

# Interest Rate Peg, Wealth Effects and Price Level Determinacy

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## Abstract

This paper analyses the issues of price level determinacy in an optimising general equilibrium model with overlapping generations. It is shown that under a pure interest rate peg, wealth effects rule out nominal indeterminacy but give rise to solution multiplicity.

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## 1 Introduction

Absolute price level determinacy has arguably been one of the most debated issues in monetary economics. A well-known result since Wicksell

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(1965)[1898] is that a pure interest rate peg monetary rule leads to nominal price indeterminacy. Sargent and Wallace (1975) show, in a rational expectations monetary model, that pure interest rate pegging provides no monetary anchor and no mechanism to determine the price level. However, as shown by Patinkin (1949) the absolute price level can be determined if the excess-demand equations for goods depend on the amount of real money balances and on other nominal non-monetary assets in the economy. Turnovsky (1980) and Sargent (1987) generalise Patinkin's result in a rational expectations model with wealth effects.

Following the analysis of McCallum (1986, 2001) it is important to distinguish between two different types of aberrational price level behaviour: nominal indeterminacy and solution multiplicity or non-uniqueness. Nominal indeterminacy arises when the model fails to determine nominal variables for a given path of real variables. Solution multiplicity refers to price level behaviours involving bubbles, thus an infinite number of paths for the price level and for real allocations satisfies all the conditions of the model. The distinction between nominal indeterminacy and solution multiplicity should leave no room for confusion. Nevertheless, according to McCallum (2001) the lack of consensus on this semantic issue could sometimes be source of misleading interpretation about what is really going on in a monetary model.

This paper examines the issue of price level determinacy under an interest rate peg in an optimising framework with overlapping generations. It is shown that wealth effects, due to heterogeneity across generations, solve the problem of nominal indeterminacy, which typically arises in the representative agent framework. However, a multiplicity of equilibrium paths towards

the steady state is also generated by wealth effects.

Section 2 presents the monetary model; Section 3 describes the main results and Section 4 concludes.

## 2 The Monetary Model with Wealth Effects

The demand side of the economy is described by a monetary version of the Yaari (1965)-Blanchard (1985) overlapping generation model with real money balances entering the utility function. Forward looking consumers face the same instantaneous probability of death  $\delta$  and receive the same sequence of after tax endowments. There is no bequest motive and wealth of newly born agents is zero. Birth and death rates are identical and total population is normalized to one. The representative consumer of the generation born at time  $s$  chooses the optimal sequence for consumption,  $c(s, t)$ , and real money balances,  $m(s, t)$ , in order to solve the optimization problem

$$\max_{\{c(s,v), m(s,v)\}} \int_t^\infty \log[c(s, v)^\eta m(s, v)^{1-\eta}] e^{-(\rho+\delta)(v-t)} dv, \quad (1)$$

subject to the instantaneous budget constraint

$$\dot{a}(s, t) = [r(t) + \delta]a(s, t) + y(s, t) - \tau(s, t) - c(s, t) - i(t)m(s, t), \quad (2)$$

and to the transversality condition

$$\lim_{v \rightarrow \infty} a(s, v) e^{-\int_t^v [r(u) + \delta] du} = 0, \quad (3)$$

where  $\rho$  is the constant rate of time preference and  $0 < \eta < 1$ ;  $a(s, t)$ ,  $y(s, t)$  and  $\tau(s, t)$  denote real financial wealth, endowment, lump-sum taxation at

time  $t$  of an agent born at time  $s$ , respectively;  $r(t)$  is the real interest rate,  $i(t)$  is the nominal interest rate and  $i(t) = r(t) + \pi(t)$ , where  $\pi(t)$  denotes the inflation rate. Financial wealth,  $a(s, t)$ , is held in the form of real money balances and real government bonds,  $b(s, t)$ . At each instant agents receive a premium payment of  $\delta a(s, t)$  from a competitive life insurance company in exchange for their financial wealth at the time of their death.

Solving the optimization problem and aggregating over all generations, where the aggregate value of a generic economic variable,  $x(s, t)$ , is obtained as  $X(t) = \int_{-\infty}^t x(s, t) \delta e^{\delta(s-t)} ds$ , yield the aggregate consumption function<sup>1</sup>

$$C(t) = [(\delta + \rho)/(1 + \xi)] [A(t) + \int_t^{\infty} [Y(v) - T(v)] e^{-\int_t^v [r(u) + \delta] du} dv], \quad (4)$$

and the optimal portfolio balance condition

$$M(t) = \xi C(t)/i(t), \quad (5)$$

where  $\xi \equiv (1 - \eta)/\eta$ . The dynamic equation for real financial wealth is

$$\dot{A}(t) = r(t)A(t) + Y(t) - T(t) - C(t) - i(t)M(t). \quad (6)$$

The dynamic equation of consumption is given by

$$\dot{C}(t) = [r(t) - \rho] C(t) - \delta \Omega A(t), \quad (7)$$

where  $\Omega \equiv \frac{\delta + \rho}{1 + \xi}$ . The public sector is described by a consolidated monetary and fiscal authority facing a budget constraint in real terms of the form

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<sup>1</sup>See the Appendix.

$$\dot{B}(t) = r(t)B(t) - T(t) - \mu(t)M(t) + G(t), \quad (8)$$

and the transversality condition

$$\lim_{v \rightarrow \infty} B(v)e^{-\int_t^v r(u)du} = 0, \quad (9)$$

where  $G$  denotes government spending and  $\mu$  is the rate of nominal money growth.

Setting  $Y$  and  $G$  constant over time, for simplicity, equilibrium in the goods market requires  $\dot{C}(t) = 0$ . From (7) the real interest rate is

$$r(t) = \rho + \delta C^{-1} \Omega [B(t) + M(t)], \quad (10)$$

Under a pure interest rate peg,  $i(t) = i$  and the nominal stock of money is endogenously determined in order to satisfy the portfolio balance condition (5). Under the assumption that the level of aggregate consumption is constant over time equilibrium in the money market requires that  $\dot{M}(t) = 0$ . It follows that equation (10) can be re-written as

$$r(t) = \rho + \delta C^{-1} \Omega [B(t) + \xi C/i]. \quad (11)$$

The real interest rate depends on the time path of the real government debt. It follows that the dynamics of the economy is entirely described by the behavior of the public debt. Combining equations (8) with (11) yields<sup>2</sup>

$$\dot{B}(t) = \delta C^{-1} \Omega B(t)^2 + (\rho + 2\delta \xi \Omega i^{-1}) B(t) - T(t) - (1 - \rho/i - \delta \xi \Omega i^{-2}) \xi C + G. \quad (12)$$

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<sup>2</sup>Full derivation is reported in the Appendix.

The government is assumed to adopt a fiscal rule of the form

$$T(t) = \gamma B(t) + Z, \quad (13)$$

where  $Z = G - (1 - \rho/i - \delta\xi\Omega i^{-2}) \xi C$  and the parameter  $\gamma$  is positive.

Substituting (13) into (12), real government debt dynamics can be expressed as

$$\dot{B}(t) = \delta C^{-1} \Omega B(t)^2 + (\rho + 2\delta\xi\Omega i^{-1} - \gamma) B(t), \quad (14)$$

which is a Bernoulli's differential equation with constant coefficients. We assume that  $\gamma > \rho + 2\delta\xi\Omega i^{-1}$ . This condition ensures that in steady state the public debt is non-negative<sup>3</sup>. The path of the real government debt is<sup>4</sup>

$$B(t) = \Psi B_0 / [\delta C^{-1} \Omega B_0 + (\Psi - \delta C^{-1} \Omega B_0) e^{\Psi t}], \quad (15)$$

where  $\Psi \equiv \gamma - \rho - 2\delta\xi\Omega i^{-1}$  and  $B_0$  is the initial level of the real stock of public debt, defined as  $B_0 = B_0^N / P_0$ , with  $B_0^N$  being the initial nominal stock of debt (predetermined) and  $P_0$  the initial price level. The initial price level affects the time path of the public debt and the stability properties of the process<sup>5</sup>.

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<sup>3</sup>The process described by equation (14) presents two steady states: one stable at  $B = 0$  and one unstable at  $B = (\gamma - \rho - 2\delta\xi\Omega i^{-1}) / (\delta\Omega/C)$ .

<sup>4</sup>See the Appendix.

<sup>5</sup>The public debt in fact is at its unstable steady state when  $P_0 = (\delta\Omega/C\Psi) B_0^N$ , when  $P_0 > (\delta\Omega/C\Psi) B_0^N$  the public debt converges to zero and explodes otherwise.

### 3 Nominal Determinacy and Multiple Equilibria

In a representative agent framework, where  $\delta = 0$ , there is nominal indeterminacy in the sense of McCallum (1986, 2001), since there is no nominal anchor to tie down the initial price level. The unique equilibrium for real variables ( $r(t) = \rho$  and  $\pi(t) = \pi$ ) is compatible with infinite possible combinations of the nominal money stock and prices.

By contrast, in our model with overlapping generations,  $\delta > 0$ , wealth effects rule out nominal indeterminacy, since for a given path of the real variables there is only one initial price level. The paths for inflation and the real interest rate are however no longer unique. The economic intuition for the existence of multiple equilibria in the case of wealth effects can be explained as follows. Consider the initial level of the inflation rate

$$\pi_0 = i - \rho - \delta C^{-1} \Omega (B_0^N / P_0 + \xi C / i), \quad (16)$$

which crucially depends on the initial level of the real public debt and thus on the initial price level,  $P_0$ . An upward perturbation in the initial price level implies a lower initial real interest rate and a higher inflation rate, although the unique steady state equilibrium values of all real variables are not affected by the perturbation. Conversely, a downward perturbation in the initial price level increases real wealth, implying a higher initial real interest rate and a lower inflation rate. In other words, in the presence of an initial stock of debt, the model displays multiple equilibria, but there exists a unique price level for each path of all real variables.

In the absence of wealth effects the model presents nominal indeterminacy but a unique equilibrium. In that case, in fact, given an initial stock of nominal assets and the sequences of exogenous variables, the equilibrium conditions determine a unique level of inflation rate.

## 4 Conclusions

This paper presents a model with wealths effects in a continuous time optimising general equilibrium framework with overlapping generations. It is shown that under a pure interest rate pegging monetary rule there exists a unique price level for a given path of real variables, but there are multiple equilibria. The overlapping generations structure of the model via wealth effects solves the nominal indeterminacy problem, but introduces multiple equilibria. Future inflation and the current price level are jointly determined, so that a continuum of equilibrium price sequences arises. A deviation of the initial price level determines a different future inflation and affects the path of the real interest rate. Higher initial price levels determine higher initial inflation rates. A sunspot in the initial price level brings the economy on a specific path towards the steady state and produces real effects. In conclusion, the nominal indeterminacy problem associated to an interest rate peg is solved by the presence of wealth effects, but a multiplicity of the equilibrium paths towards the unique long run equilibrium arises.

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## Appendix

### Solution to the Consumer's Optimization Problem

The representative consumer of the generation born at time  $s$  chooses the optimal sequence for consumption,  $c(s, t)$ , and real money balances,  $m(s, t)$ , in order to maximize her lifetime utility function

$$\int_t^\infty \log[c(s, v)^\eta m(s, v)^{1-\eta}] e^{-(\rho+\delta)(v-t)} dv, \quad (1A)$$

subject to the budget constraint

$$\dot{a}(s, t) = [r(t) + \delta]a(s, t) + y(s, t) - \tau(s, t) - c(s, t) - i(t)m(s, t), \quad (2A)$$

and to the transversality condition

$$\lim_{v \rightarrow \infty} a(s, v) e^{-\int_t^v [r(u) + \delta] du} = 0, \quad (3A)$$

where all the variables are defined as in the main text.

Since preferences are separable and the utility function is homothetic, the consumer's problem can be solved by using a two-stage budgeting procedure. Total consumption at time  $t$  of an agent born at time  $s$ ,  $u(s, t)$ , is defined as the sum of consumption plus the interest foregone on real money holdings

$$u(s, t) \equiv c(s, t) + i(t)m(s, t). \quad (4A)$$

In the first stage, the representative consumer of generation  $s$  chooses the optimal mix of consumption and real money holdings so as to maximize the

instantaneous utility function,  $\log[c(s, t)^\eta m(s, t)^{1-\eta}]$  for a given level of total consumption,  $u(s, t)$ . In the optimum the following condition must hold

$$\xi c(s, t) = i(t)m(s, t), \quad (5A)$$

where  $\xi \equiv \frac{1-\eta}{\eta}$ . Combining (5A) with (4A) yields consumption and real money balances in terms of total consumption

$$c(s, t) = \eta u(s, t), \quad (6A)$$

$$i(t)m(s, t) = (1 - \eta)u(s, t), \quad (7A)$$

In the second stage, the representative consumer of generation  $s$  derives the optimal time path of total consumption  $u(s, t)$  in order to maximize her lifetime utility function. Substituting equations (6A) and (7A) into the lifetime utility function (1A) gives

$$\int_t^\infty [\log u(s, v) - \log Q(v)] e^{-(\rho+\delta)(v-t)} dv, \quad (8A)$$

where  $Q(t) \equiv (\eta)^{-\eta} [i(t)/(1 - \eta)]^{1-\eta}$  is the ideal cost-of-living index of the basket of physical goods and real money balances at time  $t$ . Agents maximize (8A) given the budget constraint (2A) and the solvency condition (3A). The solution to the individual's optimization problem yields a closed form solution for total consumption

$$u(s, t) = (\delta + \rho) [a(s, t) + h(s, t)], \quad (9A)$$

where  $h(s, t)$  is human wealth defined as

$$h(s, t) = \int_t^\infty [y(s, v) - \tau(s, v)] e^{-\int_t^v [r(u) + \delta] du} dv. \quad (10A)$$

Substituting (9A) into (6A) yields the consumption function

$$c(s, t) = \Omega [a(s, t) + h(s, t)], \quad (11A)$$

where,  $\Omega \equiv (\delta + \rho) / (1 + \xi)$ .

### Derivation of equation (10)

Equilibrium in the goods market requires

$$Y(t) = C(t) + G(t). \quad (12A)$$

Setting  $Y$  and  $G$  constant over time implies that  $\dot{C}(t) = 0$ . From (7) of the paper

$$[r(t) - \rho] C(t) - \delta \Omega [B(t) + M(t)] = 0. \quad (13A)$$

Rearranging yields an expression for the real interest rate

$$r(t) = \rho + \delta C^{-1} \Omega [B(t) + M(t)], \quad (14A)$$

where .

### Derivation of equation (12)

The equilibrium in the goods market and the interest rate rule imply that the portfolio balance condition becomes

$$\xi (Y - G) = Mi, \quad (15A)$$

therefore the equilibrium in the money market requires  $\dot{M}(t) = 0$ , i.e.  $\mu(t) = \pi(t)$ .

Given equation (11) of the main text and the equilibrium conditions in the money and in the goods markets, the budget constraint of the consolidated monetary and fiscal authority can be re-written as

$$\dot{B}(t) = [\rho + \delta C^{-1} \Omega (B(t) + \xi C i^{-1})] B(t) - T(t) - \pi(t) \xi C i^{-1} + G. \quad (16A)$$

The inflation rate is given by the Fisher equation

$$\pi(t) = i - r(t) = i - \rho - \delta C^{-1} \Omega [B(t) + \xi C i^{-1}]. \quad (17A)$$

Substituting (17A) into (16A) yields

$$\dot{B}(t) = \delta \frac{1}{C} \Omega B(t)^2 + (\rho + 2\delta \xi \Omega i^{-1}) B(t) - T(t) - (i - \rho - \delta \xi i^{-1} \Omega) \xi C i^{-1} + G. \quad (18A)$$

This last expression can be re-written as

$$\dot{B}(t) = \delta C^{-1} \Omega B(t)^2 + (\rho + 2\delta \xi \Omega i^{-1}) B(t) - T(t) - (1 - \rho i^{-1} - \delta \xi \Omega i^{-2}) \xi C + G, \quad (19A)$$

## Derivation of equation (15)

Consider the Bernoulli's differential equation (14) which can be re-written as

$$\dot{B}(t) + \Psi B(t) = \delta C^{-1} \Omega B(t)^2, \quad (20A)$$

where  $\Psi \equiv \gamma - \rho - 2\delta\xi\Omega i^{-1}$ .

Divide (20A) by  $B(t)^2$  to get

$$B(t)^{-2}\dot{B}(t) + \Psi B(t)^{-1} = \delta C^{-1}\Omega, \quad (20A)$$

Define  $X(t) = B(t)^{-1}$ , so that  $\dot{X}(t) = -B(t)^{-2}\dot{B}(t)$ . It follows that (20A) can be expressed as follows

$$\dot{X}(t) = \Psi X(t) - \delta C^{-1}\Omega, \quad (21A)$$

which is a linear differential equation whose general solution is

$$X(t) = \Pi e^{\Psi t} + \delta\Omega/C\Psi, \quad (22A)$$

where  $\Pi$  is an arbitrary constant. The general solution of equation (19A) is then

$$B(t) = C\Psi / (\delta\Omega + C\Psi\Pi e^{\Psi t}). \quad (23A)$$

At  $t = 0$

$$B_0(\delta\Omega + C\Psi\Pi) = C\Psi. \quad (24A)$$

Solve (24A) for  $\Pi$  and then substitute the result into (23A) to get

$$B(t) = \Psi B_0 / [\delta C^{-1}\Omega B_0 + (\Psi - \delta C^{-1}\Omega B_0)e^{\Psi t}]. \quad (25A)$$