l / \partial y > 0$ ,  $I_i \triangleq \frac{dI_t}{di} < 0$  and  $dm_t/dt = dr_t/dt = 0$ . The equilibrium values are expressed as  $\bar{y}$ ,  $\bar{r}$  and  $\bar{m}$ .

Taking the linear approximation of differential equations (S1) to (S3) about the equilibrium, where  $\sigma$  is denoted as  $1 - \alpha(1 - \tau) > 0$ , will yield:

$$(S4) \quad \sigma \Delta y - \alpha(1 - \tau) \Delta r = 0,$$

$$(S5) \quad l_y \Delta y + l_r \Delta r = \Delta m,$$

$$(S6) \quad -\tau \Delta y + (1 - \tau) \Delta r = 0.$$

For the stationary stabilization with the deviations of a target variables  $y$  and  $m$ , the policy instruments  $g$  and  $i$  with  $\pi$  are set to be constants to assign fiscal and monetary policies.

$$(S7) \quad \lim_{t \rightarrow \infty} (y_t - \bar{y}_{t-1}) \pi_1 = \frac{dg}{dt}, \text{ where } \pi_1 < 0,$$

$$(S8) \quad \lim_{t \rightarrow \infty} (m_t - \bar{m}_{t-1}) \pi_2 = \frac{di}{dt}, \text{ where } \pi_2 > 0.$$

The static effects of changes in the values of  $g$  and  $i$  upon  $y$ ,  $r$ , and  $m$  will be:

$$(S9) \quad \begin{bmatrix} \sigma & 0 & -\alpha(1 - \tau) \\ l_y & 1 & l_r \\ \tau & 0 & \tau - 1 \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta m \\ \Delta r \end{bmatrix} = \begin{bmatrix} 1 & I_i \\ 0 & -l_i \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta g \\ \Delta i \end{bmatrix},$$

(S9) is solved for  $\Delta y$  and  $\Delta m$  as:

$$(S10) \quad \Delta y = \Delta g + \frac{I_i \Delta i}{1 - \alpha},$$

$$(S11) \quad \Delta m = (l_y - l_r) \Delta g + m_i \Delta i, \text{ where } m_i = l_i + \left( l_y + \frac{\tau}{1 - \tau} l_r \right) \frac{I_i}{1 - \alpha} < 0.$$

0.

Combining (S10) and (S11) with the policy in (S7) and (S8) will yield a stable system so that  $g$  and  $i$  converge asymptotically to stationary values in (S12) and the inequality and equality are expressed in (S13) and (S14) :

$$(S12) \quad \begin{bmatrix} \frac{dg}{dt} \\ \frac{di}{dt} \end{bmatrix} = \begin{bmatrix} \pi_1 & \frac{\pi_1 I_i}{1 - \alpha} \\ \pi_2 (l_y - l_r) & \pi_2 m_i \end{bmatrix} \begin{bmatrix} \Delta g \\ \Delta i \end{bmatrix},$$

$$(S13) \quad \pi_1 + \pi_2 m_i < 0,$$

$$(S14) \quad \begin{bmatrix} \pi_1 & \frac{\pi_1 I_i}{1 - \alpha} \\ \pi_2 (l_y - l_r) & \pi_2 m_i \end{bmatrix} = \pi_1 \pi_2 \left[ l_i + \left( \frac{l_r l_i}{(1 - \alpha)(1 - \tau)} \right) \right].$$

Variables  $y$ ,  $b$ , and  $m$  behave in an unstable manner into (S7) and (S8), we get:

$$(S15) \quad \sigma \frac{dy}{dt} - \alpha(1 - \tau) \frac{dr}{dt} = I_i \frac{di}{dt} + \frac{dg}{dt},$$

$$(S16) \quad l_y \frac{dy}{dt} + l_r \frac{dr}{dt} - \frac{dm}{dt} = -l_i \frac{di}{dt},$$

$$(S17) \quad \sigma \frac{dy}{dt} - \alpha(1 - \tau) \frac{dr}{dt} = \pi_2 I_i y + \pi_1 \Delta y,$$

$$(S18) \quad l_y \frac{dy}{dt} + l_r \frac{dr}{dt} - \frac{dm}{dt} = -\pi_2 l_i \Delta m,$$

where  $\Delta y = y - \bar{y}$ ,  $\Delta r = r - \bar{r}$ , and  $\Delta m = m - \bar{m}$  for given  $g$  and  $i$ .  
 Matrix form for (S15) to (S18) can be written as:

$$(S19) \quad \begin{bmatrix} 0 & i^{-1} & 1 \\ \sigma & -\alpha(1-\tau) & 0 \\ l_y & l_r & -1 \end{bmatrix} \begin{bmatrix} \frac{dy}{dt} \\ \frac{dr}{dt} \\ \frac{dm}{dt} \end{bmatrix} = \begin{bmatrix} -\tau\Delta y + (1-\tau)\Delta r \\ \pi_1\Delta y + \pi_2 I_i \Delta m \\ -\pi_2 l_i \Delta m \end{bmatrix}$$

The left hand side of above equation determines the  $y$ ,  $r$ , and  $m$  denoting  $A$  the coefficient matrix.

$$(S20) \quad \begin{bmatrix} \frac{dy}{dt} \\ \frac{dr}{dt} \\ \frac{dm}{dt} \end{bmatrix} = A^{-1} \begin{bmatrix} -\tau & 1-\tau & 0 \\ \pi_1 & 0 & \pi_2 I_i \\ 0 & 0 & -\pi_2 l_i \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta r \\ \Delta m \end{bmatrix},$$

$$= \frac{1}{|A|} \begin{bmatrix} \pi_1 H - \tau(1-\sigma) & (1-\sigma)(1-\tau) & \pi_2(I_i H - (1-\sigma)l_i) \\ -\sigma\tau - \pi_1 l_y & \sigma(1-\tau) & -\pi_2 K \\ \pi_1 l_y i^{-1} - \tau J & (1-\tau)J & \pi_2 i^{-1} K \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta r \\ \Delta m \end{bmatrix},$$

where  $|A| = (1-\sigma)l_y + \sigma H > 0$ ,  $H = l_b + i^{-1} > 0$ ,  $J = (1-\sigma)l_y + \sigma l_b > 0$ , and  $K = l_y I_i + \sigma l_b > 0$ .

The system above is asymptotically stable if and only if the following conditions are satisfied.

$$(S21) \quad (1-\tau)(1-\alpha) + \pi_1 H + \pi_2 i^{-1} K < 0,$$

$$(S22) \quad \left| A^{-1} \begin{bmatrix} -\tau & -\alpha(1-\tau) & 0 \\ \pi_1 & 1-\tau & \pi_2 I_i \\ 0 & 0 & -\pi_2 l_i \end{bmatrix} \right| = |A|^{-1}(1-\tau)\pi_1\pi_2 l_i < 0,$$

$$(S23) \quad \left| \begin{bmatrix} \pi_1 H + (1-\tau)(1-\sigma) & -\pi_2 K & \pi_2(-I_i H) + (1-\sigma)l_i \\ (1-\tau)J & \pi_1 H + \pi_2 i^{-1} K - \tau(1-\sigma) & (1-\sigma)(1-\tau) \\ \tau J - \pi_1 l_y i^{-1} & -\sigma\tau - \pi_1 l_y & \sigma(1-\tau) + \pi_2 i^{-1} K \end{bmatrix} \right| < 0.$$

In order to hold inequality, (S20) is conditional on (S21) to (S23)) assuming it depends on the initial weights of instrument vector together with the combined inequality signs of parameters. The values of parameters predetermined as in (S21) for  $H$ ,  $K$ , and  $J$  with  $\pi_1$  and  $\pi_2$ . The simulated results show that (S20) does not behave stably because the condition in (S21) and (S22) among the three are shown not to be held. The inequality may be biased due to the composite parameters, although it will not change the result.

Numerical solutions with predetermined values from the assumptions in (S7), (S8) and (S20) reconfirm this.

$$K := -1; \quad J := 1; \quad H := 1; \quad \tau := 0.2; \quad \pi_1 := -0.5; \quad \pi_2 := 1.5; \quad i := 0.05; \quad l_y := -0.5; \quad l_i := -0.6; \quad \sigma := 0.4.$$

$$Det(M) = \left( \left[ \begin{bmatrix} \pi_1 H + (1-\tau)(1-\sigma) & -\pi_2 K & -I\pi_2 H + (1-\sigma)l \\ (1-\tau)J & \pi_1 H + \frac{\pi_2 K}{i} - \tau(1-\sigma) & (1-\tau)(1-\sigma) \\ \tau J - \frac{\pi_1 l_y}{i} & -\sigma\tau - \pi_1 l_y & \sigma(1-\tau) + \frac{\pi_2 K}{i} \end{bmatrix} \right] \right)$$

$$Factor(Det(M)) = -28.65280 - 211.1400I.$$

APPENDIX 2. Dynamics of Optimal Path: IS-LM-IR with Excess Reserves

